International Trade in Durable Goods: Understanding Volatility, Cyclicality, and Elasticities *

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Abstract

Data for OECD countries document: 1. imports and exports are about three times as volatile as GDP; 2. imports and exports are pro-cyclical, and positively correlated with each other; 3. net exports are counter-cyclical. Standard models fail to replicate the behavior of imports and exports, though they can match net exports relatively well. Inspired by the fact that a large fraction of international trade is in durable goods, we propose a two-country two-sector model in which durable goods are traded across countries. Our model can match the business cycle statistics on the volatility and comovement of the imports and exports relatively well. The model is able to match many dimensions of the data, which suggests that trade in durable goods may be an important element in open-economy macro models.

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Appendix not for publication

A.1 Standard Models

In this section, we describe the standard models used in Section 5.1.1.

A.1.1 IRBC Model

The standard IRBC model is the bond-economy model in Heathcote and Perri (2002). There are two symmetric countries, Home and Foreign. In each country, there are two sectors, the intermediate and final good sectors. Because of symmetry, we focus on the Home country in describing the model. Intermediate goods are produced from capital and labor with a Cobb-Douglas technology

$$Y_{Ht}^H + Y_{Ft}^H = A_{Ht} K_{Ht}^{\theta} L_{Ht}^{1-\theta}, \tag{A.1.1}$$

where Y_{Ht}^H is the Home intermediate goods used in the Home country and Y_{Ft}^H is the Home intermediate goods used in the Foreign country. A_{Ht} is the TFP shock, K_{Ht} is capital and L_{Ht} is labor supply. Capital follows the law of motion

$$K_{Ht+1} = (1-\delta) K_{Ht} + I_{Ht}. \tag{A.1.2}$$

Final goods are produced from Home and Foreign intermediate goods

$$Y_{Ht} = \left[\alpha^{\frac{1}{\gamma}} (Y_{Ht}^H)^{\frac{\gamma-1}{\gamma}} + (1-\alpha)^{\frac{1}{\gamma}} (Y_{Ht}^F)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}. \tag{A.1.3}$$

All prices and wage are flexible. The representative household maximizes expected lifetime utility given those prices

$$E_t \sum_{j=0}^{\infty} \beta^j u_{Ht},$$

where the period utility function u_{Ht} takes the form

$$u_{Ht} = \frac{1}{1-\sigma} \left[C_{Ht}^{\mu} (1 - L_{Ht})^{1-\mu} \right]^{1-\sigma}. \tag{A.1.4}$$

The Home and Foreign country can trade real bonds of the Home country's intermediate goods. To make the model stationary, we assume a small bond holding cost as in Heathcote and Perri (2002). We calibrate the model with the same parameter values as in Heathcote and Perri (2002). Our simulation results are very close to those reported in their paper.

A.1.2 DSGE Model

It is a two-country symmetric model. We will focus on the Home country in describing our model. There is a continuum of differentiated intermediate goods indexed by $i \in [0,1]$. The Home intermediate good i ($Y_H(i)$) is produced by a single firm with capital $K_t(i)$ and labor $L_t(i)$ in the Home country. Capital and labor are not internationally mobile. Intermediate goods are aggregated into an intermediate good composite according to a CES function

$$Y_{Ht} = \left[\int_0^1 Y_{Ht}^{\frac{\phi-1}{\phi}}(i) di \right]^{\frac{\phi}{\phi-1}} \tag{A.1.5}$$

The intermediate-good market is monopolistically competitive. The firms choose prices to maximize expected discounted profit. We follow Calvo staggered price setting in this sticky-price model. In each period, the firm has a probability of $1-\lambda$ of changing its price. When $\lambda = 0$, the model reduces to the flexible price setup.

Final goods are produced from the Home and Foreign intermediate good composites according to a CES function

$$Y_t = \left[\alpha^{\frac{1}{\gamma}} Y_{Ht}^{\frac{\gamma-1}{\gamma}} + (1-\alpha)^{\frac{1}{\gamma}} Y_{Ft}^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \tag{A.1.6}$$

where α is the percentage of Home goods in final goods, and γ is the elasticity of substitution between Home and Foreign goods. The final goods market is competitive with flexible prices.

The household chooses sequences of consumption C_t, capital accumulation I_t, labor supply L_t, Home and Foreign nominal bonds (B_{Ht+1} and B_{Ft+1}) to maximize expected lifetime utility

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t u_t(C_t, 1-L_t) \right], \tag{A.1.7}$$

where $u_t = \frac{\left[C_t^{\mu}(1-L_t)^{1-\mu}\right]^{1-\sigma}}{1-\sigma}$, subject to the budget constraint

$$\begin{aligned} C_t &+ \frac{B_{Ht+1}}{(1+i_t)P_t} + \frac{S_t B_{Ft+1}}{(1+i_t^*)P_t} + I_t + \frac{1}{2}\Phi\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \\ &+ \frac{1}{2}\phi_d\left(\frac{B_{Ht+1}}{P_t}\right)^2 + \frac{1}{2}\phi_f\left(\frac{S_t B_{Ft+1}}{P_t}\right)^2 \\ &\leq \frac{W_t L_t}{P_t} + \frac{R_t K_t}{P_t} + \frac{B_{Ht}}{P_t} + \frac{B_{Ft} S_t}{P_t} + \frac{\Pi_t}{P_t}, \end{aligned} \tag{A.1.8}$$

where $\frac{1}{2}\Phi\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$ is the capital adjustment cost, $\frac{1}{2}\phi_d\left(\frac{B_{Ht+1}}{P_t}\right)^2$ and $\frac{1}{2}\phi_f\left(\frac{S_t B_{Ft+1}}{P_t}\right)^2$ are bond holding costs for the Home and Foreign nominal bonds. Π_t is the profit of intermediate-good firms. The nominal interest rate follows the Taylor rule

$$i_t = i + \Xi_\pi log(\pi_t/\pi) + \Xi_y log(gdp_t/gdp), \tag{A.1.9}$$

where π_t is the inflation rate at time t.

The first order conditions of the household approximately imply uncovered interest rate parity. In the UIP model, we break this condition with an uncovered interest rate parity shock by following Kollmann (2004). The values that we use to calibrate the DSGE model are listed in Table 7. Those parameter values are standard in the literature.

Table 7: Calibration of DSGE Model

Parameter	Value	Description
Intermediate Goods Sector		
ψ	0.36	Capital Share in Production
ϕ	6	Elasticity of Substitution between Differentiated Tradable Goods
λ	0.75	Probability of Not Changing Price.
δ	0.025	Depreciation Rate of Capital
Final Goods Sector		
α	0.85	Share of Home Goods in Final Good
γ	1.5	Elasticity of Substitution between Home and Foreign Goods
Household		
β	0.99	Subjective Discount Factor
Φ	3.2	Investment Adjustment Cost
ϕ_d	0.0001	Domestic Bond Holding Cost
ϕ_f	0.0003	Foreign Bond Holding Cost
σ	2	Preference Parameter
μ	0.36	Preference Parameter
Exogenous Shocks		
$\xi_{11} = \xi_{22}$	0.906	Technology shock AR(1) coefficient
$\xi_{12} = \xi_{21}$	0.088	Technology spillovers
σ_ε	0.0085	Standard Deviation of Productivity Shock

A.2 Benchmark Model

In this section, we give more details about our benchmark model in Section 3. We first list equations that define the equilibrium of the model, then solve for its nonstochastic steady state.

