Long-Horizon Forecasts of Asset Prices when the Discount Factor is Close to Unity*

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July 28, 2010

Abstract

Engel and West (EW, 2005) argue that as the discount factor gets closer to one, present-value asset pricing models place greater weight on future fundamentals. Consequently, current fundamentals have very weak forecasting power and asset prices in these models appear to follow approximately a random walk. We connect the Engel-West explanation to the studies of long-horizon regressions. As expected, we find that under EW's assumption that fundamentals are I(1) and observable to the econometrician, long-horizon regressions generally do not have significant forecasting power when the discount factor is large. However, when EW's assumptions are violated in a particular way, our analytical and simulation results show that long-horizon regressions can have substantial power, even when the discount factor is close to one and the power of short-horizon regressions is low. One example for the substantial power improvement at long horizons is the existence of unobservable stationary fundamentals, such as the risk premium, in present-value asset pricing models. Consistent with our model's prediction, we find that the risk premium calculated from survey data can forecast exchange rates at long horizons. These results suggest that the presence of stationary unobservable fundamentals may have played a large role in the power improvement of long-horizon regressions found in empirical studies.

JEL Classifications: F31, F41, G12, G15
Keywords: long-horizon regressions, present-value asset pricing models, exchange rates and fundamentals

*We thank Sirio Aramonte, Menzie Chinn, Bruce Hansen, Ken West, Jianfeng Yu and seminar participants at the Dallas Fed and SED Meeting for helpful comments. We thank Aaron Game and Janet Koech for excellent research assistance. All views are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Dallas, the Board of Governors or the Federal Reserve System.

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1 Introduction

It is well documented that economic variables have low power in forecasting asset returns especially at short horizons. Meese and Rogoff (1983a, 1983b) find that a simple random walk model can perform as well as various structural and time series exchange rate models based on out-of-sample forecasting accuracy criteria. Bossaerts and Hillion (1999) find that conventional predictors such as dividend yield, bond yield, and the stock market's price-earnings ratio have a low power of forecasting out-of-sample excess stock returns in 14 countries. While some studies find evidence of predictability, most results remain fragile or quantitatively moderate.[1] Engel and West (2005) take a new line of attack on this puzzle. They show that present-value asset-pricing models predict that asset prices behave like a random walk when the fundamentals are I(1) and the discount factor is large (close to one) and hence it is not surprising that fundamentals cannot forecast asset returns in these models.[2]

In this paper, we examine whether asset prices can be better distinguished from a random walk process at long horizons, under the Engel-West explanation (E-W explanation henceforth). Long-horizon predictability of asset returns has been an active empirical research topic in the last few decades. Perhaps the most well-known application is Fama and French (1988). They find that the power of dividend yields to forecast stock returns increases with the return horizon. Campbell (1991) and Cochrane (1992), among others, also find that the aggregate dividend yields can predict the excess return, and the predictability is stronger at longer horizons. The early claims of success in long-horizon exchange rate predictability include Mark(1995), Chinn and Meese (1995), and MacDonald and Taylor (1994). However, these positive findings have been challenged in other studies. For instance, Ang and Bekaert (2007) find that dividend yields predict excess returns only at short to medium horizons, but not at long horizons. Kilian (1999) finds no long-horizon exchange rate predictability after updating the dataset used by Mark (1995). He also shows that the long-horizon predictability in Mark (1995) could come from the misspecification of the bootstrap inference method. Berkowitz and Giorgianni (2001) argue that the long-horizon predictability in Mark (1995) may be driven by the assumption that the exchange rate and fundamentals are cointegrated.

In spite of these challenges, long-horizon predictability remains a relatively robust finding in the literature.

[1] Fama and Schwert (1977) use nominal interest rates to predict stock returns for US data. Examples of using dividend yields to predict stock returns include Shiller (1984), Keim and Stambaugh (1986), Fama and French (1988), Campbell and Shiller (1988), Cochrane (1992), Campbell (1991), and Hodrick (1991), among others. More recently, Lettau and Ludvigson (2001) find that the aggregate consumption-wealth ratio can predict stock returns. See Hodrick (1987) for a survey of using the forward premium to study the efficiency of the foreign exchange market. Recent contributions to exchange rate predictability include Mark (1995), Groen (2000), Mark and Sul (2001), Kilian and Taylor (2003), Faust, Rogers, and Wright (2003), Cheung, Chinn and Pascual (2005), Molodtsova and Papell (2009), and Wang and Wu (2008), among others.

[2] Engel and West (2005) also show that a wide range of exchange rate models can be written in the form of such a model, and discount factors estimated from monthly or quarterly data are indeed close to one. As a result, exchange rate models should not be judged by whether they can predict exchange rates more accurately than the random walk. Rossi (2005) gives another example in which exchange rates approximately follow a random walk even if they are determined by economic fundamentals.

Lettau and Ludvigson (2001) find that the aggregate consumption-wealth ratio can predict stock returns, especially at long horizons. Recently, Engel, Mark, and West (2007) show that out-of-sample forecasting power of models can be increased in long-horizon forecasts, especially when using panel data. Wang and Wu (2008) find long-horizon predictability of exchange rates in the context of interval forecasting. Campbell (2001) also finds that long-horizon regressions have power advantages when there is a persistent predictable component in the asset return. Mark and Sul (2006) study the same model of Campbell (2001) and find that long-horizon regressions have an asymptotic power advantage over the short-horizon regressions if the regressor is endogenous.

The focus of this paper is on whether there is some underlying basis for long-horizon predictability that can be reconciled with the E-W explanation. We study the population R-squared of short- and long-horizon regressions in the context of a present-value asset pricing model as in Engel and West (2005). Two situations are considered. First, we assume that economic fundamentals are nonstationary, following Engel and West (2005). If fundamentals are I(1), we formally show that the theoretical R-squared of long-horizon regressions converges to zero for all time horizons, when the discount factor approaches one. That is, for a large discount factor, the long-horizon regressions do not have significant power advantage over a random walk. This is consistent with the E-W explanation and the intuition is straightforward. According to the E-W explanation, the change of the asset price in present-value models converges (in probability) to a white noise process as the discount factor approaches one, if economic fundamentals are I(1). In this case, the h-period change of the asset price is the sum of h white noises, and therefore a white noise process as well.

The more interesting case is when Engel and West's (2005) assumptions that fundamentals are I(1) do not hold, so that some economic fundamentals are I(0) rather than I(1). We consider this case because some well-known present-value asset pricing models may include stationary fundamentals. Campbell and Shiller (1988) develop a useful approximation to the present value formula. In their model, one of the fundamentals is the risk premium. The risk premium is usually treated as a stationary variable, for instance as in Campbell (2001). Pastor, Sinha and Swaminathan (2008) calculate a proxy for the risk premium from the implied cost of capital. The variable appears to be stationary and persistent. For exchange rate models, it is well-known that the Uncovered Interest Parity (UIP) condition does not hold in the data. If we allow a risk premium in UIP, the risk premium enters the present-value model as an unobservable variable.[3] We calculated this risk premium from Consensus Forecasts survey data and find statistical evidence of stationarity. In addition, economic fundamentals in some exchange rate models, such as bilateral output gap and inflation differentials in a model based on a Taylor

[3]Engel and West (2004) find that while observable fundamentals can account for a sizable fraction of exchange rate variation, there is still substantial unexplained variation that may be accounted for by unobservable fundamentals, such as the risk premium in Uncovered Interest Parity (UIP).

Rule for monetary policy, are theoretically stationary.[4]

In contrast to the case with only I(1) fundamentals, we find that the R-squared of short- and long-horizon regressions no longer converges to zero as the discount factor converges to one. Furthermore, we derive a reasonably general analytical condition under which long-horizon regressions have more power than short-horizon regressions. This condition is a generalization of the example in Engel, Mark, and West (2007), in which a stationary, but persistent, unobservable fundamental brings substantial power improvements in long-horizon regressions.

To illustrate the result with data, we consider two exchange rate models – a standard monetary model, and one based on a Taylor Rule for monetary policy. Both models include stationary fundamentals, including a risk premium. For seven foreign currency-U.S. dollar exchange rates, we calculate the risk premium from Consensus Forecasts of exchange rates, assuming the survey data is an appropriate measure of market expectations. The risk premium and other variables were used to form a dataset of economic fundamentals. We then estimate a Vector-Autoregression (VAR) model of the fundamentals. The estimated model is used to simulate fundamentals and the exchange rate for each country. We find that the population R-squared of long-horizon regressions on simulated data is substantially higher than those of short-horizon regressions for several countries.

Our theory says that long-horizon predictability of asset returns comes from the stationary fundamentals. The long-run levels of asset prices in the model are determined by the I(1) fundamentals. However, due to the existence of stationary fundamentals, the asset price can substantially deviate from its long-run trend and converge back only gradually. In a simple example, we show that the stationary fundamental is the short-run deviation of the asset price from its long-run equilibrium level and therefore can forecast the long-horizon movement of the asset price. Motivated by this result, we use the risk premium calculated from the Consensus Forecasts survey data to forecast exchange rates. We find strong evidence that the risk premiums can predict exchange rates at long horizons, even though the predictability is low at the short horizon.

Taken together, our results suggest that stationary and potentially unobservable fundamentals, such as the risk premium, may play an important role in reconciling the E-W explanation and the empirical evidence of long-horizon predictability. This result is related to several recent studies in the literature. Clarida, Sarno, Taylor, and Valente (2003) find that the term structure of forward premia may contain information about the risk premium that is useful for forecasting exchange rates. Adrian, Etula, and Shin (2009) find that fluctuations in risk premia captured by the variation in the aggregate balance sheets of financial intermediaries are useful

[4] In data, evidence of stationarity of these variables are mixed, but it is also well known that unit root tests often suffer from low power.

in forecasting exchange rates.[5] Pastor, Sinha and Swaminathan (2008) find that the implied cost of capital (as a proxy for the conditional expected return) is useful in capturing time variation in stock returns. Campbell (2001) studies long-horizon predictability of stock returns. There is indeed a close analogy of his model to the case with stationary fundamentals of this paper and this connection is detailed in the appendix.[6] If exchange rate predictability is coming from the risk premium, perhaps panel estimation would perform better since there may be a common component to the risk premium across dollar exchange rates. Groen (2000) and Mark and Sul (2001) find exchange rate predictability by using panel data. Rogoff and Stavrakeva (2008) find forecast improvement after allowing for common cross-country shocks in their panel forecasting specification, although the improvement is not entirely robust to the forecast window.

The remainder of the paper is organized as follows: Section 2 provides a brief introduction to the E-W explanation. In Section 3, we derive the long-horizon regressions from present-value asset pricing models and study the power of long-horizon regressions, particularly when some fundamentals are stationary. Section 4 presents our simulation results and the results of using the risk premium to predict exchange rates. Section 5 summarizes major findings and concludes.

2 Asset Pricing Model and E-W Explanation

A wide range of linear present-value asset pricing models can be written as a special case of a general form

$$s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t\left(f_{1,t+j} + u_{1,t+j}\right) + b\sum_{j=0}^{\infty} b^j E_t\left(f_{2,t+j} + u_{2,t+j}\right), \tag{1}$$

where s_t is the (log) asset price, $0 < b < 1$ is the discount factor, $\{f_{1,t}, f_{2,t}\}$ are observable fundamentals and $\{u_{1,t}, u_{2,t}\}$ are unobservable fundamentals. In equation (1), the asset price equals the sum of the expected present value of future fundamentals. Some well-known applications of the present-value model include Campbell and Shiller (1987, 1988) and West (1988). The present value formula can be interpreted as a log-linear approximation to a general asset pricing relationship with stochastic discount factors. The discount factor b in the formula (1) is not the pricing kernel, but instead is equal to e^{g-h}, where g is the mean dividend growth rate and h is the mean stock return.[7] Engel and West (2005) demonstrate analytically that if the discount factor b is close to unity, and either (1) $f_{1,t} + u_{1,t} \sim I(1)$ and $f_{2,t} + u_{2,t} = 0$, or (2) $f_{2,t} + u_{2,t} \sim I(1)$ with $f_{1,t} + u_{1,t}$ unrestricted, then s_t approximately follows a random walk.

[5] See Chen and Tsang (2009) for a recent study along this line of research.
[6] See Appendix A.1 for details.
[7] See Campbell and Shiller (1988).

To develop intuition, consider a simple example where $u_{1,t} = f_{2,t} = u_{2,t} = 0$. Equation (1) reduces to:

$$s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t(f_{1,t+j}). \tag{2}$$

If the fundamental $f_{1,t}$ follows a random walk, it is straightforward from the above equation that $s_t = f_{1,t}$, which means that the exchange rate is also a random walk. However, first differences of the fundamentals are typically serially correlated in the data. For simplicity, we assume that the first difference of $f_{1,t}$ follows an AR(1) process:

$$\Delta f_{1,t+1} = \phi \Delta f_{1,t} + \varepsilon_{t+1}, \tag{3}$$

where $|\phi| < 1$. From equations (2) and (3), we obtain:

$$\Delta s_{t+1} = \frac{\phi(1-b)}{1-b\phi}\Delta f_{1,t} + \frac{1}{1-b\phi}\varepsilon_{t+1}. \tag{4}$$

In equation (4), the first difference of the asset price is a weighted average of the first difference of the fundamental at time t and "news" arriving at time $t+1$. Δs_{t+1} is serially correlated because $\Delta f_{1,t}$ is serially correlated. However, when b approaches one, the coefficient in front of $\Delta f_{1,t}$ converges to zero. As a result, Δs_{t+1} converges to $\frac{1}{1-\phi}\varepsilon_{t+1}$, which is simply a white noise. The intuition for this result is that more weight is given to the new information rather than the changes of current fundamentals when the discount factor is large. Engel and West (2005) show that the discount factor estimated from data is indeed sufficiently close to one in standard exchange rate models that the theorem is potentially useful in accounting for exchange rate behavior. They argue that the exchange rate may be determined by fundamentals, but cannot be distinguished from a random walk. The same argument applies to other asset prices.

3 Long-Horizon Regressions under E-W Explanation

In this section, we derive long-horizon regressions from the linear present-value models studied in Engel and West (2005). We investigate conditions under which the E-W explanation is consistent with the empirical findings that economic models can forecast asset prices at long horizons.