A.2.1 Equilibrium Conditions

We divide all prices by the price of nondurable consumption goods (P^N_{Ht} in the Home country and P^N_{Ft} in the Foreign). That is, we use the nondurable consumption good as numeraire. We use a hat above all prices to denote that they are relative prices in terms of nondurable consumption. The equilibrium of the benchmark model is defined by the following equations.

Nondurable Good Sector

$$1 = (A^N_{Ht})^{-1}(\hat{R}^N_{Ht})^{\chi}\hat{W}^{1-\chi}_{Ht}\chi^{-\chi}(1-\chi)^{\chi-1}, \tag{A.2.1}$$

where $\hat{R}^N_{Ht}$ is defined as

$$\hat{R}^N_{Ht} = \left[\alpha(\hat{R}^{NH}_{Ht})^{1-\gamma} + (1-\alpha)(\hat{R}^{NF}_{Ht})^{1-\gamma}\right]^{\frac{1}{1-\gamma}}.$$

$$K^{NH}_{Ht} = \alpha\left(\frac{\hat{R}^{NH}_{Ht}}{\hat{R}^N_{Ht}}\right)^{-\gamma} K^N_{Ht} \tag{A.2.2}$$

$$K^{NF}_{Ht} = (1-\alpha)\left(\frac{\hat{R}^{NF}_{Ht}}{\hat{R}^N_{Ht}}\right)^{-\gamma} K^N_{Ht} \tag{A.2.3}$$

$$K^N_{Ht} = \chi Y^N_{Ht}/\hat{R}^N_{Ht} \tag{A.2.4}$$

$$L^N_{Ht} = (1-\chi)Y^N_{Ht}/\hat{W}_{Ht} \tag{A.2.5}$$

Symmetric conditions hold in the Foreign country

$$1 = (A^N_{Ft})^{-1}(\hat{R}^N_{Ft})^{\chi}\hat{W}^{1-\chi}_{Ft}\chi^{-\chi}(1-\chi)^{\chi-1}, \tag{A.2.6}$$

where $\hat{R}^N_{Ft}$ is defined as

$$\hat{R}^N_{Ft} = \left[\alpha(\hat{R}^{NF}_{Ft})^{1-\gamma} + (1-\alpha)(\hat{R}^{NH}_{Ft})^{1-\gamma}\right]^{\frac{1}{1-\gamma}}.$$

$$K_{Ft}^{NF} = \alpha \left(\frac{\hat{R}_{Ft}^{NF}}{\hat{R}_{Ft}^{N}} \right)^{-\gamma} K_{Ft}^{N} \tag{A.2.7}$$

$$K_{Ft}^{NH} = (1-\alpha) \left(\frac{\hat{R}_{Ft}^{NH}}{\hat{R}_{Ft}^{N}} \right)^{-\gamma} K_{Ft}^{N} \tag{A.2.8}$$

$$K_{Ft}^{N} = \chi Y_{Ft}^{N} / \hat{R}_{Ft}^{N} \tag{A.2.9}$$

$$L_{Ft}^{N} = (1-\chi) Y_{Ft}^{N} / \hat{W}_{Ft} \tag{A.2.10}$$

Durable Good Sector

$$\hat{P}_{Ht}^{DH} = (A_{Ht}^{D})^{-1} (\hat{R}_{Ht}^{D})^{\epsilon} \hat{W}_{Ht}^{1-\epsilon} \epsilon^{-\epsilon} (1-\epsilon)^{\epsilon-1}, \tag{A.2.11}$$

where $\hat{R}_{Ht}^{D}$ is defined as

$$\hat{R}_{Ht}^{D} = \left[\alpha (\hat{R}_{Ht}^{DH})^{1-\gamma} + (1-\alpha)(\hat{R}_{Ht}^{DF})^{1-\gamma} \right]^{\frac{1}{1-\gamma}} .$$

$$K_{Ht}^{DH} = \alpha \left(\frac{\hat{R}_{Ht}^{DH}}{\hat{R}_{Ht}^{D}} \right)^{-\gamma} K_{Ht}^{D} \tag{A.2.12}$$

$$K_{Ht}^{DF} = (1-\alpha) \left(\frac{\hat{R}_{Ht}^{DF}}{\hat{R}_{Ht}^{D}} \right)^{-\gamma} K_{Ht}^{D} \tag{A.2.13}$$

$$K_{Ht}^{D} = \epsilon \hat{P}_{Ht}^{DH} Y_{Ht}^{D} / \hat{R}_{Ht}^{D} \tag{A.2.14}$$

$$L_{Ht}^{D} = (1-\epsilon) \hat{P}_{Ht}^{DH} Y_{Ht}^{D} / \hat{W}_{Ht} \tag{A.2.15}$$

Symmetric conditions hold in the Foreign country.

$$\hat{P}_{Ft}^{DF} = (A_{Ft}^{D})^{-1} (\hat{R}_{Ft}^{D})^{\epsilon} \hat{W}_{Ft}^{1-\epsilon} \epsilon^{-\epsilon} (1-\epsilon)^{\epsilon-1}, \tag{A.2.16}$$

where $\hat{R}_{Ft}^{D}$ is defined as

$$\hat{R}_{Ft}^{D} = \left[\alpha (\hat{R}_{Ft}^{DF})^{1-\gamma} + (1-\alpha)(\hat{R}_{Ft}^{DH})^{1-\gamma} \right]^{\frac{1}{1-\gamma}} .$$

$$K_{Ft}^{DF} = \alpha \left(\frac{\hat{R}_{Ft}^{DF}}{\hat{R}_{Ft}^{D}} \right)^{-\gamma} K_{Ft}^{D} \tag{A.2.17}$$

$$K_{Ft}^{DH} = (1-\alpha) \left(\frac{\hat{R}_{Ft}^{DH}}{\hat{R}_{Ft}^{D}} \right)^{-\gamma} K_{Ft}^{D} \tag{A.2.18}$$

$$K_{Ft}^{D} = \epsilon \hat{P}_{Ft}^{DF} Y_{Ft}^{D} / \hat{R}_{Ft}^{D} \tag{A.2.19}$$

$$L_{Ft}^{D} = (1-\epsilon) \hat{P}_{Ft}^{DF} Y_{Ft}^{D} / \hat{W}_{Ft} \tag{A.2.20}$$

Households

The law of motion for durable consumption

$$D_{Ht+1}^{H} = (1-\delta_D) D_{Ht}^{H} + d_{Ht}^{H} \tag{A.2.21}$$

$$D_{Ht+1}^{F} = (1-\delta_D) D_{Ht}^{F} + d_{Ht}^{F}. \tag{A.2.22}$$

The law of motion for capital

$$K_{Ht+1}^{NH} = (1-\delta) K_{Ht}^{NH} + I_{Ht}^{NH} \tag{A.2.23}$$

$$K_{Ht+1}^{NF} = (1-\delta) K_{Ht}^{NF} + I_{Ht}^{NF} \tag{A.2.24}$$

$$K_{Ht+1}^{DH} = (1-\delta) K_{Ht}^{DH} + I_{Ht}^{DH} \tag{A.2.25}$$

$$K_{Ht+1}^{DF} = (1-\delta) K_{Ht}^{DF} + I_{Ht}^{DF}. \tag{A.2.26}$$

The budget constraint

$$\begin{aligned} C_{Ht} &+ \hat{P}_{Ht}^{DH} \left(d_{Ht}^{H} + \Delta_{Ht}^{H} + I_{Ht}^{NH} + \Lambda_{Ht}^{NH} + I_{Ht}^{DH} + \Lambda_{Ht}^{DH} + \frac{B_{Ht+1}}{1+i_t} + \frac{1}{2} \Phi B_{Ht+1}^2 \right) \\ &+ \frac{Q_t \hat{P}_{Ft}^{DF} \hat{P}_{Ht}}{\hat{P}_{Ft}(1-\tau)} \left(d_{Ht}^{F} + \Delta_{Ht}^{F} + I_{Ht}^{NF} + \Lambda_{Ht}^{NF} + I_{Ht}^{DF} + \Lambda_{Ht}^{DF} \right) \\ &\leq \hat{W}_{Ht} L_{Ht} + \hat{P}_{Ht}^{DH} B_{Ht} + \hat{R}_{Ht}^{NH} K_{Ht}^{NH} + \hat{R}_{Ht}^{NF} K_{Ht}^{NF} + \hat{R}_{Ht}^{DH} K_{Ht}^{DH} + \hat{R}_{Ht}^{DF} K_{Ht}^{DF}, \end{aligned} \tag{A.2.27}$$

where Q_t is the CPI real exchange rate. The consumer price index (CPI) is defined as

$$\hat{P}_{Ht} = (\hat{P}_{Ht}^{DH})^{\omega_2} \left(\frac{Q_t \hat{P}_{Ft}^{DF} \hat{P}_{Ht}}{\hat{P}_{Ft}(1-\tau)} \right)^{1-\omega_1-\omega_2}, \tag{A.2.28}$$

where ω_1 is the expenditure share of nondurable consumption. ω_2 and ω_3 are the expenditure shares of Home and Foreign durable consumption respectively.

The followings are the first order conditions from the household's lifetime utility maximization problem.

$$\hat{P}^{DH}_{Ht}\left(1+\frac{\partial\Delta^H_{Ht}}{\partial d^H_{Ht}}\right)=E_t\left[\beta\frac{\partial u_{t+1}/\partial D^H_{Ht+1}}{\partial u_t/\partial C_{Ht}}-\Gamma_{t,t+1}\hat{P}^{DH}_{Ht+1}\left(\frac{\partial\Delta^H_{Ht+1}}{\partial D^H_{Ht+1}}-(1-\delta_D)(1+\frac{\partial\Delta^H_{Ht+1}}{\partial d^H_{Ht+1}})\right)\right], \tag{A.2.29}$$

where $\Gamma_{t,t+1}=\beta\frac{\partial u_{t+1}/\partial C_{Ht+1}}{\partial u_t/\partial C_{Ht}}$ is the intertemporal elasticity of substitution of nondurable consumption goods. The left hand side of the equation is the cost in terms of nondurable goods, of increasing one unit durable consumption. The right hand side is the benefit, which includes three parts: 1. the increase of period $t+1$ utility due to the increase of durable consumption stock ($\beta\frac{\partial u_{t+1}/\partial D^H_{Ht+1}}{\partial u_t/\partial C_{Ht}}$); 2. the adjustment cost associated with the increased consumption stock ($\Gamma_{t,t+1}\hat{P}^{DH}_{Ht+1}\frac{\partial\Delta^H_{Ht+1}}{\partial D^H_{Ht+1}}$);[31] 3. the value of undepreciated durable consumption goods $\Gamma_{t,t+1}\hat{P}^{DH}_{Ht+1}(1-\delta_D)(1+\frac{\partial\Delta^H_{Ht+1}}{\partial d^H_{Ht+1}})$. In equilibrium, the marginal cost of increasing durable consumption stock is equal to its marginal benefit.

$$\begin{aligned}&\frac{Q_t\hat{P}^{DF}_{Ft}\hat{P}_{Ht}}{\hat{P}_{Ft}(1-\tau)}\left(1+\frac{\partial\Delta^F_{Ht}}{\partial d^F_{Ht}}\right)\\&=E_t\left[\beta\frac{\partial u_{t+1}/\partial D^F_{Ht+1}}{\partial u_t/\partial C_{Ht}}-\Gamma_{t,t+1}\frac{Q_{t+1}\hat{P}^{DF}_{Ft+1}\hat{P}_{Ht+1}}{\hat{P}_{Ft+1}(1-\tau)}\left(\frac{\partial\Delta^F_{Ht+1}}{\partial D^F_{Ht+1}}-(1-\delta_D)(1+\frac{\partial\Delta^F_{Ht+1}}{\partial d^F_{Ht+1}})\right)\right]\end{aligned} \tag{A.2.30}$$

$$\frac{\partial u_t}{\partial L_{Ht}}+\hat{W}_{Ht}\frac{\partial u_t}{\partial C_{Ht}}=0 \tag{A.2.31}$$

$$\hat{P}^{DH}_{Ht}\left(1+\frac{\partial\Lambda^{NH}_{Ht}}{\partial I^{NH}_{Ht}}\right)=E_t\left[\Gamma_{t,t+1}\left(\hat{R}^{NH}_{Ht+1}-\hat{P}^{DH}_{Ht+1}\left(\frac{\partial\Lambda^{NH}_{Ht+1}}{\partial K^{NH}_{Ht+1}}-(1-\delta)(1+\frac{\partial\Lambda^{NH}_{Ht+1}}{\partial I^{NH}_{Ht+1}})\right)\right)\right] \tag{A.2.32}$$