As noted in the introduction, long-horizon regressions have been widely used to test the efficiency of asset markets. Let $s_{t+h} - s_t$ be the h-period change of the asset price and z_t be the deviation of the asset price from its long-run equilibrium level. Typically, in-sample or out-of-sample forecasts of $s_{t+h} - s_t$ from the model:

$$s_{t+h} - s_t = c_h + \beta_h z_t + \upsilon_{t+h} \tag{5}$$

are compared with in-sample or out-of-sample forecasts from the random walk model (equation (5) with the restriction $\beta_h = 0$).[8] If the R-squared of equation (5) increases, or the out-of-sample relative Mean Square Prediction Error (MSPE) decreases with time horizon h, it is taken as evidence that the asset price converges to its long-run value over time and therefore is predictable at long horizons.

We derive long-horizon regressions from the following linear present-value model:

$$s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t f_{1,t+j} + b\sum_{j=0}^{\infty} b^j E_t (f_{2,t+j} + u_{2,t+j}). \tag{6}$$

Compared to equation (1), u_{1t} was omitted in the model to simplify the set up. The well-known stock price model in Campbell and Shiller (1988) is a special case of equation (6). In Appendix A.1, we show that the stock price model in Campbell and Shiller (1988) and Campbell (2001) can be written into

$$p_t = (1-\rho)d_t + \rho E_t p_{t+1} - \lambda x_t, \tag{7}$$

where p_t is the log of the stock price, d_t is the log dividend, and x_t is the expected one-period-ahead return. Rewrite equation (6) and let $f_{2,t} = 0$, we have

$$s_t = (1-b) f_{1,t} + b E_t s_{t+1} + b u_{2,t}. \tag{8}$$

The two models in equations (7) and (8) are identical under the mapping of notations that $s_t = p_t$, $f_{1,t} = d_t$, $bu_{2t} = -\lambda x_t$ and $b = \rho$.

Two notable exchange rate models in the literature also fall under the setup in equation (6): the Monetary model and one based on a Taylor Rule for monetary policy (Taylor Rule model henceforth).[9] In the Monetary model,

$$s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t \left(m_{t+j} - m^*_{t+j} - \gamma(y_{t+j} - y^*_{t+j}) + q_{t+1} - (v_{t+j} - v^*_{t+j})\right) - b\sum_{j=1}^{\infty} b^j E_t \rho_{t+j}, \tag{9}$$

[8] One can also restrict $c_h = 0$ to obtain the driftless random walk model.
[9] See Appendix A.2 for details of these two models.

where m_t and y_t are logarithms of the domestic money stock and output, respectively. The superscript $*$ denotes the foreign country. $q_t \equiv s_t + p_t^* - p_t$ is the real exchange rate, v_t is a money demand shock, and $\rho_t \equiv E_t s_{t+1} - s_t - (i_t - i_t^*)$ is the deviation from UIP. We interpret this deviation as a risk premium in currency markets. In this model $f_{1,t} = m_t - m_t^* - \gamma(y_t - y_t^*) + q_t - (v_t - v_t^*)$, $f_{2,t} = 0$, and $u_{2,t} = -\rho_t$.

In the Taylor Rule model,

$$s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t(p_{t+j} - p_{t+j}^*) - b\sum_{j=0}^{\infty} b^j E_t\left(r_g(y_{t+j}^g - y_{t+j}^{*g}) + r_\pi(\pi_{t+j} - \pi_{t+j}^*) + v_{t+j} - v_{t+j}^* + \rho_t\right), \tag{10}$$

where p_t, y_t^g, π_t and v_t are domestic aggregate price, output gap, the inflation rate and monetary shock, respectively.[10] In this case $f_{1,t} = p_t - p_t^*$, and $f_{2,t} + u_{2,t} = -\left(r_g(y_t^g - y_t^*) + r_\pi(\pi_t - \pi_t^*) + v_t - v_t^* + \rho_t\right)$.

In the studies of exchange-rate markets, the left-hand side of equation (5), $s_{t+h} - s_t$, is the h-horizon change of log exchange rates. In the studies of stock markets, h-horizon excess stock returns, instead of h-horizon changes of log stock prices, are usually used on the left-hand side of long-horizon regressions. Appendix A.1 shows with a simple example that there is indeed a close analogy between these two types of long-horizon regressions in the literature.

Throughout the paper, we maintain the assumption that $f_{1,t}$ is I(1) and $u_{2,t}$ is I(0). The nonstationarity of $f_{1,t}$ is supported by empirical evidence of asset pricing models studied in the literature. For instance, the log dividend in the stock price model is usually found nonstationary in the data. The same is true for $f_{1,t}$ in the Monetary and Taylor Rule models of the exchange rate. There are two reasons to assume that $u_{2,t}$ is stationary. First, the assumption that $u_{2,t} \sim I(0)$ is maintained to guarantee that long-horizon regressions are not spurious.[11] More importantly, some proxies of $u_{2,t}$ calculated from the data suggest that this variable is stationary as we have mentioned in the introduction.

Let the deviation of the asset price from its long-run equilibrium level (z_t) be $z_t \equiv s_t - f_{1,t} - \frac{1}{1-b} f_{2,t}$. Under equation (6), s_t is I(1) and cointegrated with fundamentals, with z_t being a stationary error-correction term. $s_{t+h} - s_t$ is also stationary because s_t is an I(1). Therefore, linear projections of $s_{t+h} - s_t$ on z_t are valid under the setup in equation (6) and assumptions on fundamentals.

Valid projections does not imply that z_t can predict $s_{t+h} - s_t$. To study how the power of long-horizon regressions changes with the time horizon, we introduce additional assumptions and derive long horizon regressions explictly from (6). As shown in the above examples, the fundamentals in equation (6) are linear functions of

[10] Engel and West (2005, 2006), Mark (2009), and Molodtsova and Papell (2009) develop models of the nominal exchange rate based on a Taylor Rule for monetary policy. This specification comes from Engel and West (2005).

[11] By definition $u_{2,t}$ is unobservable, hence not included as a regressor in long-horizon regressions. If this "left-out" variable is non-stationary, long-horizon regressions may be spurious.

economic variables. Collect the economic variables in an $n \times 1$ vector ($\mathbf{X}_t$) and assume that the first difference of $\mathbf{X}_t$ follows a stationary VAR(p) process:

$$\varepsilon_t = \Delta\mathbf{X}_t - \Phi_1\Delta\mathbf{X}_{t-1} - ... - \Phi_p\Delta\mathbf{X}_{t-p} \equiv (I_n - \Phi(L))\Delta\mathbf{X}_t \tag{11}$$

with $E_t\varepsilon_{t+j} = 0, \forall j > 0$ and $E(\varepsilon_t\varepsilon_t^{'}) = \Omega$.[12] Since some economic variables are I(0) (e.g., risk premium), $\mathbf{X}_t$ contains both I(1) economic variables as well as sums of I(0) variables across time.

Let $\alpha_1, \alpha_{21}, \alpha_{22}$ be $n \times 1$ coefficient vectors. We assume that:

$$\begin{aligned} f_{1,t} &= \alpha_1^{'}\mathbf{X}_t \sim I(1) \neq 0 \\ u_{2,t} &= \alpha_{22}^{'}\Delta\mathbf{X}_t \sim I(0). \end{aligned} \tag{12}$$

For $f_{2,t}$, two cases are considered:

Case 1.

$$f_{2,t} = u_{2,t} = 0 \text{ or } f_{2,t} = \alpha_{21}^{'}\mathbf{X}_t \sim I(1) \text{ with } \alpha_{22} \text{ unrestricted;} \tag{13}$$

Case 2.

$$f_{2,t} = \alpha_{21}^{'}\Delta\mathbf{X}_t \sim I(0) \text{ with } f_{2,t} + u_{2,t} \neq 0. \tag{14}$$

In Case 1, all observable fundamentals (either $f_{1,t}$ or $\{f_{1,t}, f_{2,t}\}$) are I(1). Also, $f_{2,t} + u_{2,t}$ is either I(1) or zero, so Engel and West's (2005) assumptions are satisfied. Case 2, however, implies that $f_{2,t} + u_{2,t} \sim I(0) \neq 0$, which violates Engel and West's assumptions. This case is motivated by asset pricing models studied in the literature and has high empirical relevance. For instance, in the equity return model of Campbell and Shiller (1988) and the Monetary model of equation (9), $f_{2,t}$ equals zero, but $u_{2,t}$ is non-zero. In the Taylor Rule model of equation (10), $f_{2,t}$ and u_{2t} are both theoretically stationary and non-zero.

We derive long-horizon regressions and discuss the properties of the population R-squared of these regressions,

[12] Engel and West (2005) consider a more general setup (MA(∞)) for $\Delta\mathbf{X}_t$. The reason to consider a less general setup in this paper is purely technical.

under both Cases 1 and 2. In both cases, we define

$$\begin{aligned} z_t &\equiv s_t - f_{1,t} - \frac{b}{1-b} f_{2,t} \\ R^2(h) &\equiv \frac{\{Cov\,(s_{t+h} - s_t, z_t)\}^2}{Var(z_t)Var(s_{t+h} - s_t)}. \end{aligned} \tag{15}$$

This definition of z_t is consistent with most empirical studies on long-horizon regressions, including Mark (1995), Chinn and Meese (1995), Engel, Mark and West (2007), and Molodtsova and Papell (2009). $R^2(h)$ are population R-squareds from long-horizon OLS regressions with z_t as the only regressor. Finally, we use following definitions:

$$\underset{np \times np}{F} \equiv \begin{bmatrix} \Phi_1 & \Phi_2 & ... & \Phi_{p-1} & \Phi_p \\ I_n & 0 & ... & 0 & 0 \\ 0 & I_n & ... & 0 & 0 \\ \vdots & \vdots & ... & \vdots & \vdots \\ 0 & 0 & ... & I_n & 0 \end{bmatrix}, \quad \underset{np \times 1}{\Delta \mathbf{Y}_t} \equiv \begin{bmatrix} \Delta \mathbf{X}_t \\ \Delta \mathbf{X}_{t-1} \\ \vdots \\ \Delta \mathbf{X}_{t-p+1} \end{bmatrix}, \quad \underset{np \times 1}{\mathbf{v}_t} \equiv \begin{bmatrix} \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \underset{np \times n}{\iota} \equiv \begin{bmatrix} I_n \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

and let $\mathbf{Q} \equiv E(\mathbf{v}_t \mathbf{v}_t^{'})$. Notice that $\Delta \mathbf{Y}_t = F \Delta \mathbf{Y}_{t-1} + \mathbf{v}_t$. Define $F(b) \equiv bF(I_{np} - bF)^{-1}$ for $b \in (0, 1]$. $F(b)$ exists because all eigenvalues of F lie inside the unit circle by the assumption that $\Delta \mathbf{X}_t$ is a stationary process.

3.1 Long-Horizon Regression under Case 1

In Appendix A.3, we show that under Case 1,

$$s_{t+h} - s_t = \rho_h z_t - \alpha_{22}^{'} \iota^{'} \left(\sum_{k=0}^{h-1} F^k \right) \Delta \mathbf{Y}_t + \upsilon_{t+h} \tag{16}$$

can be derived from equations (6) and (11), where $\{z_t, \Delta \mathbf{Y}_t\}$ and υ_{t+h} do not correlate. Formulas for ρ_h and υ_{t+h} can be found in the appendix. In the absence of unobservable economic variables ($\alpha_{22} = 0$), the long-horizon regression reduces to the one that is typically used in empirical studies. Our result generalizes Campbell and Shiller (1987) and Nason and Rogers (2008), which derive the short-horizon regression (i.e. $h = 1$) from a present-value model with only observable fundamentals.

In addition, we show that the power of long-horizon regressions, as measured by $R^2(h)$, converges to zero for all time horizons h as the discount factor b approaches one. That is, the predictive power of long-horizon regressions becomes negligible for all time horizons if the discount factor is close to one. As a result, economic models have no significant power advantage over the random walk in forecasting asset prices even at long horizons.

This is a straightforward extension of the Engel-West Theorem to the cases where $h > 1$. To illustrate $R^2(h)$ under Case 1, consider a simple numerical example:

Example 1. Let $f_{2,t} = u_{2,t} = 0$, $\alpha_1 = 1$, and

$$\begin{aligned} n = 1 \quad &, \quad p = 1 \\ \mathbf{X}_t = X_{1,t} \quad &, \quad \varepsilon_t = \varepsilon_{1,t} \\ \Phi_1 = \phi_1 \quad &, \quad \Omega = \sigma_1^2 \end{aligned}$$

This example falls under Case 1, with $f_{1,t} = X_{1,t}, z_t = s_t - f_{1,t}$. Using our results in Appendix A.3:

$$\Delta s_{t+1} = \frac{1-b}{b} z_t + \frac{1}{1-b\phi_1}\varepsilon_{1,t+1}. \tag{17}$$

Comparing equations (17) and (16), $\rho_1 = \frac{1-b}{b}$ and $\upsilon_{t+1} = \frac{1}{1-b\phi_1}\varepsilon_{1,t+1}$. After some algebra, we have:

$$Var(z_t) = \left(\frac{b\phi_1}{1-b\phi_1}\right)^2 \frac{\sigma_1^2}{1-\phi_1^2}, \text{ and } Var(\upsilon_{t+1}) = \frac{\sigma_1^2}{(1-b\phi_1^2)}. \tag{18}$$

When the discount factor $b \to 1$, $\rho_1 \to 0$ while $Var(z_t)$ and $Var(\upsilon_{t+1})$ converge to constants. Hence, $R^2(1) \to 0$. For instance, if $b = 0.95$, ρ_1 is approximately equal to 0.05. Assuming $\phi_1 = 0.5, \sigma_1^2 = 1$ and using equations (18), $R^2(1) \approx 0.001$.

Now consider different horizons h. Using the formula of ρ_h derived in Appendix A.3 and the fact that $\rho_1 = \frac{1-b}{b}$, we have[13]

$$\rho_h = \rho_1 \frac{1-\phi_1^h}{1-\phi_1} \equiv \rho_1 \mu_h. \tag{19}$$

The multiplier $\mu_h \equiv \frac{1-\phi_1^h}{1-\phi_1} > 1$ increases monotonically in h and ϕ_1 for $\phi_1 \in (0,1)$. As a result, ρ_h is an increasing function of the time horizon h. However, for a large discount factor, say $b = 0.95$, the level and increases in ρ_h are small unless ϕ_1 is unrealistically large.[14]

Since $u_{2t} = 0$, $R^2(h) = \rho_h^2 Var(z_t)/(\rho_h^2 Var(z_t) + Var(\upsilon_{t+h}))$, and it is straightforward albeit tedious to show that $Var(\upsilon_{t+h})$ can be written as:

$$Var(\upsilon_{t+h}) = \frac{\phi_1^2(1-b)^2\sigma_1^2}{(1-b\phi_1)^2} \left\{ \sum_{j=1}^{h-1} \left(\frac{1}{\phi_1(1-b)} + \frac{1-\phi_1^j}{1-\phi_1} \right)^2 + \frac{1}{\phi_1^2(1-b)^2} \right\} \tag{20}$$

[13] This result is also derived in Berkowitz and Giorgianni (2001). However, they do not derive it from the present-value asset pricing model. Instead, they derive their result by assuming z_t in the long-horizon regressions follows an AR(1) process.