$$\begin{aligned}&\frac{Q_t\hat{P}^{DF}_{Ft}\hat{P}_{Ht}}{\hat{P}_{Ft}(1-\tau)}\left(1+\frac{\partial\Lambda^{NF}_{Ht}}{\partial I^{NF}_{Ht}}\right)\\&=E_t\left[\Gamma_{t,t+1}\left(\hat{R}^{NF}_{Ht+1}-\frac{Q_{t+1}\hat{P}^{DF}_{Ft+1}\hat{P}_{Ht+1}}{(1-\tau)\hat{P}_{Ft+1}}\left(\frac{\partial\Lambda^{NF}_{Ht+1}}{\partial K^{NF}_{Ht+1}}-(1-\delta)(1+\frac{\partial\Lambda^{NF}_{Ht+1}}{\partial I^{NF}_{Ht+1}})\right)\right)\right]\end{aligned} \tag{A.2.33}$$

$$\hat{P}^{DH}_{Ht}\left(1+\frac{\partial\Lambda^{DH}_{Ht}}{\partial I^{DH}_{Ht}}\right)=E_t\left[\Gamma_{t,t+1}\left(\hat{R}^{DH}_{Ht+1}-\hat{P}^{DH}_{Ht+1}\left(\frac{\partial\Lambda^{DH}_{Ht+1}}{\partial K^{DH}_{Ht+1}}-(1-\delta)(1+\frac{\partial\Lambda^{DH}_{Ht+1}}{\partial I^{DH}_{Ht+1}})\right)\right)\right] \tag{A.2.34}$$

$$\begin{aligned}&\frac{Q_t\hat{P}^{DF}_{Ft}\hat{P}_{Ht}}{\hat{P}_{Ft}(1-\tau)}\left(1+\frac{\partial\Lambda^{DF}_{Ht}}{\partial I^{DF}_{Ht}}\right)\\&=E_t\left[\Gamma_{t,t+1}\left(\hat{R}^{DF}_{Ht+1}-\frac{Q_{t+1}\hat{P}^{DF}_{Ft+1}\hat{P}_{Ht+1}}{(1-\tau)\hat{P}_{Ft+1}}\left(\frac{\partial\Lambda^{DF}_{Ht+1}}{\partial K^{DF}_{Ht+1}}-(1-\delta)(1+\frac{\partial\Lambda^{DF}_{Ht+1}}{\partial I^{DF}_{Ht+1}})\right)\right)\right]\end{aligned} \tag{A.2.35}$$

$$\frac{1}{1+i_t}+\Phi B_{Ht+1}=E_t\left[\Gamma_{t,t+1}\frac{\hat{P}^{DH}_{Ht+1}}{\hat{P}^{DH}_{Ht}}\right] \tag{A.2.36}$$

[31] It is useful to note that this term is negative.

Symmetric conditions hold in the Foreign country. From Walras' law, one equation is redundant. We eliminate the Foreign country's budget constraint from our equation system. So equations in the Foreign country are

$$D^F_{Ft+1} = (1-\delta_D)D^F_{Ft} + d^F_{Ft} \tag{A.2.37}$$

$$D^H_{Ft+1} = (1-\delta_D)D^H_{Ft} + d^H_{Ft} \tag{A.2.38}$$

$$K^{NF}_{Ft+1} = (1-\delta)K^{NF}_{Ft} + I^{NF}_{Ft} \tag{A.2.39}$$

$$K^{NH}_{Ft+1} = (1-\delta)K^{NH}_{Ft} + I^{NH}_{Ft} \tag{A.2.40}$$

$$K^{DF}_{Ft+1} = (1-\delta)K^{DF}_{Ft} + I^{DF}_{Ft} \tag{A.2.41}$$

$$K^{DH}_{Ft+1} = (1-\delta)K^{DH}_{Ft} + I^{DH}_{Ft} \tag{A.2.42}$$

$$\hat{P}_{Ft} = (\hat{P}^{DF}_{Ft})^{\omega_2}\left(\frac{\hat{P}^{DH}_{Ht}\hat{P}_{Ft}}{Q_t\hat{P}_{Ht}(1-\tau)}\right)^{1-\omega_1-\omega_2} \tag{A.2.43}$$

$$\hat{P}^{DF}_{Ft}\left(1+\frac{\partial\Delta^F_{Ft}}{\partial d^F_{Ft}}\right) = E_t\left[\beta\frac{\partial u^*_{t+1}/\partial D^F_{Ft+1}}{\partial u^*_t/\partial C_{Ft}} - \Gamma^*_{t,t+1}\hat{P}^{DF}_{Ft+1}\left(\frac{\partial\Delta^F_{Ft+1}}{\partial D^F_{Ft+1}} - (1-\delta_D)(1+\frac{\partial\Delta^F_{Ft+1}}{\partial d^F_{Ft+1}})\right)\right] \tag{A.2.44}$$

$$\begin{aligned}&\frac{\hat{P}^{DH}_{Ht}\hat{P}_{Ft}}{Q_t\hat{P}_{Ht}(1-\tau)}\left(1+\frac{\partial\Delta^H_{Ft}}{\partial d^H_{Ft}}\right)\\ &= E_t\left[\beta\frac{\partial u^*_{t+1}/\partial D^H_{Ft+1}}{\partial u^*_t/\partial C_{Ft}} - \Gamma^*_{t,t+1}\frac{\hat{P}^{DH}_{Ht+1}\hat{P}_{Ft+1}}{Q_{t+1}\hat{P}_{Ht+1}(1-\tau)}\left(\frac{\partial\Delta^H_{Ft+1}}{\partial D^H_{Ft+1}} - (1-\delta_D)(1+\frac{\partial\Delta^H_{Ft+1}}{\partial d^H_{Ft+1}})\right)\right]\end{aligned} \tag{A.2.45}$$

$$\frac{\partial u^*_t}{\partial L_{Ft}} + \hat{W}_{Ft}\frac{\partial u^*_t}{\partial C_{Ft}} = 0 \tag{A.2.46}$$

$$\hat{P}^{DF}_{Ft}\left(1+\frac{\partial\Lambda^{NF}_{Ft}}{\partial I^{NF}_{Ft}}\right) = E_t\left[\Gamma^*_{t,t+1}\left(\hat{R}^{NF}_{Ft+1} - \hat{P}^{DF}_{Ft+1}\left(\frac{\partial\Lambda^{NF}_{Ft+1}}{\partial K^{NF}_{Ft+1}} - (1-\delta)(1+\frac{\partial\Lambda^{NF}_{Ft+1}}{\partial I^{NF}_{Ft+1}})\right)\right)\right] \tag{A.2.47}$$

$$\begin{aligned}&\frac{\hat{P}^{DH}_{Ht}\hat{P}_{Ft}}{Q_t\hat{P}_{Ht}(1-\tau)}\left(1+\frac{\partial\Lambda^{NH}_{Ft}}{\partial I^{NH}_{Ft}}\right)\\ &= E_t\left[\Gamma^*_{t,t+1}\left(\hat{R}^{NH}_{Ft+1} - \frac{\hat{P}^{DH}_{Ht+1}\hat{P}_{Ft+1}}{Q_{t+1}(1-\tau)\hat{P}_{Ht+1}}\left(\frac{\partial\Lambda^{NH}_{Ft+1}}{\partial K^{NH}_{Ft+1}} - (1-\delta)(1+\frac{\partial\Lambda^{NH}_{Ft+1}}{\partial I^{NH}_{Ft+1}})\right)\right)\right]\end{aligned} \tag{A.2.48}$$

$$\hat{P}^{DF}_{Ft}\left(1+\frac{\partial\Lambda^{DF}_{Ft}}{\partial I^{DF}_{Ft}}\right) = E_t\left[\Gamma^*_{t,t+1}\left(\hat{R}^{DF}_{Ft+1} - \hat{P}^{DF}_{Ft+1}\left(\frac{\partial\Lambda^{DF}_{Ft+1}}{\partial K^{DF}_{Ft+1}} - (1-\delta)(1+\frac{\partial\Lambda^{DF}_{Ft+1}}{\partial I^{DF}_{Ft+1}})\right)\right)\right] \tag{A.2.49}$$

$$\begin{aligned}&\frac{\hat{P}^{DH}_{Ht}\hat{P}_{Ft}}{Q_t\hat{P}_{Ht}(1-\tau)}\left(1+\frac{\partial\Lambda^{DH}_{Ft}}{\partial I^{DH}_{Ft}}\right)\\ &= E_t\left[\Gamma^*_{t,t+1}\left(\hat{R}^{DH}_{Ft+1} - \frac{\hat{P}^{DH}_{Ht+1}\hat{P}_{Ft+1}}{Q_{t+1}(1-\tau)\hat{P}_{Ht+1}}\left(\frac{\partial\Lambda^{DH}_{Ft+1}}{\partial K^{DH}_{Ft+1}} - (1-\delta)(1+\frac{\partial\Lambda^{DH}_{Ft+1}}{\partial I^{DH}_{Ft+1}})\right)\right)\right]\end{aligned} \tag{A.2.50}$$

$$\frac{1}{1+i_t} + \frac{\Phi B_{Ft+1}}{1-\tau} = E_t\left[\Gamma^*_{t,t+1}\frac{\hat{P}^{DH}_{Ft+1}}{\hat{P}^{DH}_{Ft}}\right], \tag{A.2.51}$$

where $\hat{P}^{DH}_{Ft} = \frac{\hat{P}_{Ft}\hat{P}^{DH}_{Ht}}{(1-\tau)Q_t\hat{P}_{Ht}}$.

Market Clearing Conditions

The model is closed with market clearing conditions

$$Y_{Ht}^{N} = C_{Ht} \tag{A.2.52}$$

$$Y_{Ft}^{N} = C_{Ft} \tag{A.2.53}$$

$$Y_{Ht}^{D} = d_{Ht}^{H} + \Delta_{Ht}^{H} + I_{Ht}^{NH} + \Lambda_{Ht}^{NH} + I_{Ht}^{DH} + \Lambda_{Ht}^{DH} + \frac{1}{2}\Phi B_{Ht+1}^{2} + \frac{d_{Ft}^{H} + \Delta_{Ft}^{H} + I_{Ft}^{NH} + \Lambda_{Ft}^{NH} + I_{Ft}^{DH} + \Lambda_{Ft}^{DH} + \frac{1}{2}\Phi B_{Ft+1}^{2}}{1-\tau} \tag{A.2.54}$$

$$Y_{Ft}^{D} = d_{Ft}^{F} + \Delta_{Ft}^{F} + \frac{d_{Ht}^{F} + \Delta_{Ht}^{F}}{1-\tau} + I_{Ft}^{NF} + \Lambda_{Ft}^{NF} + I_{Ft}^{DF} + \Lambda_{Ft}^{DF} + \frac{I_{Ht}^{NF} + \Lambda_{Ht}^{NF} + I_{Ht}^{DF} + \Lambda_{Ht}^{DF}}{1-\tau} \tag{A.2.55}$$

$$L_{Ht}^{N} + L_{Ht}^{D} = L_{Ht} \tag{A.2.56}$$

$$L_{Ft}^{N} + L_{Ft}^{D} = L_{Ft} \tag{A.2.57}$$

$$B_{Ht} + B_{Ft} = 0. \tag{A.2.58}$$

There are 10 equations in the nondurable good sector (from equation (A.2.1) to (A.2.10)), and 10 equations in the durable good sector (from equation (A.2.11) to (A.2.20)). We have 31 equations from the household's problem (from equation (A.2.21) to (A.2.51)). In addition, we have 7 equations in this section (from equation (A.2.52) to (A.2.58)). As a total, we have 58 equations. Those equations define equilibrium conditions for the following 58 variables.

16 variables in nondurable good sector:

$$\begin{bmatrix} \hat{R}_{Ht}^{NH} & \hat{R}_{Ht}^{NF} & \hat{W}_{Ht} & K_{Ht}^{NH} & K_{Ht}^{NF} & K_{Ht}^{N} & L_{Ht}^{N} & Y_{Ht}^{N} \\ \hat{R}_{Ft}^{NF} & \hat{R}_{Ft}^{NH} & \hat{W}_{Ft} & K_{Ft}^{NF} & K_{Ft}^{NH} & K_{Ft}^{N} & L_{Ft}^{N} & Y_{Ft}^{N} \end{bmatrix}$$

16 variables in durable good sector

$$\begin{bmatrix} \hat{P}_{Ht}^{DH} & \hat{R}_{Ht}^{DH} & \hat{R}_{Ht}^{DF} & K_{Ht}^{DH} & K_{Ht}^{DF} & K_{Ht}^{D} & L_{Ht}^{D} & Y_{Ht}^{D} \\ \hat{P}_{Ft}^{DF} & \hat{R}_{Ft}^{DF} & \hat{R}_{Ft}^{DH} & K_{Ft}^{DF} & K_{Ft}^{DH} & K_{Ft}^{D} & L_{Ft}^{D} & Y_{Ft}^{D} \end{bmatrix}$$

26 variables in household's problem

$$[D_{Ht}^{H} \quad d_{Ht}^{H} \quad D_{Ht}^{F} \quad d_{Ht}^{F} \quad I_{Ht}^{NH} \quad I_{Ht}^{NF} \quad I_{Ht}^{DH} \quad I_{Ht}^{DF} \quad C_{Ht} \quad \hat{P}_{Ht} \quad \hat{B}_{Ht} \quad L_{Ht}$$

$$D_{Ft}^{F} \quad d_{Ft}^{F} \quad D_{Ft}^{H} \quad d_{Ft}^{H} \quad I_{Ft}^{NF} \quad I_{Ft}^{NH} \quad I_{Ft}^{DF} \quad I_{Ft}^{DH} \quad C_{Ft} \quad \hat{P}_{Ft} \quad \hat{B}_{Ft} \quad L_{Ft}$$

$$i_t \quad Q_t].$$

A.2.2 Solving Steady State of Benchmark Model

From the household's problem, we can solve the return to capital

$$\hat{R}_H^{NH} = \hat{R}_H^{DH} = \hat{P}_H^{DH}\left[\frac{1}{\beta} - (1-\delta)\right]. \tag{A.2.59}$$

In the steady state, the return to capital in the nondurable-good sector is the same as that in the durable-good sector. This result is intuitive since there is no long-run restriction on moving capital between these two sectors. Similarly, we can find that the return to Foreign capital goods is also the same across these two sectors

$$\hat{R}_H^{NF} = \hat{R}_H^{DF} = \frac{\hat{R}_H^{NH}}{1-\tau}. \tag{A.2.60}$$

Because of the trade cost, the return to the Foreign-good capital has to be higher than the return to the Home-good capital. As shown in the calibration, this trade cost generates home bias endogenously in the durable good sector.