[14] Even if $\phi_1 = 0.8$, a level difficult to justify given the data, $\lim_{h\to\infty}\mu_h = 5$ and $\lim_{h\to\infty}\rho_h = 0.26$.

Substituting the expression for $Var(z_t)$ in equation (18) and $Var(\upsilon_{t+h})$ in equation (20) into the formula of $R^2(h)$, we can calculate $R^2(h)$ numerically. But since $R^2(h) \approx 0$ across h for reasonable values of ϕ_1 and $b \approx 1$, in order to compare $R^2(h)$ more meaningfully across h, we calculate the ratio of $R^2(h)/R^2(1)$ where $h = 1, ..., 16$. Figure 1 plots $R^2(h)/R^2(1)$ under different ϕ_1's. When $\Delta f_{1,t}$ is not very persistent (e.g., $\phi_1 = 0.4$), the ratio is less than one for all horizons h and decreases monotonically with h. When $\Delta f_{1,t}$ is more persistent, there is a humped-shaped relation between R-squareds and time horizon: long-horizon regressions have greater power than the short-horizon (i.e. $h = 1$) regressions but the power of long-horizon regressions eventually dies out with the increase of h. We see that long-horizon regressions generally have lower power than short-horizon regressions when $h > 10$.

In order to understand the humped-shaped pattern, Figure 2 plots ρ_h and $Var(\upsilon_{t+h})$ across h when $\phi_1 = 0.8$. As predicted by equation (19), ρ_h increases with the time horizon h, but at a decreasing rate. $Var(\upsilon_{t+h})$, on the other hand, increases at an approximately constant rate. In this example, before the fourth horizon, the numerator of $R^2(h)$ increases faster than the denominator and hence $R^2(h)$ increases in h. However, after this horizon the growth rate of $var(\upsilon_{t+h})$ eventually catches up and exceeds that of ρ_h. As a result, $R^2(h)$ starts to decline with h. ■

In summary, under Case 1, long-horizon regressions derived from the present-value model of asset prices have R-squareds that converge to zero as the discount factor converges to one. An example shows that if the changes of economic fundamentals are persistent, the power of long-horizon regressions can be higher than the power of short-horizon regressions, even though it is small in absolute terms.

3.2 Long-Horizon Regressions under Case 2

We also derive the long-horizon regressions in the form of equation (16) under Case 2, but with different expressions for ρ_h and υ_{t+h} (see Appendix A.3). However, $R^2(h)$'s are generally non-zero even when the discount factor equals one – highlighting the fact that the existence of stationary fundamentals (f_{2t} and u_{2t}) opens up the possibility of detecting exchange rate predictability in long-horizon regressions even for large discount factors.

Let $b = 1$. Define:

$$\begin{aligned}
\mathbf{P}_{t+k} &\equiv E_t(f_{2,t+k} + u_{2,t+k}) \\
\Xi_{1,t+k} &\equiv f_{2,t+k} + u_{2,t+k} - E_t(f_{2,t+k} + u_{2,t+k}) \\
\Xi_{2,t+k} &\equiv E_{t+k}\left(\sum_{j=0}^{\infty} \Delta f_{1,t+k+j} + f_{2,t+k+j} + u_{2,t+k+j}\right) - E_{t+k-1}\left(\sum_{j=0}^{\infty} \Delta f_{1,t+k+j} + f_{2,t+k+j} + u_{2,t+k+j}\right),
\end{aligned}$$

then, it turns out that under Case 2,

$$\frac{1-R^2(h)}{R^2(h)} = \frac{Var\left(\sum_{k=0}^{h-1}(\Xi_{1,t+k} - \Xi_{2,t+k+1})\right)}{Var(\sum_{k=0}^{h-1}\mathbf{P}_{t+k})}.$$

Notice that $\mathbf{P}_{t+k}$ is the time t forecast of $f_{2,t+k}+u_{2,t+k}$, while $\Xi_{1,t+k}$ and $\Xi_{2,t+k}$ are forecast errors, where $\Xi_{1,t+k}$ is the forecast error induced by predicting $f_{2,t+k}+u_{2,t+k}$ at time t, and $\Xi_{2,t+k}$ is the forecast error induced by predicting $\sum_{j=0}^{\infty}\Delta f_{1,t+k+j}+f_{2,t+k+j}+u_{2,t+k+j}$ at time $t+k-1$ instead of $t+k$. Therefore, $R^2(h)$ will increase with h when the following condition needs to be satisfied:

$$\frac{Var\left(\sum_{k=0}^{h}(\Xi_{1,t+k} - \Xi_{2,t+k+1})\right)}{Var\left(\sum_{k=0}^{h-1}(\Xi_{1,t+k} - \Xi_{2,t+k+1})\right)} \leq \frac{Var(\sum_{k=0}^{h}\mathbf{P}_{t+k})}{Var(\sum_{k=0}^{h-1}\mathbf{P}_{t+k})}. \tag{21}$$

Equation (21) says that $R^2(h)$ increases with the forecasting horizon if the percentage increase in the variance of the forecast errors is smaller than the percentage increase in the variance of the forecasts. Intuitively, long-horizon forecasts are useful relative to the random walk when $f_{2t}+u_{2t}$ is sufficiently persistent, while on the other hand errors that are induced in the forecasting process are reasonably small. Appendix A.3 further shows that given our model set up, the variance of forecasts grows at a dimishing rate, while the variance of forecast errors will approximately grow at a linear rate for large h. Taken together, these results indicates that humped-shaped $R^2(h)$ is possible, and that as h increases, $R^2(h)$ converges to zero as the forecast errors accumulate and overwhelm the predictable component.

Although equation (21) gives the condition for long-horizon regressions to have more power than the short-horizon regressions, the extent of power improvement depends on the structure of the fundamentals. Engel, Mark, and West (2007) develop a simple example (identical to one in Cochrane (2001)) to show that long-horizon regressions can have substantial power improvement even when the discount factor is close to one. We use a similar example here to develop intuition.

Example 2. Consider the setup in equation (6). Assuming $f_{2,t}=0$,

$$s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t f_{1,t+j} + b\sum_{j=0}^{\infty} b^j E_t u_{2,t+j} \tag{22}$$

We further assume $f_{1,t}$ follows a random walk[15] and $u_{2,t}$ follows an AR(1) process:

$$\begin{aligned} f_{1,t+1} &= f_{1,t} + \varepsilon_{t+1} \\ u_{2,t+1} &= \phi u_{2,t} + v_{t+1}, \end{aligned}$$

where $|\phi| < 1$. After some algebra, we obtain:

$$s_t = f_{1,t} + \frac{bu_{2,t}}{1 - b\phi}. \tag{23}$$

In equation (23), the log asset price is determined by a permanent component $f_{1,t}$, and a transitory component $\frac{bu_{2,t}}{1-b\phi}$. Similarly, the h-period change of s_t is also determined by the change in the permanent component and the change in the transitory component:

$$\begin{aligned} s_{t+h} - s_t &= f_{1,t+h} - f_{1,t} + \frac{b}{1 - b\phi}\left(u_{2,t+h} - u_{2,t}\right) \\ &= P_{t+h} + T_{t+h}, \end{aligned} \tag{24}$$

where $P_{t+h} \equiv f_{1,t+h} - f_{1,t}$ is asset price movements due to the change of the permanent component, and $T_{t+h} \equiv \frac{b}{1-b\phi}\left(u_{2,t+h} - u_{2,t}\right)$ is asset price movements due to the change of the transitory component. We can further write these expressions as:

$$\begin{aligned} P_{t+h} &= \sum_{j=1}^{h} \varepsilon_{t+j} \\ T_{t+h} &= \frac{b}{1 - b\phi}\left[(\phi^h - 1)u_{2,t} + \sum_{j=0}^{h-1} \phi^j v_{t+h-j}\right]. \end{aligned} \tag{25}$$

It is helpful to recognize that the right-hand-side of long-horizon regressions is proportional to $u_{2,t}$:

$$z_t = s_t - f_{1t} = \frac{bu_{2,t}}{1 - b\phi}. \tag{26}$$

As a result, the term containing $u_{2,t}$ in T_{t+h} can be predicted by z_t. If one regresses T_{t+h} on $u_{2,t}$ (and similarly on z_t, since z_t is proportional to $u_{2,t}$), the population R-squared equals $\frac{1-\phi^h}{2}$, which increases with time horizon h with an upper bound of 0.5. When $u_{2,t}$ is very persistent (ϕ is close to one), T_{t+1} is difficult to predict at the

[15] Allowing $f_{1,t}$ to have a stationary component does not change our results. The stationary, or transitory, component has negligible effects on long-horizon predictability if $f_{1,t}$ is I(1) and the discount factor is large.

short horizon because the R-squared ($\frac{1-\phi}{2}$) of the is small. However, this R-squared increases with h, and the transitory part T_{t+h} is helpful for finding long-horizon predictability. Note that P_t is not predictable from z_t and asset price changes are increasingly driven by P_t with the increase of time horizon h. As a result, the R-squared is humped shaped if one regress h-horizon changes of the asset price $(P_{t+h} + T_{t+h})$ on z_t. Depending on the underlying data generating process, the R-squared could be substantially higher than zero at some horizon $h > 0$. In contrast, if $u_{2t} = 0$, this predictability will not be there. ■

Under Case 2, long-horizon regressions have the potential to produce good forecasts relative to the random walk, even when the discount factor is one. Furthermore, we found that R-squareds of long horizon regressions increase with horizon as long as the variance of forecasts of the stationary fundamentals f_{2t} and u_{2t} increases faster than the variance of forecast errors. A simple example attributes the predictability in long horizon regressions to the predictability of the transitory component of the asset price that is present when there are stationary fundamentals.

4 Simulations and Empirical Results

According to the results for Case 2, stationary fundamentals opens up the possibility of finding a non-trivial R-squared in long-horizon regressions even when the discount factor b is close to one. The actual power of long-horizon regressions, however, depends on the structure of the underlying economic variables. We explore the power in the following two exercises. First, we simulate two models in the exchange rate determination literature: the Monetary and Taylor Rule models. Using data on economic variables and fitted model coefficients, we calculate the population R-squared of long-horizon regression at various horizons using a very large sample of simulated exchange rates. In the second exercise, we use the risk premium calculated from the survey data as the regressor in long-horizon regressions to forecast exchange rates.

4.1 Simulations

In the Monetary model with stationary fundamentals of equation (9), the matrix $\mathbf{\Delta X_t}$ is defined as:

$$\Delta \mathbf{X}_t \equiv \begin{bmatrix} \Delta m_t & \Delta m_t^* & \Delta y_t & \Delta y_t^* & \Delta q_t & \Delta v_t & \Delta v_t^* & \rho_t \end{bmatrix}. \tag{27}$$

The stationary fundamental in this model is $u_{2t} = -\rho_t$, which is typically unobservable to the econometrician. The risk premium is:

$$\rho_t = E_t^m s_{t+1} - s_t - (f_t - s_t), \tag{28}$$

where $E_t^m s_{t+1}$ is the market expectation of the exchange rate at time $t+1$, s_t is the spot exchange rate, and f_t is the forward exchange rate.

The risk premium is unobservable because the market expectation of the exchange rate is typically unobservable to the econometrician. For our exercise, we assume that expectations are formed as in the surveys. That is, we calculate the risk premium from survey data (Consensus Forecasts) by assuming that the survey data is a correct measure of market expectations of the exchange rate. 3-month forecasts of the exchange rate are available for 8 countries: Canada, Denmark, Germany, Japan, Norway, Switzerland, UK, and US during 1989Q4-2007Q2.[16] For each country, ρ_t is calculated from equation (28) with $E_t^m s_{t+1}$ replaced by 3-month forecasts of the exchange rate. Figure 6 shows the risk premium that is calculated using the survey data. The risk premium appears to be stationary, and stationarity is further affirmed by unit root tests: using the Augmented Dickey-Fuller test, we reject the unit root hypothesis for the risk premium at a 1% significance level for all exchange rates (Table 1).

Most remaining data are obtained from the G10+ dataset provided by Haver Analytics. The money demand shock v_t is recovered from the money demand function:

$$m_t - p_t = \alpha + \gamma y_t + \beta i_t + v_t. \tag{29}$$

The money stock m_t is the seasonally adjusted M2 for all countries except Japan, for which it is M2 plus CDs.[17] p_t is the CPI index and y_t is GDP. The short-term nominal interest rate is measured using 3-month Treasure Bill rate in Canada and the US, and three-month interbank offer rate for Denmark, Germany, Norway, Switzerland, and the UK. The short-term interest rate in Japan is measured using the 3-month Certificate of Deposit (Gensaki) rate.

We cannot reject the unit root hypothesis for m_t, p_t, y_t, and i_t and the null hypothesis that these variables are not cointegrated at the 5% significance level for most countries. We proceed to take first differences when estimating (29).[18] The OLS regression errors are recovered as Δv_t. Together with other economic variables,

[16]See Appendix A.4.2 for more details about the data.

[17]Norway's M2 data is from International Financial Statistics and seasonally adjusted using EViews.

[18]Using the Augmented Dickey-Fuller test with a constant and time trend, we fail to reject unit roots for m_t, p_t, y_t, and i_t at various lags for most countries. Using the same test, we fail to reject the null hypothesis that m_t, p_t, y_t, and i_t are not cointegrated at a 1% significance level for all countries.

we build $\Delta \mathbf{X}_t$ and estimate a VAR(1) process for $\Delta \mathbf{X}_t$.[19] The coefficient and variance-covariance matrix of residuals estimates, together with a large number of Gaussian innovations, are used to simulate pseudo-data for $\Delta \mathbf{X}_t$, which in turn are used to simulate exchange rates with $b = 0.97$.[20] Long-horizon regressions in equation (5) are estimated with the simulated exchange rate data, where the deviation of the exchange rate from its long-run equilibrium level is defined as $z_t = s_t - (m_t - m_t^* - \gamma(y_t - y_t^*) + q_t - (v_t - v_t^*))$.