Substituting equations (A.2.59) and (A.2.60) into the definitions of $\hat{R}_H^N$ and $\hat{R}_H^D$, we find $\hat{R}_H^N = \hat{R}_H^D$, which says that the return to aggregate capital is the same in those two sectors. Now we assume that the production structure is the same in these two sectors by equalizing the capital share in both sectors ($\chi = \epsilon$).[32] If we compare equations (A.2.1) and (A.2.11), the above results and the assumption of equal capital share imply $\hat{P}_H^{DH} = 1$.[33] That is, in the steady state, durable goods have the same price as nondurable goods. The intuition comes from the fact that the production costs of durable and nondurable goods are the same in the steady state: the same production structure, same cost of capital and same cost of labor. This result gives us the solution to the Home-good capital return

$$\hat{R}_H^{NH} = \hat{R}_H^{DH} = \left[\frac{1}{\beta} - (1-\delta)\right]. \tag{A.2.61}$$

The return to Foreign-good capital can be solved from equation (A.2.60). The returns to the aggregate capital in both sectors can be solved from their definitions.

$$\hat{R}_H^N = \left[\alpha(\hat{R}_H^{NH})^{1-\gamma} + (1-\alpha)(\hat{R}_H^{NF})^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{A.2.62}$$

$$\hat{R}_H^D = \left[\alpha(\hat{R}_H^{DH})^{1-\gamma} + (1-\alpha)(\hat{R}_H^{DF})^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{A.2.63}$$

[32]This assumption is also used in other two-sector models, for instance, Erceg and Levin (2006), Whelan (2003).
[33]The steady-state productivity shock is equal to 1 in both sectors.

Now we can solve the wage from equation (A.2.1)

$$\hat{W}_H = \left[(\hat{R}_H^N)^{-\chi}\chi^{\chi}(1-\chi)^{1-\chi}\right]^{\frac{1}{1-\chi}}. \tag{A.2.64}$$

So far, all prices have been solved and we move to solve quantities of the model.

The utility function in our model is too complicated for us to obtain a recursive solution to quantity variables. Instead we solve them numerically. From the labor demand function in both sectors (equations (A.2.5) and (A.2.15)), we have

$$L_H = L_H^N + L_H^D = (1-\chi)(Y_H^N + Y_H^D)/\hat{W}_H, \tag{A.2.65}$$

in which we used the condition of $\chi = \epsilon$. We assume that in the steady state, labor supply is one third. This gives us the first equation that we will use to solve some variables numerically

$$C_H + Y_H^D = \frac{\hat{W}_H}{3(1-\epsilon)}. \tag{A.2.66}$$

We also used the market clearing condition of nondurable goods ($C_H = Y_H^N$) to get the above equation.

In the symmetric equilibrium, the real exchange rate Q_t is equal to one. The prices of durable goods are the same across countries $\hat{P}_H^{DH} = \hat{P}_F^{DF}$. From the household's first order conditions (equations (A.2.29) and (A.2.30)), we have

$$\frac{1}{\beta} - (1-\delta_D) = \frac{\partial u_t/\partial D_H^H}{\partial u_t/\partial C_H} \tag{A.2.67}$$

$$\frac{1}{1-\tau}\left[\frac{1}{\beta} - (1-\delta_D)\right] = \frac{\partial u_t/\partial D_H^F}{\partial u_t/\partial C_H}. \tag{A.2.68}$$

From the market clearing condition of durable goods, we have

$$Y_H^D = \delta_D\left(D_H^H + \frac{D_H^F}{1-\tau}\right) + \frac{\delta\chi}{\hat{R}_H^N}\left(C_H + Y_H^D\right)\left[\alpha\left(\frac{\hat{R}_H^{NH}}{\hat{R}_H^N}\right)^{-\gamma} + \frac{1-\alpha}{1-\tau}\left(\frac{\hat{R}_H^{NF}}{\hat{R}_H^N}\right)^{-\gamma}\right]. \tag{A.2.69}$$

From the tradeoff between consumption and labor, we have

$$\frac{\partial u_t}{\partial L_{Ht}} + \hat{W}_{Ht}\frac{\partial u_t}{\partial C_{Ht}} = 0. \tag{A.2.70}$$

Equations (A.2.66), (A.2.67), (A.2.68), (A.2.69) and (A.2.70) are used to solve jointly for consumption

(C_H), durable output (Y_H^D), stock of home-good durable consumption (D_H^H), stock of foreign-good durable consumption (D_H^F) and parameter ρ. ρ is chosen such that the steady state labor supply is one third. With solutions to these variables, we can solve other variables recursively.

$$Y_H^N = C_H \tag{A.2.71}$$

$$K_H^N = \frac{\epsilon}{\hat{R}_H^N} Y_H^N \tag{A.2.72}$$

$$K_H^{NH} = \alpha \left(\frac{\hat{R}_H^{NH}}{\hat{R}_H^N} \right)^{-\gamma} K_H^N \tag{A.2.73}$$

$$K_H^{NF} = (1-\alpha) \left(\frac{\hat{R}_H^{NF}}{\hat{R}_H^N} \right)^{-\gamma} K_H^N \tag{A.2.74}$$

$$K_H^N = \chi Y_H^N / \hat{R}_H^N \tag{A.2.75}$$

$$L_H^N = (1-\chi) Y_H^N / \hat{W}_H \tag{A.2.76}$$

$$K_H^{DH} = \alpha \left(\frac{\hat{R}_H^{DH}}{\hat{R}_H^D} \right)^{-\gamma} K_H^D \tag{A.2.77}$$

$$K_H^{DF} = (1-\alpha) \left(\frac{\hat{R}_H^{DF}}{\hat{R}_H^D} \right)^{-\gamma} K_H^D \tag{A.2.78}$$

$$K_H^D = \epsilon Y_H^D / \hat{R}_H^D \tag{A.2.79}$$

$$L_H^D = (1-\epsilon) Y_H^D / \hat{W}_H \tag{A.2.80}$$

$$d_H^H = \delta_D D_H^H \tag{A.2.81}$$

$$d_H^F = \delta_D D_H^F \tag{A.2.82}$$

$$I_H^{NH} = \delta K_H^{NH} \tag{A.2.83}$$

$$I_H^{NF} = \delta K_H^{NF} \tag{A.2.84}$$

$$I_H^{DH} = \delta K_H^{DH} \tag{A.2.85}$$

$$I_H^{DF} = \delta K_H^{DF} \tag{A.2.86}$$

$$B_H = 0 \tag{A.2.87}$$

$$i = \frac{1}{\beta} - 1 \tag{A.2.88}$$

A.3 Traded Nondurable Model

In this model, we allow home and foreign countries to trade part of their nondurable consumption. We use the home country to describe our model. Symmetric conditions hold in the foreign country. The production function in the nondurable goods sector is the same as in our benchmark model

$$Y_{Ht}^{N} = A_{Ht}^{N} K_{Ht}^{N\chi} L_{Ht}^{N(1-\chi)}. \tag{A.3.1}$$

Nondurable goods produced in the home country are used for nontraded nondurable consumption (C_{Ht}^{NN}), traded nondurable consumption in the home country (C_{Ht}^{NH}), and traded nondurable consumption in the foreign country (C_{Ft}^{NH})

$$Y_{Ht}^{N} = C_{Ht}^{NN} + C_{Ht}^{NH} + C_{Ft}^{NH}. \tag{A.3.2}$$

Home and foreign traded nondurable consumption is aggregated into traded nondurable consumption

$$C_{Ht}^{NT} = \left[n^{\frac{1}{\kappa}} \left(C_{Ht}^{NH} \right)^{\frac{\kappa-1}{\kappa}} + (1-n)^{\frac{1}{\kappa}} \left(C_{Ht}^{NF} \right)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}. \tag{A.3.3}$$

Traded nondurable consumption and nontraded nondurable consumption are aggregated into nondurable consumption

$$C_{Ht} = \frac{\left(C_{Ht}^{NN} \right)^{v} \left(C_{Ht}^{N} T \right)^{1-v}}{v^{v} (1-v)^{1-v}}. \tag{A.3.4}$$

We assume that the law of one price (LOP) holds for traded nondurable goods, and traded and nontraded nondurable goods have the same price

$$P_{Ht}^{NN} = P_{Ht}^{NH} = P_{Ft}^{NH} / S_t, \tag{A.3.5}$$

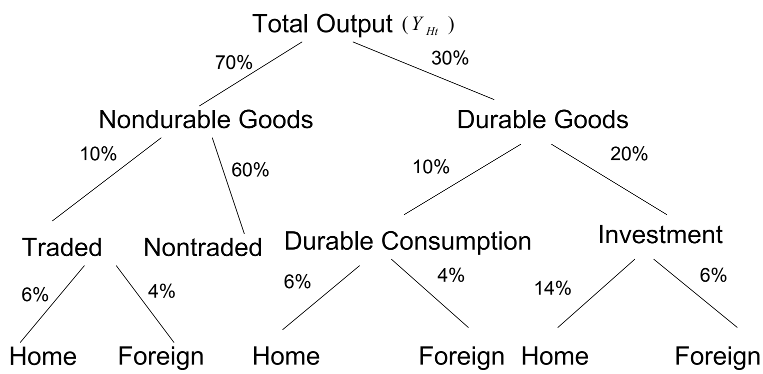
where P_{Ht}^{NN} is the price of nontraded nondurable goods, P_{Ht}^{NH} is the price of home-country-produced traded nondurable goods in the home country, and P_{Ft}^{NH} is the price of home-country-produced trade nondurable goods in the foreign country. We also abstract from iceberg trade costs for nondurable goods.

The rest of the model follows the same setup as our benchmark model. We have added four new variables into the model: C_{Ht}^{NN}, C_{Ht}^{NH}, C_{Ht}^{NF}, and P_{Ht}^{NN}. The following new equations are added to close our model

$$P_{Ht}^{NN}C_{Ht}^{NN} = vP_{Ht}^{N}C_{Ht}$$
$$P_{Ht}^{NN}C_{Ht}^{NH} = n(1-v)P_{Ht}^{N}C_{Ht}$$
$$P_{Ht}^{NN}C_{Ht}^{NF} = (1-n)(1-v)P_{Ht}^{N}C_{Ht}$$
$$P_{Ht}^{N} = (P_{Ht}^{NN})^{v}(P_{Ht}^{NT})^{1-v}$$

The model is calibrated to match the structure in Figure 4. In this economy, the trade share is 14% of total output. The share of durable goods in trade is 70%, which is in line with what we found in OECD countries. Capital goods account for 43% of trade, which matches the US data well.

Figure 4: Structure of Trade Nondurable Model



Note:
Numbers in this figure are percentage of total output.

A.4 Elasticity Puzzle

In this section, we use a simple example to give more details about the elasticity puzzle in the literature. Suppose the final output is a CES composite of Home and Foreign goods. From the CES aggregation function, we can find demands for imports and the Home goods

$$Y_{Ht}^{F} = (1-\alpha)\left(\frac{P_{Ht}^{F}}{P_{Ht}}\right)^{-\gamma} Y_{Ht} \tag{A.4.1}$$

$$Y_{Ht}^{H} = \alpha\left(\frac{P_{Ht}^{H}}{P_{Ht}}\right)^{-\gamma} Y_{Ht}, \tag{A.4.2}$$

where Y_{Ht}^F is the demand for Foreign goods. P_{Ht}^F is the price of Foreign goods and P_{Ht} is the aggregate price. Y_{Ht} is the aggregate demand. Y_{Ht}^H is the demand for Home goods and P_{Ht}^H is the price of Home goods. Dividing one of the above equations by the other, we have

$$\frac{Y_{Ht}^F}{Y_{Ht}^H} = \frac{1-\alpha}{\alpha}\left(\frac{P_{Ht}^F}{P_{Ht}^H}\right)^{-\gamma}. \tag{A.4.3}$$

By definition, the elasticity of substitution between the Home and Foreign goods is

$$-\frac{dlog(Y_{Ht}^F/Y_{Ht}^H)}{dlog(P_{Ht}^F/P_{Ht}^H)} = \gamma. \tag{A.4.4}$$

If $log(Y_{Ht}^F/Y_{Ht}^H)$ and $log(P_{Ht}^F/P_{Ht}^H)$ are stationary, we can regress $log(Y_{Ht}^F/Y_{Ht}^H)$ on $log(P_{Ht}^F/P_{Ht}^H)$ to find the elasticity. This exercise has been done with industrial-level data in several papers, for instance, Reinert and Roland-Holst (1992), Blonigen and Wilson (1999), Reinert and Shiells (1993). The estimates from quarterly data are usually small with an average of around 0.85. In aggregate models, this parameter is usually calibrated in the range between 0.5 to 2. Bergin (2006) estimates a two-country general equilibrium model. His estimate is 1.13. Heathcote and Perri (2002) also estimate this parameter with aggregate data and find an estimate of 0.9. So at the business cycle frequency, the empirical findings both at disaggregate and aggregate levels point to a small elasticity of substitution between home and foreign goods.