Figure 4(a) shows $R^2(h)$ at various horizons for different countries. Two interesting findings are noted. First, the R-squared is generally small in the short-horizon regression: it is about 0.05 or less in 5 out of 7 countries. Second, in 6 out of 7 countries, $R^2(h)$ displays a humped shape across h. In some countries, the increase of $R^2(h)$ across h is substantial. It rises from about 0.07 to more than 0.3 for Germany, and from about 0.11 to 0.28 for Switzerland. These results confirm our analysis under Case 2 that $R^2(h)$ does not converge to zero even when the discount factor equals one. The humped shape of $R^2(h)$ supports the notion that long-horizon predictability can exist even when short-horizon predictability is low. It also illustrates the result that whether or not $R^2(h)$ increases with h depends on the condition in (21): as described previously, for large h, this condition will be violated, thus explaining the humped shape.

In a robustness check, we set up the simulation with cross-country differences of economic variables entering $\Delta \mathbf{X}_t$:

$$\Delta \mathbf{X}_t = \left[\begin{array}{ccccc} \Delta(m_t - m_t^*) & \Delta(y_t - y_t^*) & \Delta q_t & \Delta(v_t - v_t^*) & \rho_t \end{array} \right].$$

We conduct the rest of the simulations the same way as before. Results are reported in Figure 4(b). The shapes of the $R^2(h)$'s are qualitatively similar to those found in 4(a), though the $R^2(h)$ for Germany is now significantly larger.

Next, the Taylor Rule model in equation (10) is simulated. In this example, $f_{1,t} = p_t - p_t^*$ and $f_{2,t} + u_{2,t} = -\left(r_g(y_t^g - y_t^{*g}) + r_\pi(\pi_t - \pi_t^*) + v_t - v_t^* + \rho_t\right)$. Data exhibits strong evidence that p_t and p_t^* are I(1) and ρ_t is I(0). Unit root test results for other variables, however, are mixed. As it may be difficult for unit root tests to distinguish between an I(1) process and a persistent I(0) process, we follow our setup in Section 3.2 and assume that these variables are also I(0).

[19] We restrict the lag length in the VAR to 1 due to the short sample size and the large number of parameters that need to be estimated.

[20] This process is detailed in Appendix A.4.

Following the definition of $\Delta \mathbf{X}_t$ in Section 3.2, we define:

$$\Delta \mathbf{X}_t = \begin{bmatrix} y_t^g & y_t^{g*} & \pi_t & \pi_t^* & v_t & v_t^* & \rho_t \end{bmatrix}. \tag{30}$$

Notice that since p_t and p_t^* are logarithms of prices, Δp_t and Δp_t^* are the same as π_t and π_t^*, and we do not include Δp_t and Δp_t^* in $\Delta \mathbf{X}_t$. GDP gaps y_t^g and y_t^{g*} are quadratically detrended GDP.[21] v_t and v_t^* are residuals of regressing the policy rate on the output gap and CPI inflation in each country. Pseudo data for $\Delta \mathbf{X}_t$ and s_t are generated using the same method as the simulations for the Monetary model. z_t in long-horizon regressions is defined in four different ways, depending on the specification of $f_{2,t}$:

- Case 1: $z_t = s_t - p_t + p_t^* - \frac{b}{1-b}\left(r_g(y_t^g - y_t^{*g}) + r_\pi(\pi_t - \pi_t^*) + v_t - v_t^* + \rho_t\right)$;
- Case 2(a): $z_t = s_t - p_t + p_t^* - \frac{b}{1-b}\left(r_g(y_t^g - y_t^{*g}) + r_\pi(\pi_t - \pi_t^*)\right)$;
- Case 2(b): $z_t = s_t - p_t + p_t^* - \frac{b}{1-b}\left(r_g(y_t^g - y_t^{*g}) + r_\pi(\pi_t - \pi_t^*) + v_t - v_t^*\right)$
- Case 3: $z_t = s_t - p_t + p_t^*$.

These four cases differ in terms of how the stationary fundamental $f_{2,t}$ is defined. For instance, Case 2(a) sets $f_{2,t} = -\left(r_g(y_t^g - y_t^{*g}) + r_\pi(\pi_t - \pi_t^*)\right)$ and $u_{2,t} = -(v_t - v_t^* + \rho_t)$, a case considered by Molodtsova and Papell (2009). In Case 3, $(u_{2,t} = -\left(r_g(y_t^g - y_t^{*g}) + r_\pi(\pi_t - \pi_t^*) + v_t - v_t^* + \rho_t\right))$ is used in studies of long-run Purchasing Power Parity (PPP), for instance, Chinn and Meese (1995).

Figure 5 shows simulation results for each case. We notice several interesting findings in the long-horizon regressions:

- As in the Monetary model, $R^2(h)$ is small (less than 0.1) in most short-horizon regressions.
- In most cases, the long-horizon regressions have much larger $R^2(h)$ than the short-horizon regressions for some time horizon $h > 1$, in particular for Cases 2(b) and 3.
- Case 1 performs the best in all countries at the short horizon. This is intuitive as f_{2t} includes all stationary fundamentals, leaving nothing unobserved by the econometrician.
- At very long horizons, the R-squared remains relative high in the Taylor Rule model compared to that in the Monetary model. This is because the stationary component $f_{2t} + u_{2t}$ is very persistent in the Taylor Rule model. Because this simulation is simply an illustration of the long-horizon predictability of Taylor

[21] Using other detrending methods, such as the H-P filter, does not change our results qualitatively.

Rule fundamentals, we shy away from the fact that some fundamentals are borderline non-stationary in the data. Simulation results come close to what we find in the Monetary model if, say, Δy_t^g and Δy_t^{g*} are used in f_{2t} instead of their levels.

4.2 Forecasting Exchange Rates with the Risk Premium

In Example 2, the deviation of the exchange rate from its long-run level, z_t is a linear function of $u_{2,t}$. In other words, the unobservable fundamental $u_{2,t}$ can predict the convergency of the exchange rate to its long-run level over time. In this simple example, the error correction term z_t used in empirical studies is a perfect measure of $u_{2,t}$. However, when there are measurement errors for $f_{1,t}$, z_t may behave differently from u_{2t}. In this case, if we can find a direct proxy for $u_{2,t}$, long-horizon regressions may perform better.

The Monetary model in equation (9) falls under the setup in Example 2, with $u_{2,t} = -\rho_t$ being the risk premium from UIP. As in the previous section, although u_{2t} is typically unobserved, we can recover a proxy for u_{2t} using Consensus Forecasts data. With motivation driven by Example 2, we investigate the empirical performance of long-horizon regressions, using u_{2t} as the predictor. We use the same set of countries as in the simulations: Canada, Denmark, Germany, Japan, Norway, Switzerland, and UK. For each country and horizon, we report regression coefficient, its statistical significance as measured by the bootstrapped p-values of Newey and West (1987) t-statistics, and regression R^2s.[22]

The estimation and inference used is straight-forward and follows the setup of Mark (1995). Long-horizon regressions in the form of equation (5) are conducted for horizons $h = 1, 2, ..., 20$, with z_t being the risk premium. $\widehat{\beta}_h$, $R^2(h)$ and $t-$statistic with Newey-West standard errors ($t^{NW}(h)$) are computed. To conduct inference on the signficance of $\widehat{\beta}_h$, we use a bootstrapped distribution of $t^{NW}(h)$ rather than standard normal critical values due to well-known finite sample distortions present in long-horizon regressions.[23] Bandwidth used in standard error estimation ranges from 4 to 20. Our analysis shows that evidence of predictability is the strongest when the bandwidth is the smallest as the standard error is closest to the regular $t-$statistic standard error. To be conservative, we select a large bandwidth of 20.

We use a Gaussian bootstrap to generate the critical values for $t^{NW}(h)$ in the following way. First, we

[22]We do not report out-of-sample forecasts due to the short time series and the fact that we have focused on in-sample measures in this paper.

[23]See Mark (1995) and Berkowitz and Giorgianni (2001), for instance.

estimate the DGP under the null that β_h is zero:

$$\begin{aligned} \Delta s_t &= \widehat{c}_0 + \widehat{v}_t \\ z_t &= \widehat{b}_0 + \sum_{j=1}^{p} \widehat{b}_j z_{t-j} + \widehat{e}_t. \end{aligned} \tag{31}$$

And let $\widehat{\Sigma}_{ve}$ be the variance-covariance matrix estimate of $(\widehat{v}_t, \widehat{e}_t)'$. The lag p is selected by Bayesian Information Criterion (BIC). Using estimates $\widehat{c}_1, \widehat{b}_0, ..., \widehat{b}_p, \widehat{\Sigma}_{ve}$ and bivariate Gaussian innovations $(v_t^*, e_t^*)'$,[24] we generate a pseudo data set of s_t and z_t based on (31), and computed $t^{NW}(h)$. This process is repeated 5000 times to obtain a null distribution for $t^{NW}(h)$. The p-value is the quantile of the null distribution at which $t^{NW}(h)$ from unrestricted estimation lies.

There are three things we should note about the inference procedure. First, even though we use the same bootstrap setup as Mark (1995), we believe that our application is not subject to the criticism that the stationarity assumption on z_t biases the results towards predictability, as argued by Kilian (1999) and Berkowitz and Giorgianni (2001). More specifically, in Mark's bootstrap, z_t is the error correction term between exchange rate, relative money supply, and relative output. When these variables are not cointegrated, possibly due to measurement errors in money supply and output, z_t will not be stationary. In such cases, bootstrapping the null distribution under the assumption that z_t is stationary and comparing it to $t^{NW}(h)$ from data, where z_t may actually be nonstationary, may produce spurious results. In our case, however, z_t is a proxy for risk premium, and we have documented strong evidence that z_t is stationary (see Table 1). Therefore, imposing that z_t is a stationary AR(p) process is a reasonable approach. Second, we have also conducted asymptotic inference by using the standard normal as the null distribution. We found much lower $p-$values (i.e., more evidence for predictability) relative to bootstrap distribution. We understand that finite sample distortions may render these $p-$values inaccurate, and therefore we rely on the more conservative bootstrap test. Finally, we conducted a suite of sensitivity analyses including using nonparametric bootstrap, changing the number of lags in the process of equation (31), and increasing the number of bootstrap iterations. We did not find the results to be qualitatively different.

Table 2 and Figure 6 summarizes the results. We observed that:

- For all countries, $\widehat{\beta}(h)$ in Table 2 are generally negative. This finding is consistent with Example 2, in which $\widehat{\beta}(h) = \phi^h - 1 < 0$ since $|\phi| < 1$.

[24] We generated the same number of bivariate Gaussian innovations vector as the sample size plus 100, to account for start-up effects.

- The $p-$values on $t^{NW}(h)$ in Table 2 generally decrease significantly from horizon one to horizon twenty for all countries except Japan and the UK. In five out of seven countries, the $p-$value falls below 0.1 at some horizon greater than 8.

- $R^2(h)$ increases with h for all countries, although it remain quantitatively small in Canada and the UK even at long horizons. Figure 6 plots $R^2(h)$ across h. We see evidence of humped-shape $R^2(h)$ in most countries as illustrated in previous examples and simulations. In five out of seven countries, R-squared can rise to about 0.3 in long-horizon regressions from less than 0.05 in short-horizon regressions.

In summary, we find evidence of long-horizon predictability for exchange rates using the risk premium calculated from Consensus Forecasts survey date. This finding is consistent with our example in which the stationary and potentially unobservable fundamentals can substantially increase the power of long-horizon regressions. These results suggest that the stationary and potentially unobservable fundamentals, such as the risk premium, may have played an important role in the power improvement of long-horizon regressions found in empirical studies.

5 Conclusion

Engel and West (2005) propose an explanation to empirical findings that economic fundamentals have low power in forecasting asset prices especially at short horizons. With a simple and reasonable modification to their nonstationarity assumption about economic fundamentals, we find that the E-W explanation is also consistent with the finding that fundamental-based models can forecast asset prices at long horizons. When we allow stationary, and potentially unobservable, fundamentals in the present-value asset pricing models studied in Engel and West (2005), the population R-squareds of long-horizon regressions can be high although the R-squared is close to zero in the short-horizon regression. A potential candidate for the stationary fundamental in some standard exchange rate models is the UIP risk premium. We calculate the risk premium from survey data and find strong evidence of stationarity. The risk premium, along with other fundamentals, and the model coefficients estimated from the data are used to simulate exchange rates from two standard exchange rate models. The fundamentals can forecast the simulated exchange rates at long horizons, but not at short horizons. Consistent with our theory, we also find empirical evidence that the risk premium calculated from the survey data can forecast exchange rates at long-horizons.

Although we focus on exchange rate models in the paper, our findings apply to other asset prices too, for instance, stock prices. For simplicity, we only consider linear asset pricing models. Incorporating other features, such as nonlinearity, has also been found to be successful in forecasting exchange rates more accurately. For instance, Kilian and Taylor (2003) find strong evidence of predictability at horizons of 2 to 3 years, but not at shorter horizons. Faust, Rogers and Wright (2003) and Molodtsova, Nikolsko-Rzhevskyy and Papell (2008) also find exchange rate predictability using real-time, but not revised, data. Engel and West (2005) and Chen, Rogoff, and Rossi (forthcoming) find the connection between exchange rates and fundamentals in the opposite direction: exchange rates are helpful to forecast fundamentals. Further empirical research along these lines may be fruitful.

We focus on the population R-squareds of long-horizon regressions in the theoretical part of the paper and do not study the case of a finite sample. In small samples, long-horizon regressions may have serious size distortions when asymptotic critical values are used. For instance, see Mark (1995) and Campbell (2001). However, we do consider the small sample bias in our empirical work when using the risk premium to forecast exchange rates. We find that the power advantage of the long-horizon regressions will remain after size is corrected in our empirical study. Throughout the paper, we do not consider out-of-sample forecasting. Note that the purpose of our paper is not to find a model or economic fundamental that can forecast the exchange rate out of sample. Instead, we reconcile Engel and West's result that the exchange rate approximately follows a random walk in present-value models when the discount factor is close to unity and empirical findings that exchange rates are predictable at long horizons. We clarify some plausible conditions under which long-horizon regressions work even when the discount factor is large.

Table 1: Unit Root Tests for the Risk Premium

	Canada	Denmark	Germany	Japan	Norway	Switzerland	UK
ADF test with constant and time trend	-7.552	-7.535	-8.451	-6.982	-8.842	-7.916	-7.368
ADF test with constant only	-8.657	-5.006	-5.599	-5.133	-5.669	-5.321	-7.201

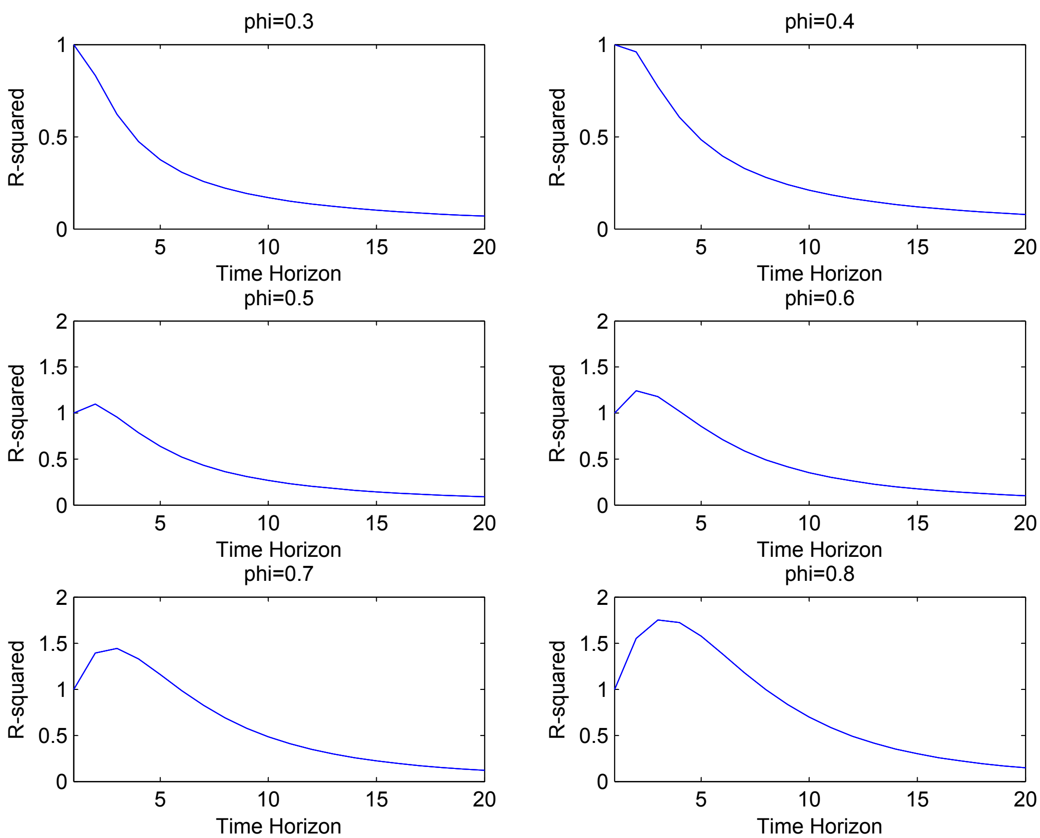
Note:
–Entries are Augmented Dicky-Fuller t-statistics. The unit root hypothesis is rejected at a 99% confidence level in all cases.
–4 lags are included in the above tests. The results do not change qualitatively with the number of lags.

Table 2: Long-horizon Regressions with Risk Premium as Predictor

	h	$\widehat{\beta}_h$	$R^2(h)$	$t^{NW}(h)$	$p-$value
Canada	1	-0.153	0.003	-1.017	0.216
	4	-0.006	0.000	-0.011	0.497
	8	-0.231	0.001	-0.175	0.440
	12	-2.877	0.045	-1.854	0.093
	20	-5.328	0.060	-2.223	0.065
Denmark	1	-0.102	0.002	-0.642	0.305
	4	-0.304	0.005	-0.799	0.277
	8	-1.267	0.040	-2.081	0.095
	12	-2.660	0.129	-1.801	0.128
	20	-5.545	0.356	-4.722	0.012
Germany	1	-0.120	0.003	-1.237	0.179
	4	-0.206	0.003	-0.743	0.292
	8	-1.412	0.058	-1.918	0.114
	12	-2.810	0.170	-1.868	0.137
	20	-5.268	0.369	-2.991	0.078
Japan	1	0.408	0.033	2.704	0.967
	4	0.061	0.000	0.127	0.583
	8	-1.236	0.039	-1.199	0.278
	12	-2.583	0.137	-1.992	0.166
	20	-2.616	0.150	-1.639	0.257
Norway	1	-0.374	0.028	-1.502	0.126
	4	-1.050	0.060	-1.961	0.095
	8	-1.734	0.085	-3.202	0.037
	12	-2.721	0.157	-2.711	0.071
	20	-5.981	0.450	-5.407	0.013
Switzerland	1	-0.202	0.009	-1.269	0.171
	4	-0.279	0.005	-1.070	0.213
	8	-1.060	0.040	-1.344	0.177
	12	-2.434	0.158	-1.989	0.100
	20	-4.494	0.385	-5.058	0.009
United Kingdom	1	-0.406	0.018	-1.672	0.113
	4	-0.779	0.019	-2.189	0.066
	8	-0.357	0.002	-0.288	0.409
	12	-1.302	0.022	-0.553	0.341
	20	-1.235	0.017	-0.576	0.321

Note:
–Results from long horizon regressions in the form of $s_{t+h} - s_t = c_h + \beta_h z_t + \upsilon_{t+h}$, where z_t is a proxy for risk premium constructed using Consensus Forecasts.
–$t^{NW}(h)$ are $t-$statistics calculated with Newey-West standard errors with a bandwidth of 20.
–$p-$values are constructed using the bootstrapped methodology outlined in section 4.2.

Figure 1: Asymptotic R-squared with AR(1) Fundamental



Note:

The R-squared for horizons greater than 1 is normalized by the short-horizon R-squared ($R^2(1)$).

Figure 2: ρ_h and $\sigma^2_{\nu_{t+h}}$ at Different Time Horizons

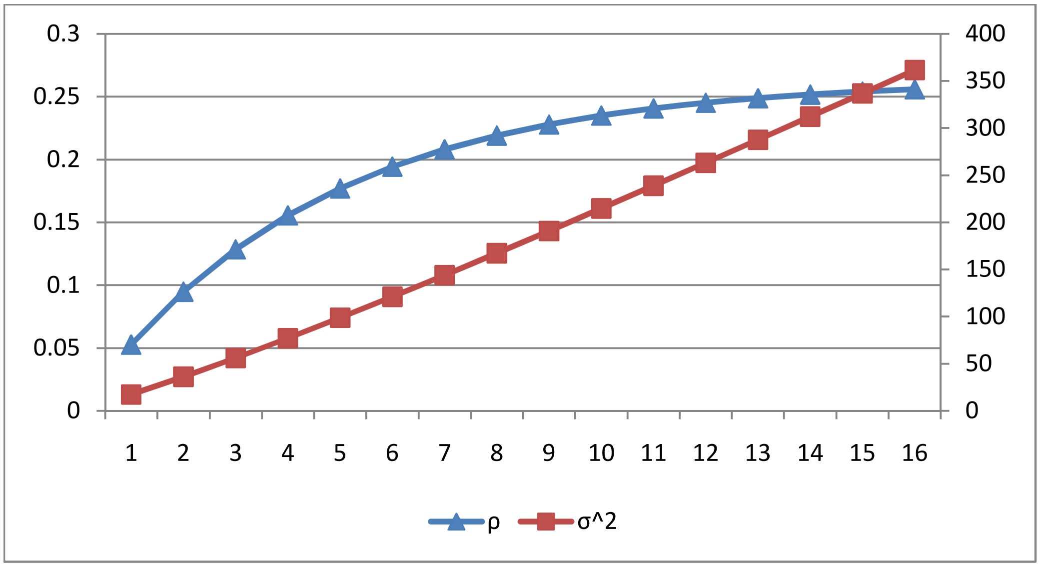


Figure 3: Exchange Rate Risk Premium Calculated From Survey Data

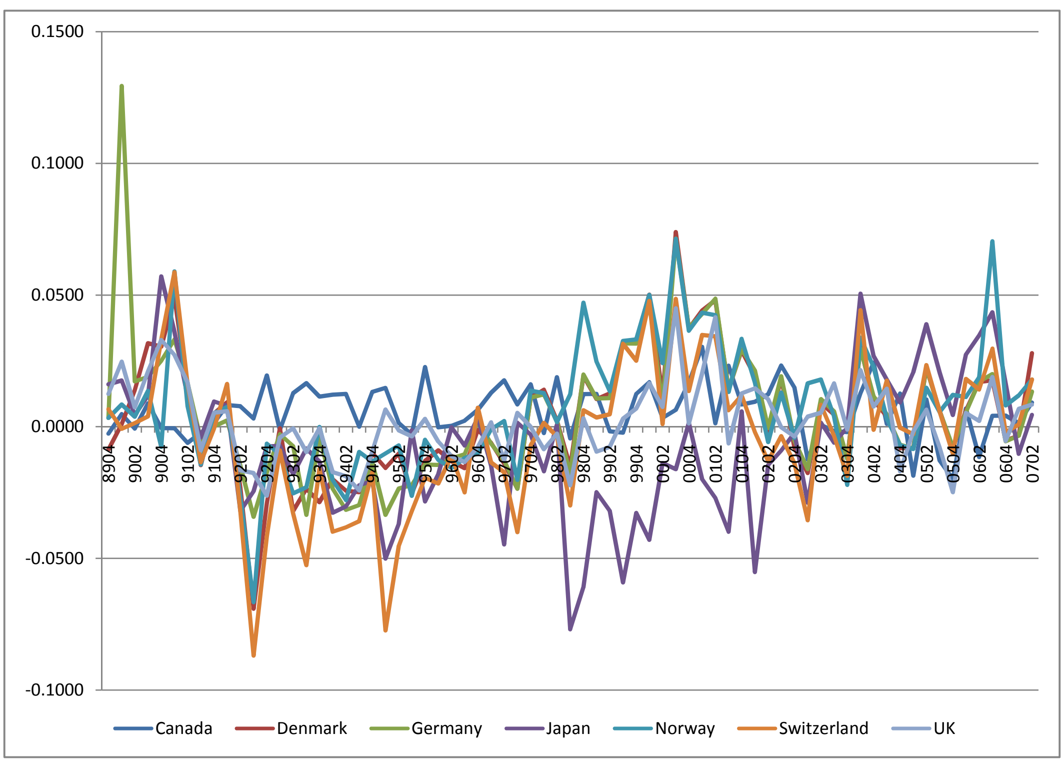
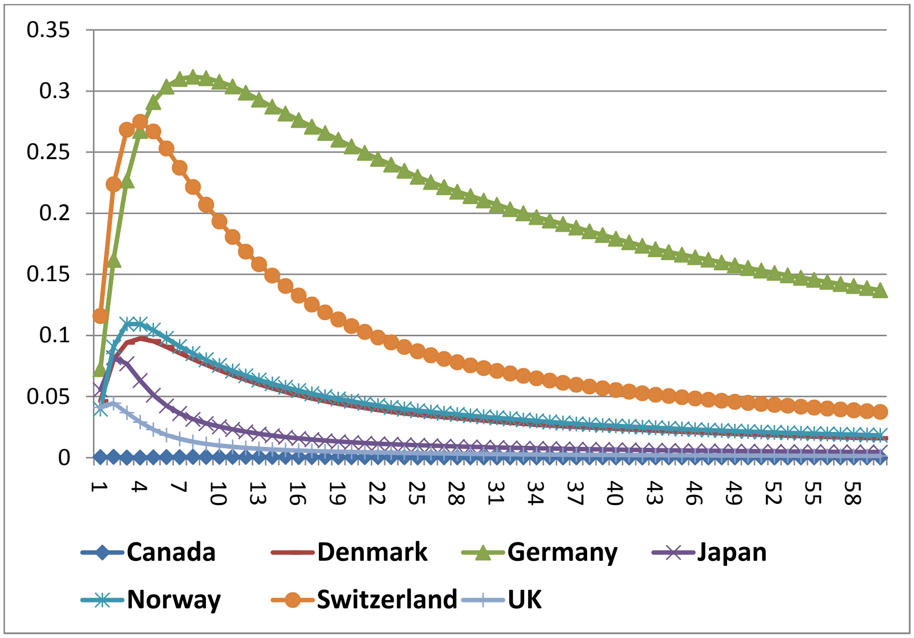
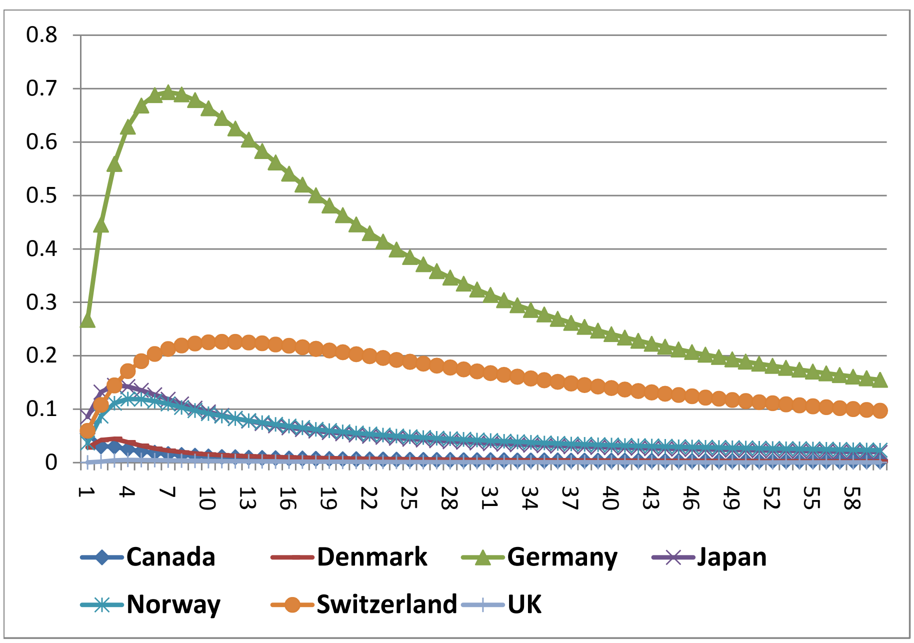


Figure 4: Population R-squared with VAR(1) Fundamental: Monetary Model



(a) Monetary Model 1

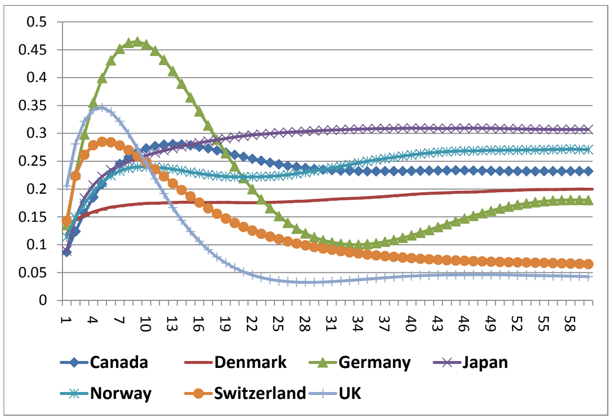


(b) Monetary Model 2

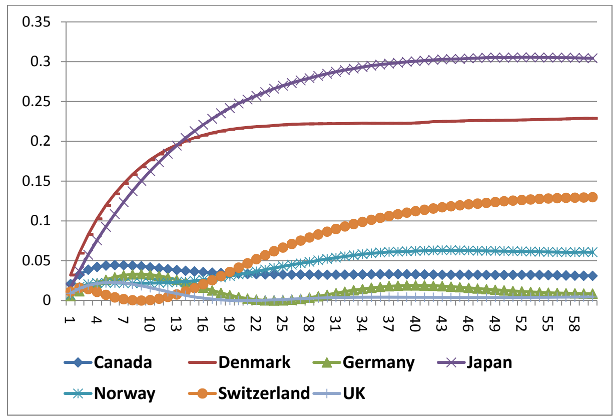
–Figure 4(a): $\mathbf{\Delta X_t} \equiv \begin{bmatrix} \Delta m_t & \Delta m_t^* & \Delta y_t & \Delta y_t^* & \Delta q_t & \Delta v_t & \Delta v_t^* & \rho_t \end{bmatrix}$.