Ruhl (2005) uses a regression to find the elasticity of substitution in his model. Drozd and Nosal (2007) propose a measurement that follows this spirit, but they do not run a regression. Instead, from equation (A.4.3), the standard deviation of $log(Y_{Ht}^F/Y_{Ht}^H)$ divided by the standard deviation of $log(P_{Ht}^F/P_{Ht}^H)$ is also equal to γ under the model's setup. We use both methods in our benchmark model and find our results are robust.

The second strand of literature estimates the elasticity of substitution through the (long-run) response of trade flows to permanent relative price changes. One example is a tariff reduction. Let's assume that the rate of tariff is τ and the law of one price holds after taking into account the tariff, that is,

$$P_{Ht}^F = (1+\tau)S_t P_{Ft}^F, \tag{A.4.5}$$

where P_{Ft}^F is the price of foreign goods in the foreign country and S_t is the exchange rate. Substitute this to equation (A.4.3), we have

$$\begin{aligned}\frac{Y_{Ht}^F}{Y_{Ht}^H} &= \frac{1-\alpha}{\alpha}\left(\frac{P_{Ht}^F}{P_{Ht}^H}\right)^{-\gamma} \\ &= \frac{1-\alpha}{\alpha}\left((1+\tau)S_t\frac{P_{Ft}^F}{P_{Ht}^H}\right)^{-\gamma}.\end{aligned} \tag{A.4.6}$$

We use variables without time scripts to denote their steady state values. From equation (A.4.6), we have

$$\Delta log\left(\frac{Y_H^F}{Y_H^H}\right) = -\gamma\Delta log(1+\tau) - \gamma\Delta log\left(\frac{SP_F^F}{P_H^H}\right). \tag{A.4.7}$$

Under the assumption that there is no change of relative price $\frac{SP_F^F}{P_H^H}$, the increase of trade share is determined by the change of tariff and the elasticity of substitution γ. The equation is identified by regressing it across different industries

$$\Delta log\left(\frac{Y_{iH}^F}{Y_{iH}^H}\right) = \alpha - \gamma\Delta log(1+\tau_i) + \varepsilon_i, \tag{A.4.8}$$

where i is the index of industries. The estimates from industrial level data usually give a large γ, which ranges from 6 to 15. For instance, see Feenstra and Levinsohn (1995), Head and Ries (2001), and Lai and Trefler (2002). Yi (2003) shows that the trade share of output increased substantially for a small decrease in tariffs. He points out that to replicate those findings in a general equilibrium model, the elasticity of substitution between the home and foreign goods must be very large. These results are strikingly different from those obtained from first stand of literature, though under the setup of our example, they are estimating the same parameter γ. This discrepancy has been labeled the elasticity puzzle in the literature.

A.5 Backus-Smith Puzzle

In this section, we describe how to calculate the utility-based real exchange rate used in Section 5.2. The calculation is straightforward if we ignore the frictions in the economy. Suppose that the household is renting the durable consumption from a competitive market instead of owning it. The rent cost in terms of the nondurable goods will be the marginal utility of durable consumption divided by the marginal utility of nondurable consumption

$$\hat{P}_{Ht}^{RH} = \frac{\partial u_{Ht}/\partial D_{Ht}^{H}}{\partial u_{Ht}/\partial C_{Ht}} \tag{A.5.1}$$

$$\hat{P}_{Ht}^{RF} = \frac{\partial u_{Ht}/\partial D_{Ht}^{F}}{\partial u_{Ht}/\partial C_{Ht}}, \tag{A.5.2}$$

where $\hat{P}_{Ht}^{RH}$ and $\hat{P}_{Ht}^{RF}$ are respectively the rental prices for Home- and Foreign-good durable consumption. All prices with a hat are in terms of nondurable goods.

The aggregate durable consumption stock is a CES function of the Home- and Foreign-good durable consumption stocks

$$D_{Ht} = \left[\psi^{\frac{1}{\theta}}(D_{Ht}^{H})^{\frac{\theta-1}{\theta}} + (1-\psi)^{\frac{1}{\theta}}(D_{Ht}^{F})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{A.5.3}$$

where ψ is the weight of Home durable goods in the durable consumption composite, and θ is the elasticity of substitution between the Home and Foreign durable goods. It is straightforward for us to find the shadow price of D_{Ht}

$$\hat{P}_{Ht}^{R} = \left[\psi(\hat{P}_{Ht}^{RH})^{1-\theta} + (1-\psi)(\hat{P}_{Ht}^{RF})^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{A.5.4}$$

The utility consumption is a CES function of durable and nondurable consumption

$$UC_{Ht} = \left(\mu^{\frac{1}{\zeta}} D_{Ht}^{\frac{\zeta-1}{\zeta}} + (1-\mu)^{\frac{1}{\zeta}} C_{Ht}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}. \tag{A.5.5}$$

It is easy for us to find the utility-based price index

$$\hat{P}_{Ht}^{UC} = \left[\mu(\hat{P}_{Ht}^{R})^{1-\zeta} + (1-\mu)\right]^{\frac{1}{1-\zeta}}. \tag{A.5.6}$$

Symmetric equations hold in the Foreign country, from which we can find the utility-based price index in the Foreign country $\hat{P}_{Ft}^{UC}$. The utility-based real exchange rate is

$$Q_t^{UC} = \frac{Q_t \hat{P}_{Ft}^{UC} \hat{P}_{Ht}}{\hat{P}_{Ht}^{UC} \hat{P}_{Ft}}, \tag{A.5.7}$$

where Q_t is the CPI-based real exchange rate. $\hat{P}_{Ht}$ is the consumer price index in the Home country and $\hat{P}_{Ft}$ is the consumer price index in the Foreign country. To see how to get this equation, it is useful to note that

$$\begin{aligned} Q_t^{UC} &= \frac{S_t P_{Ft}^{UC}}{P_{Ht}^{UC}} \\ &= \frac{\frac{S_t P_{Ft}}{P_{Ht}} \frac{P_{Ht}}{P_{Ft}} \frac{P_{Ft}^{UC}}{P_{Ft}^{N}} P_{Ft}^{N}}{\frac{P_{Ht}^{UC}}{P_{Ht}^{N}} P_{Ht}^{N}} \\ &= \frac{Q_t \hat{P}_{Ft}^{UC} \hat{P}_{Ht}}{\hat{P}_{Ht}^{UC} \hat{P}_{Ft}}, \end{aligned} \tag{A.5.8}$$

where all prices without a hat are nominal prices and P_{Ht}^{N} (P_{Ft}^{N}) is the nominal price of Home (Foreign) nondurable goods.