–Figure 4(b): $\mathbf{\Delta X_t} = \begin{bmatrix} \Delta(m_t - m_t^*) & \Delta(y_t - y_t^*) & \Delta q_t & \Delta(v_t - v_t^*) & \rho_t \end{bmatrix}$.

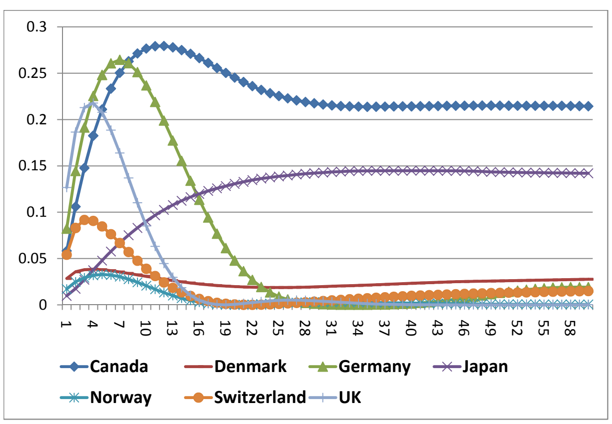
Figure 5: Population R-squared with VAR(1) Fundamental: Taylor Rule Model



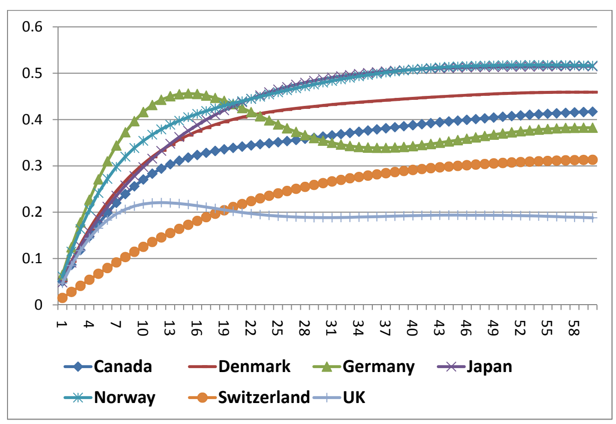
(a) Case 1



(b) Case 2(a)



(c) Case 2(b)



(d) Case 3

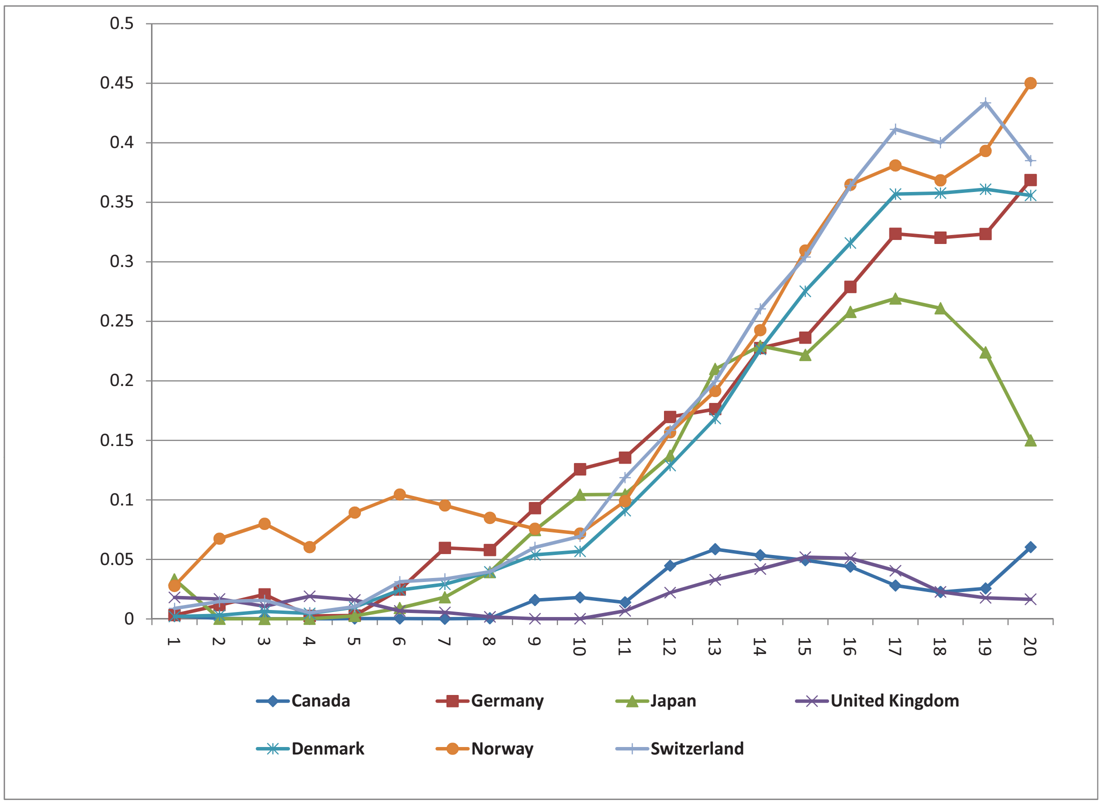
–Case 1: $z_t = s_t - p_t + p_t^* - \frac{b}{1-b}\left[r_g(y_t^g - y_t^{*g}) + r_\pi(\pi_t - \pi_t^*) + v_t - v_t^* + \rho_t\right]$

–Case 2(a): $z_t = s_t - p_t + p_t^* - \frac{b}{1-b}\left[r_g(y_t^g - y_t^{*g}) + r_\pi(\pi_t - \pi_t^*)\right]$

–Case 2(b): $z_t = s_t - p_t + p_t^* - \frac{b}{1-b}\left[r_g(y_t^g - y_t^{*g}) + r_\pi(\pi_t - \pi_t^*) + v_t - v_t^*\right]$

–Case 3: $z_t = s_t - p_t + p_t^*$.

Figure 6: $R^2(h)$ from Long-Horizon Regression Based on Risk Premium



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APPENDIX

A.1 Connection with Campbell (2001)

In this section, we draw the analogy between Campbell's (2001) long-horizon exercise and Engel, Mark, and West's (EMW, 2007) long-horizon exercise that we use in Section 3.2.

Take equation (2) of Campbell and Shiller (1988):

$$r_{t+1} = k + (1-\rho)d_t + \rho p_{t+1} - p_t, \tag{A.1.1}$$

where d_t is the log of dividends, and p_t is the log of the stock price. We have rearranged and canceled terms on the right-hand side, and changed some notation. Equation (A.1.1) is derived from an approximation for the rate of return (capital gain plus dividend yield) around the non-stochastic steady state in which dividends grow at a rate g, and h is the steady state log rate of return on stocks. The discount factor ρ in this approximation is equal to e^{g-h}.

In Campbell (2001), x_t is defined by $\lambda x_t \equiv E_t r_{t+1}$. So from equation (A.1.1) we have:

$$\lambda x_t = k + (1-\rho)d_t + \rho E_t p_{t+1} - p_t. \tag{A.1.2}$$

We can rewrite this equation (dropping the intercept for convenience) as:

$$p_t = (1-\rho)d_t + \rho E_t p_{t+1} - \lambda x_t. \tag{A.1.3}$$

Now compare to Example 2 in Section 3.2:

$$s_t = (1-b)f_{1,t} + bE_t s_{t+1} + bu_{2,t}. \tag{A.1.4}$$

Here is the mapping of notations from equation (A.1.3) of Campbell to equation (A.1.4) of EMW:

Table 3: Mapping of Campbell to EMW

Campbell (2001)	p_t	d_t	$-\lambda x_t$	ρ
EMW (2007)	s_t	$f_{1,t}$	$bu_{2,t}$	b

Campbell (2001) assumes that x_t is stationary and follows an AR(1) process:

$$x_{t+1} = \phi x_t + u_{t+1}. \tag{A.1.5}$$

Campbell (2001) closes his model by modeling r_{t+1}:

$$r_{t+1} - E_t r_{t+1} = v_{t+1}, \tag{A.1.6}$$

where v_{t+1} is the expectation error and possibly correlated with u_{t+1}. Comparing the above equation with (A.1.1), we have:

$$p_{t+1} - E_t p_{t+1} = v_{t+1}/\rho. \tag{A.1.7}$$

EMW (2007) models $u_{2,t}$ the same way as Campbell (2001) models x_t

$$u_{2,t+1} = \phi u_{2,t} + u_{t+1}. \tag{A.1.8}$$

They call $u_{2,t}$ the risk premium, which is unobservable to the econometrician. This interpretation is consistent with the Monetary model. $f_{1,t}$ is assumed to be a random walk:

$$f_{1,t+1} = f_{1,t} + \varepsilon_{t+1}. \tag{A.1.9}$$

Substituting equations (A.1.8) and (A.1.9) into (A.1.4), we have:

$$s_{t+1} - E_t s_{t+1} = \varepsilon_{t+1} + \frac{b}{1 - b\phi} u_{t+1}. \tag{A.1.10}$$

So we have that $\varepsilon_{t+1} + \frac{b}{1-b\phi} u_{t+1}$ is analogous to v_{t+1}/ρ in equation (A.1.7). Note that because EMW assume ε_{t+1} and u_{t+1} are uncorrelated, then EMW's $\varepsilon_{t+1} + \frac{b}{1-b\phi} u_{t+1}$ (analogous to Campbell's v_{t+1}) is necessarily correlated with EMW's u_{t+1} (analogous to Campbell's u_{t+1}.) Mark and Sul (2006) find that long-horizon regressions have power advantage over the short-horizon regression when the regressor is endogenous (u_{t+1} and v_{t+1} are correlated.) In this example, we show that the endogeneity exists under EMW's setup.

It is useful to recognize that the long-horizon regression that Campbell (2001) simulates does not require a model of the stochastic process of d_t (which is $f_{1,t}$ in EMW), but instead uses a model of the ex-ante risk premium, λx_t. Campbell (2001) implicitly assumes that the change in d_t is stationary. So, Campbell's simulations are consistent with any I(1) or I(0) data generating process for d_t. As we will now note, EMW's long-horizon regression does require a model of $f_{1,t}$ (which is analogous to Campbells d_t), because EMW's simulations require that we be able to solve the model for s_t, or at least $s_{t+1} - s_t$.

Next, we turn to the long-horizon regressions, which appear to be different. Campbell's long-horizon regression regresses $r_{t+1} + ... + r_{t+k}$ on x_t, while EMW regress $s_{t+k} - s_t$ on $f_{1,t} - s_t$. While these seem to be completely different regressions, under the assumptions made about the stochastic processes, they are in fact very similar. First, note that under the assumptions of EMW, we have:

$$f_{1,t} - s_t = -\frac{b}{1-b\phi} u_{2,t}.$$

So, EMW's r.h.s. variable in the long-horizon regression, $f_{1,t} - s_t$, is just proportional to their risk premium, $-bu_{2,t}$. Likewise, Campbell's r.h.s. variable, x_t, is just proportional to his risk premium, λx_t. If EMW had run the same regression for exchange rates as Campbell does for stock prices, then, their r.h.s. variable would be $(1-b\phi)(f_{1,t} - s_t)$, which is analogous to Campbell's r.h.s. variable, x_t, because Campbell normalizes λ to one.

What is different is the dependent variable. From equation (A.1.1), we have:

$$\begin{aligned} r_{t+1} + ... + r_{t+k} &= (1-\rho)d_t + \rho p_{t+1} - p_t + ... + (1-\rho)d_{t+k-1} + \rho p_{t+k} - p_{t+k-1} \\ &= p_{t+k} - p_t + (\rho - 1)\left[p_{t+k} - d_{t+k-1} + ... + p_{t+1} - d_t\right]. \end{aligned} \tag{A.1.11}$$

The l.h.s. variable in EMW would be, by analogy to equation (A.1.11) above

$$s_{t+k} - s_t + (b-1)\left[s_{t+k} - f_{1,t+k-1} + ... + s_{t+1} - f_{1,t}\right]. \tag{A.1.12}$$

It is straightforward that:

$$\begin{aligned} & s_{t+k} - s_t + (b-1)\left[s_{t+k} - f_{1,t+k-1} + ... + s_{t+1} - f_{1,t}\right] \\ &= s_{t+k} - s_t + (b-1)\left[s_{t+k} - s_t - f_{1,t+k-1} + ... + s_{t+1} - f_{1,t} + s_t\right] \\ &= b(s_{t+k} - s_t) + (b-1)\sum_{j=0}^{k-1}(s_{t+j} - f_{1,t+j}). \end{aligned} \tag{A.1.13}$$

In short, EMW's long-horizon regression regresses $s_{t+k} - s_t$ on $f_{1,t} - s_t$, but if they had followed the Campbell methodology, they would regress $b(s_{t+k} - s_t) + (b-1)\sum_{j=0}^{k-1}(s_{t+j} - f_{1,t+j})$ on $(1-b\phi)(f_{1,t} - s_t)$. Note that the two methodologies are very similar for b close to one.

Cochrane (2001) argues that long-horizon predictability of excess returns on stock prices is a result of the fact that the forecasting variable is persistent. In a simple example similar to Campbell's (2001), Cochrane (2001) shows that when expected returns are persistent, the coefficient and R-squared in long-horizon regressions increase with the time horizon. Cochrane (2001) rightfully points out that in the empirical studies of long-horizon predictability, it may be better to focus on the stationary fundamentals and ignore the stationary component of nonstationary fundamentals. [25] We confirm this conjecture and make it clear that when the discount factor is close to unity, the stationary component of nonstationary fundamentals does not help for long-horizon forecasts.

[25] It is mentioned in Cochrane (2001) that "If dividend growth (Δf_{1t} in our example) is also forecastable, then the dividend/price ratio (z_t in our example) is a combination of dividend growth (Δf_{1t}) and return forecasts (ρ_t). Actual return forecasting exercises can often benefit from cleaning up the dividend/price ratio to focus on the implied return forecast."

Appendix not for publication

A.2 Exchange Rate Models

A.2.1 Monetary Model

Assume the money market clearing condition in the home country is:

$$m_t = p_t + \gamma y_t - \alpha i_t + v_t,$$

where m_t is the log of the money supply, p_t is the log of the aggregate price, i_t is the nominal interest rate, y_t is the log of output, and v_t is a money demand shock. A symmetric condition holds in the foreign country and we use an asterisk in subscript to denote variables in the foreign country. Subtracting foreign money market clearing condition from the home, we have:

$$i_t - i_t^* = \frac{1}{\alpha}\left[-(m_t - m_t^*) + (p_t - p_t^*) + \gamma(y_t - y_t^*) + (v_t - v_t^*)\right]. \tag{A.2.1}$$

The nominal exchange rate is equal to its purchasing power value plus the real exchange rate:

$$s_t = p_t - p_t^* + q_t. \tag{A.2.2}$$

The uncovered interest rate parity condition in financial market takes the form

$$E_t s_{t+1} - s_t = i_t - i_t^* + \rho_t, \tag{A.2.3}$$

where ρ_t is the uncovered interest rate parity shock. Substituting equations (A.2.1) and (A.2.2) into (A.2.3), we have:

$$s_t = (1-b)\left[m_t - m_t^* - \gamma(y_t - y_t^*) + q_t - (v_t - v_t^*)\right] - b\rho_t + bE_t s_{t+1}, \tag{A.2.4}$$

where $b = \alpha/(1+\alpha)$. Solving s_t recursively and applying the "no-bubbles" condition, we have:

$$s_t = E_t\left\{(1-b)\sum_{j=0}^{\infty} b^j\left[m_{t+j} - m_{t+1}^* - \gamma(y_{t+j} - y_{t+1}^*) + q_{t+1} - (v_{t+j} - v_{t+j}^*)\right] - b\sum_{j=1}^{\infty} b^j \rho_{t+j}\right\}. \tag{A.2.5}$$

In the standard Monetary model, such as Mark (1999), purchasing power parity ($q_t = 0$) and uncovered interest rate parity hold ($\rho_t = 0$). Furthermore, it is assumed that the money demand shock is zero ($v_t = v_t^* = 0$) and $\gamma = 1$. Equation (A.2.5) then reduces to:

$$s_t = E_t \left\{ (1-b) \sum_{j=0}^{\infty} b^j \left(m_{t+j} - m_{t+j}^* - (y_{t+j} - y_{t+j}^*) \right) \right\}.$$

If we release the above assumptions and allow both observable and unobservable fundamentals in the model, $f_{1t} = m_t - m_t^* - \gamma(y_t - y_t^*) + q_t - (v_t - v_t^*)$, $u_{1t} = 0$, $f_{2t} = 0$, and $u_{2t} = -\rho_t$ in this example. Assume the risk premium ρ_t is stationary. By our definition of the error correction term z_t:

$$\begin{aligned} z_t \equiv s_t - f_{1t} &= s_t - (m_t - m_t^* - \gamma(y_t - y_t^*) + q_t - (v_t - v_t^*)) && \text{(A.2.6)} \\ &= p_t - p_t^* - (m_t - m_t^*) + \gamma(y_t - y_t^*) + v_t - v_t^*. && \text{(A.2.7)} \end{aligned}$$

In the second part of the above equation, we replace $s_t - q_t$ with $p_t - p_t^*$. If we assume purchasing power parity ($q_t = 0$), the error correction term becomes:

$$z_t \equiv s_t - f_{1t} = s_t - (m_t - m_t^* - \gamma(y_t - y_t^*) - (v_t - v_t^*)). \tag{A.2.8}$$

A.2.2 Taylor Rule Model

We follow Engel and West (2005) to assume that both countries follow the Taylor Rule and the foreign country targets the exchange rate in its Taylor Rule. The interest rate differential is:

$$i_t - i_t^* = r_s(s_t - \bar{s}_t^*) + r_g(y_t^g - y_t^{*g}) + r_\pi(\pi_t - \pi_t^*) + v_t - v_t^*, \tag{A.2.9}$$

where $\bar{s}_t^*$ is the targeted exchange rate. Assume that monetary authorities target the PPP level of the exchange rate: $\bar{s}_t^* = p_t - p_t^*$. Substituting this condition and the interest rate differential of the UIP condition, we have:

$$s_t = (1-b)(p_t - p_t^*) - b\left[r_g(y_t^g - y_t^{*g}) + r_\pi(\pi_t - \pi_t^*) + v_t - v_t^*\right] - b\rho_t + bE_t s_{t+1}, \tag{A.2.10}$$

where $b = \frac{1}{1+r_s}$. Comparing to our models with both observable and unobservable fundamentals, in this example, $f_{1t} = p_t - p_t^*$, $u_{1t} = 0$, and $f_{2t} + u_{2t} = -\left[r_g(y_t^g - y_t^{*g}) + r_\pi(\pi_t - \pi_t^*) + v_t - v_t^* + \rho_t\right]$. Under the condition that f_{1t} and f_{2t} are I(1) and ρ_t is I(0), s_t, f_{1t}, and f_{2t} are cointegrated with the conintegrating vector of $(1 \quad -1 \quad -\frac{b}{1-b})$.

The error correction term is $z_t \equiv s_t - f_{1t} - \frac{b}{1-b} f_{2t}$.

A.3 Long Horizon Regressions Under Cases 1 and 2

Propositions 1 and 2 contains formal the characterization of long-horizon regressions in section 3 of the paper.

Proposition 1. *Consider the case specified in (13) and a fixed $b < 1$. Under the setup in (6), (11), (12) and (15),*

(a) For ρ_h and υ_{t+h} as defined in the proof of this proposition,

$$s_{t+h} - s_t = \rho_h z_t - \alpha_{22}^{'} \iota^{'} \left(\sum_{k=0}^{h-1} F^k \right) \Delta \mathbf{Y}_t + \upsilon_{t+h} \tag{A.3.1}$$

where $(z_t, \Delta \mathbf{Y}_t)$ and υ_{t+h} do not correlate.

(b) For $np \times 1$ vectors $A_{h,b}$ and $\{B_{k,b}\}_{k=1}^{h-1}$ as defined in the proof of this proposition,

$$\frac{1 - R^2(h)}{R^2(h)} = \frac{\sum_{k=0}^{h-1} B_{k,b}^{'} \mathbf{Q} B_{k,b}}{A_{h,b}^{'} E(\Delta \mathbf{Y}_t \Delta \mathbf{Y}_t^{'}) A_{h,b}}.$$

As $b \to 1$, $A_{h,b} = O(1)$ while $B_{h,b} = O(b/(1-b))$ for all $h < \infty$.

Proof of 1(a). First note that using equation(6) and the definition of $\alpha(b)$, we have:

$$z_t = b u_{2t} + \alpha^{'}(b) \sum_{j=1}^{\infty} b^j E_t \Delta \mathbf{X}_{t+j}. \tag{A.3.2}$$

Notice that:

$$\begin{aligned} \Delta \mathbf{Y}_{t+j} &= F^j \Delta \mathbf{Y}_t + \xi_{t+j} \\ \xi_{t+j} &\equiv \sum_{i=1}^{j} F^{j-i} \mathbf{v}_{t+i}. \end{aligned} \tag{A.3.3}$$

Hence, $E_t \Delta \mathbf{X}_{t+j} = \iota^{'} F^j \Delta \mathbf{Y}_t$, and

$$z_t = b \alpha_{22}^{'} \iota^{'} \Delta \mathbf{Y}_t + \alpha^{'}(b) \iota^{'} \left(\sum_{j=1}^{\infty} b^j F^j \right) \Delta \mathbf{Y}_t \equiv \beta^{'}(b) \Delta \mathbf{Y}_t,$$

where $\beta(b) \equiv b\iota\alpha_{22} + F^{'}(b)\iota\alpha(b)$ and $\alpha(b) \equiv \alpha_1 + \frac{b}{1-b}\alpha_{21} + b\alpha_{22}$. For $b < 1$, z_t is stationary because $\Delta\mathbf{Y}_t$ is stationary. This implies that s_t, f_{1t} and f_{2t} are cointegrated.

Now consider the one-step-ahead regression. Manipulations of equation (6) yield:

$$\Delta s_{t+1} = \Delta f_{1t} + \frac{b}{1-b}\Delta f_{2t} + \alpha^{'}(b)\sum_{j=0}^{\infty} b^j \left(E_{t+1}\Delta\mathbf{X}_{t+1+j} - E_t\Delta\mathbf{X}_{t+j}\right).$$

Equation (A9) of Engel and West (2005) shows that:

$$\sum_{j=0}^{\infty} b^j \left(E_{t+1}\Delta\mathbf{X}_{t+1+j} - E_t\Delta\mathbf{X}_{t+1+j}\right) = (I_n - \Phi(b))^{-1}\varepsilon_{t+1}.$$

Substituting this and (A.3.2) into the expression for Δs_{t+1}, we have:

$$\begin{aligned}
\Delta s_{t+1} &= \alpha^{'}(b)\Delta\mathbf{X}_t - bu_{2t} + \alpha^{'}(b)\left\{\sum_{j=0}^{\infty} b^j \left(E_t\Delta\mathbf{X}_{t+1+j} - E_t\Delta\mathbf{X}_{t+j}\right) + (I_n - \Phi(b))^{-1}\varepsilon_{t+1}\right\} \\
&= \frac{1-b}{b}\alpha^{'}(b)\sum_{j=1}^{\infty} b^j E_t\Delta\mathbf{X}_{t+j} - bu_{2t} + \alpha^{'}(b)(I_n - \Phi(b))^{-1}\varepsilon_{t+1} \\
&= \frac{1-b}{b}z_t - u_{2t} + \alpha^{'}(b)(I_n - \Phi(b))^{-1}\varepsilon_{t+1}.
\end{aligned}$$

Define the following matrices:

$$\underset{n\times nh}{\Gamma_h^{'}(b)} \equiv \left[(I_n - \Phi(b))^{-1}, ..., (I_n - \Phi(b))^{-1}\right], \qquad \underset{nh\times 1}{\varepsilon_{t,h}} \equiv \left[\varepsilon_{t+1}^{'}, ..., \varepsilon_{t+h}^{'}\right]^{'}.$$

Then, by iterating the one-step-ahead regression h times, we have:

$$s_{t+h} - s_t = \frac{1-b}{b}\sum_{k=0}^{h-1} z_{t+k} - \sum_{k=0}^{h-1} u_{2t+k} + \alpha^{'}(b)\Gamma_h^{'}(b)\varepsilon_{t,h}. \tag{A.3.4}$$

Using (A.3.3) and the fact that for $b < 1$, $\beta(b)\beta^{'}(b)$ is non-singular, we obtain:

$$\begin{aligned}
z_{t+k} &= \beta^{'}(b)\Delta\mathbf{Y}_{t+k} \\
&= \beta^{'}(b)\left(F^k\Delta\mathbf{Y}_t + \xi_{t+k}\right) \\
&= \beta^{'}(b)F^k(\beta(b)\beta^{'}(b))^{-1}\beta(b)z_t + \beta^{'}(b)\xi_{t+k}.
\end{aligned}$$

After some straightforward but tedious calculations,

$$\sum_{k=0}^{h-1}\xi_{t+k}=F_h^{'}\varepsilon_{t,h}, \qquad \underset{np\times nh}{F_h^{'}}\equiv\left[\begin{array}{cccccc}\sum_{j=0}^{h-2}F^j\iota & \sum_{j=0}^{h-3}F^j\iota & ... & \sum_{j=0}^{1}F^j\iota & I_{np}\iota & 0\end{array}\right].$$

Plugging these expressions into (A.3.4), we have:

$$\begin{aligned}s_{t+h}-s_t &= \frac{1-b}{b}\beta^{'}(b)\left(\sum_{k=0}^{h-1}F^k\right)\left(\beta(b)\beta^{'}(b)\right)^{-1}\beta^{'}(b)z_t+\frac{1-b}{b}\beta^{'}(b)\sum_{k=0}^{h-1}\xi_{t+k}-\alpha_{22}^{'}\iota^{'}\sum_{k=0}^{h-1}\left(F^k\mathbf{\Delta Y}_t+\xi_{t+k}\right)+\beta^{'}(b)\Gamma_h^{'}(b)\varepsilon_{t,h}\\ &\equiv \rho_h z_t-\alpha_{22}^{'}\iota^{'}\left(\sum_{k=0}^{h-1}F^k\right)\mathbf{\Delta Y}_t+\upsilon_{t+h},\end{aligned}$$

where

$$\begin{aligned}\rho_h &\equiv \frac{1-b}{b}\beta^{'}(b)\left(\sum_{k=0}^{h-1}F^k\right)\left(\beta(b)\beta^{'}(b)\right)^{-1}\beta^{'}(b)\\ \upsilon_{t+h} &\equiv \left\{\left(\frac{1-b}{b}\beta^{'}(b)-\alpha_{22}^{'}\iota^{'}\right)F_h^{'}+\alpha^{'}(b)\Gamma_h^{'}(b)\right\}\varepsilon_{t,h}.\end{aligned}$$

■

Proof of 1(b).

Because $E\Delta\mathbf{X}_t=0$ and $E(s_{t+h}-s_t)=Ez_t=0$, we have $R^2(h)=\frac{\{E(s_{t+h}-s_t)z_t\}^2}{Ez_t^2E(s_{t+h}-s_t)^2}$. First, consider the numerator. Using $z_t=\beta^{'}(b)\mathbf{\Delta Y}_t$ and re-arranging, we have:

$$s_{t+h}-s_t=\left\{\frac{1-b}{b}\beta^{'}(b)-\alpha_{22}^{'}\iota^{'}\right\}\left(\sum_{k=0}^{h-1}F^k\right)\mathbf{\Delta Y}_t+\upsilon_{t+h}.$$

Then,

$$\begin{aligned}\{E(s_{t+h}-s_t)z_t\}^2 &= \left\{\frac{1-b}{b}\beta^{'}(b)-\alpha_{22}^{'}\iota^{'}\right\}\left(\sum_{k=0}^{h-1}F^k\right)E(\mathbf{\Delta Y}_t\mathbf{\Delta Y}_t^{'})\beta(b)\beta^{'}(b)E(\mathbf{\Delta Y}_t\mathbf{\Delta Y}_t^{'})\left(\sum_{k=0}^{h-1}F^{'k}\right)\left\{\frac{1-b}{b}\beta(b)-\iota\alpha_{22}\right\}\\ &= Ez_t^2\left\{\frac{1-b}{b}\beta^{'}(b)-\alpha_{22}^{'}\iota^{'}\right\}\left(\sum_{k=0}^{h-1}F^k\right)E(\mathbf{\Delta Y}_t\mathbf{\Delta Y}_t^{'})\left(\sum_{k=0}^{h-1}F^{'k}\right)\left\{\frac{1-b}{b}\beta(b)-\iota\alpha_{22}\right\},\end{aligned}$$

where the last equality is due to the fact that $E^2z_t=\beta^{'}(b)E(\mathbf{\Delta Y}_t\mathbf{\Delta Y}_t^{'})\beta(b)$ and $(\beta(b)\beta^{'}(b))^{-1}$ exists for $b<1$.

Now define:

$$A_{h,b} \equiv \left(\sum_{k=0}^{h-1} F^{'k}\right)\left\{\frac{1-b}{b}\beta(b) - \iota\alpha_{22}\right\}. \tag{A.3.5}$$

Then, we have:

$$\begin{aligned}
\{E(s_{t+h}-s_t)z_t\}^2 &= Ez_t^2 A_{h,b}^{'} E(\Delta\mathbf{Y}_t\Delta\mathbf{Y}_t^{'})A_{h,b} \\
A_{h,b} &\overset{b\to 1}{\to} \left(\sum_{k=0}^{h-1} F^{'k}\right)\left\{F^{'}(b)\iota\alpha_{21} - \iota\alpha_{22}\right\} \\
&= O(1).
\end{aligned}$$

Next, consider the denominator:

$$\begin{aligned}
E(s_{t+h}-s_t)^2 &= \left\{\frac{1-b}{b}\beta^{'}(b) - \alpha_{22}^{'}\iota^{'}\right\}\left(\sum_{k=0}^{h-1} F^{k}\right) E(\Delta\mathbf{Y}_t\Delta\mathbf{Y}_t^{'})\left(\sum_{k=0}^{h-1} F^{'k}\right)\left\{\frac{1-b}{b}\beta(b) - \iota\alpha_{22}\right\} + Ev_{t+h}^2 \\
&= A_{h,b}^{'} E(\Delta\mathbf{Y}_t\Delta\mathbf{Y}_t^{'})A_{h,b} + Ev_{t+h}^2 \\
Ev_{t+h}^2 &= \left\{\left(\frac{1-b}{b}\beta^{'}(b) - \alpha_{22}^{'}\iota^{'}\right)F_h^{'} + \alpha^{'}(b)\Gamma_h^{'}(b)\right\}\Omega\otimes I_{nh}\left\{F_h\left(\frac{1-b}{b}\beta(b) - \iota\alpha_{22}\right) + \Gamma_h(b)\alpha(b)\right\},
\end{aligned}$$

where $\otimes$ denotes the Kronecker product. Using the definitions of F_h and $\Gamma_h(b)$, and the fact that $\mathbf{Q} = \iota\Omega\iota^{'}$ and $\iota^{'}\iota = I_n$, we have:

$$\begin{aligned}
Ev_{t+h}^2 = \sum_{k=0}^{h-1} &\left\{\left(\frac{1-b}{b}\beta^{'}(b) - \alpha_{22}^{'}\iota^{'}\right)\left(\sum_{j=0}^{k-1} F^{j}\right) + \alpha^{'}(b)(I_n - \Phi(b))^{-1}\iota^{'}\right\}\mathbf{Q} \\
&\left\{\left(\sum_{j=0}^{k-1} F^{'j}\right)\left(\frac{1-b}{b}\beta(b) - \iota\alpha_{22}\right) + \iota(I_n - \Phi^{'}(b))^{-1}\alpha(b)\right\}.
\end{aligned}$$

Define:

$$B_{k,b} \equiv \left\{\left(\sum_{j=0}^{k-1} F^{'j}\right)\left(\frac{1-b}{b}\beta(b) - \iota\alpha_{22}\right) + \iota(I_n - \Phi^{'}(b))^{-1}\alpha(b)\right\}. \tag{A.3.6}$$

Then, we have:

$$E(s_{t+h}-s_t)^2 = A_{h,b}^{'} E(\Delta\mathbf{Y}_t\Delta\mathbf{Y}_t^{'})A_{h,b} + \sum_{k=0}^{h-1} B_{k,b}^{'}\mathbf{Q}B_{k,b},$$

and

$$\frac{1-b}{b}B_{k,b} \overset{b\to 1}{\to} \iota(I_n-\Phi^{'}(1))^{-1}\alpha_{21}=O(1)$$

Putting the numerator and denominator together, we have:

$$\frac{1}{R^2(h)}-1=\frac{E(s_{t+h}-s_t)^2}{\left\{E(s_{t+h}-s_t)z_t\right\}^2/Ez_t^2}-1=\frac{\sum_{k=0}^{h-1}B_{k,b}^{'}\mathbf{Q}B_{k,b}}{A_{h,b}^{'}E(\Delta\mathbf{Y}_t\Delta\mathbf{Y}_t^{'})A_{h,b}}$$

■

Proposition 2. *Consider the case specified in (14) and a fixed $b<1$. Under the setup in (6), (11), (12) and (15),*

(a) For a fixed $b<1$ and θ_h and ν_{t+h} as defined in the proof of this proposition,

$$s_{t+h}-s_t=\theta_h z_t-\alpha_{22}^{'}\iota^{'}\left(\sum_{k=0}^{h-1}F^k\right)\Delta\mathbf{Y}_t+\nu_{t+h}$$

where $(z_t,\Delta\mathbf{Y}_t)$ and ν_{t+h} do not correlate.

(b) Set $b=1$. For $np\times 1$ vectors C_h and $\{D_k\}_{k=0}^{h-1}$ defined in the proof,

$$\frac{1-R^2(h)}{R^2(h)}=\frac{\sum_{k=0}^{h-1}D_k^{'}\mathbf{Q}D_k}{C_h^{'}E(\Delta\mathbf{Y}_t\Delta\mathbf{Y}_t^{'})C_h}.$$

Further, if we define:

$$\begin{aligned}
\mathbf{P}_{t+k} &\equiv E_t(f_{2,t+k}+u_{2,t+k})\\
\Xi_{1,t+k} &\equiv f_{2,t+k}+u_{2,t+k}-E_t(f_{2,t+k}+u_{2,t+k})\\
\Xi_{2,t+k} &\equiv E_{t+k}\left(\sum_{j=0}^{\infty}\Delta f_{1,t+k+j}+f_{2,t+k+j}+u_{2,t+k+j}\right)-E_{t+k-1}\left(\sum_{j=0}^{\infty}\Delta f_{1,t+k+j}+f_{2,t+k+j}+u_{2,t+k+j}\right),
\end{aligned}$$

then,

$$\frac{1-R^2(h)}{R^2(h)}=\frac{Var\left(\sum_{k=0}^{h-1}(\Xi_{1,t+k}-\Xi_{2,t+k+1})\right)}{Var(\sum_{k=0}^{h-1}\mathbf{P}_{t+k})}.$$

Proof of 2(a).

The proof of Proposition 2(a) is similar to the proof of Proposition 1(a), but there are some critical differences. First, notice that using the definition of $\eta(b)$, we have:

$$z_t = \frac{-b^2}{1-b} f_{2t} + bu_{2t} + \eta^{'}(b) \sum_{j=1}^{\infty} b^j E_t \Delta \mathbf{X}_{t+j}. \tag{A.3.7}$$

Again using the fact that $E_t \Delta \mathbf{X}_{t+j} = \iota^{'} F^j \Delta \mathbf{Y}_t$, we have:

$$z_t = b\left(\alpha_{22}^{'} - \frac{b}{1-b}\alpha_{21}^{'}\right)\iota^{'}\Delta \mathbf{Y}_t + \eta^{'}(b)\iota^{'}\left(\sum_{j=1}^{\infty} b^j F^j\right)\Delta \mathbf{Y}_t = \omega^{'}(b)\Delta \mathbf{Y}_t,$$

where $\omega(b) \equiv b\iota(\alpha_{22} - \frac{b}{1-b}\alpha_{21}) + F^{'}(b)\iota\eta(b)$ and $\eta(b) \equiv \alpha_1 + b(\alpha_{21} + \alpha_{22})$. For $b < 1$, z_t is stationary because $\Delta \mathbf{Y}_t$ is stationary, implying cointegration between s_t and f_{1t}.

Using the same mechanics as the derivation for the one-step-ahead regression in Proposition 1(a), it can be shown that:

$$\Delta s_{t+1} = \frac{1-b}{b} z_t - u_{2t} + \eta^{'}(b)(I_n - \Phi(b))^{-1}\varepsilon_{t+1}.$$

Notice that the key difference between propositions 1(a) and 2(a) lies in the difference between $\alpha(b), \beta(b)$ and $\eta(b), \omega(b)$. By iterating the one-step-ahead regression ahead h times, we have:

$$s_{t+h} - s_t \quad = \quad \frac{1-b}{b} \sum_{k=0}^{h-1} z_{t+k} - \sum_{k=0}^{h-1} u_{2t+k} + \eta^{'}(b)\Gamma_h^{'}(b)\varepsilon_{t,h}. \tag{A.3.8}$$

Again, following the same mechanics as in Proposition 1(a), we have:

$$z_{t+k} = \omega^{'}(b) F^k (\omega(b)\omega^{'}(b))^{-1} \omega(b) z_t + \omega^{'}(b)\xi_{t+k}.$$

Plugging this into (A.3.8), we have:

$$\begin{aligned} s_{t+h} - s_t &= \frac{1-b}{b}\omega^{'}(b)\left(\sum_{k=0}^{h-1} F^k\right)\left(\omega(b)\omega^{'}(b)\right)^{-1}\omega^{'}(b) z_t + \frac{1-b}{b}\omega^{'}(b)\sum_{k=0}^{h-1}\xi_{t+k} - \alpha_{22}^{'}\iota^{'}\sum_{k=0}^{h-1}\left(F^k \Delta \mathbf{Y}_t + \xi_{t+k}\right) + \eta^{'}(b)\Gamma_h^{'}(b)\varepsilon_{t,h} \\ &\equiv \theta_h z_t - \alpha_{22}^{'}\iota^{'}\left(\sum_{k=0}^{h-1} F^k\right)\Delta \mathbf{Y}_t + \nu_{t+h}, \end{aligned}$$

where

$$
\begin{aligned}
\theta_h &\equiv \frac{1-b}{b}\omega^{'}(b)\left(\sum_{k=0}^{h-1}F^k\right)\left(\omega(b)\omega^{'}(b)\right)^{-1}\omega^{'}(b) \\
\nu_{t+h} &\equiv \left\{\left(\frac{1-b}{b}\omega^{'}(b)-\alpha_{22}^{'}\iota^{'}\right)F_h^{'}+\eta^{'}(b)\Gamma_h^{'}(b)\right\}\varepsilon_{t,h}.
\end{aligned}
$$

■

Proof of 2(b).

Using the same derivation as in Proposition 1(b), and the fact that $z_t = \omega^{'}(b)\Delta\mathbf{Y}_t$, we have:

$$
s_{t+h}-s_t = \left\{\frac{1-b}{b}\omega^{'}(b)-\alpha_{22}^{'}\iota^{'}\right\}\left(\sum_{k=0}^{h-1}F^k\right)\Delta\mathbf{Y}_t+\nu_{t+h}.
$$

As a result,

$$
\{E(s_{t+h}-s_t)z_t\}^2 = Ez_t^2\left\{\frac{1-b}{b}\omega^{'}(b)-\alpha_{22}^{'}\iota^{'}\right\}\left(\sum_{k=0}^{h-1}F^k\right)E(\Delta\mathbf{Y}_t\Delta\mathbf{Y}_t^{'})\left(\sum_{k=0}^{h-1}F^{'k}\right)\left\{\frac{1-b}{b}\omega(b)-\iota\alpha_{22}\right\},
$$

and

$$
\begin{aligned}
E(s_{t+h}-s_t)^2 &= \left\{\frac{1-b}{b}\omega^{'}(b)-\alpha_{22}^{'}\iota^{'}\right\}\left(\sum_{k=0}^{h-1}F^k\right)E(\Delta\mathbf{Y}_t\Delta\mathbf{Y}_t^{'})\left(\sum_{k=0}^{h-1}F^{'k}\right)\left\{\frac{1-b}{b}\omega(b)-\iota\alpha_{22}\right\}+E\nu_{t+h}^2 \\
E\nu_{t+h}^2 &= \sum_{k=0}^{h-1}\left\{\left(\frac{1-b}{b}\omega^{'}(b)-\alpha_{22}^{'}\iota^{'}\right)\left(\sum_{j=0}^{k-1}F^j\right)+\eta^{'}(b)(I_n-\Phi(b))^{-1}\iota^{'}\right\}\mathbf{Q} \\
&\quad \left\{\left(\sum_{j=0}^{k-1}F^{'j}\right)\left(\frac{1-b}{b}\omega(b)-\iota\alpha_{22}\right)+\iota(I_n-\Phi^{'}(b))^{-1}\eta(b)\right\}
\end{aligned}
$$

If $b=1$, $\frac{1-b}{b}\omega(b)|_{b=1} = -\iota\alpha_{21}$, and $\eta(b)|_{b=1} = \eta(1) = \alpha_1+\alpha_{21}+\alpha_{22}$. Therefore, if we define:

$$
C_h \equiv \left(\sum_{k=0}^{h-1}F^{'k}\right)\iota(\alpha_{21}+\alpha_{22}) \tag{A.3.9}
$$

$$
D_k \equiv \left(\sum_{j=0}^{k-1}F^{'j}\right)\iota(\alpha_{21}+\alpha_{22})+\iota(I_n-\Phi^{'}(b))^{-1}\eta(1), \tag{A.3.10}
$$

for $b=1$, we have:

$$\begin{aligned}
\frac{1}{R^2(h)}-1 &= \frac{E(s_{t+h}-s_t)^2}{\left\{E(s_{t+h}-s_t)z_t\right\}^2/Ez_t^2}-1 \\
&= \frac{\sum_{k=0}^{h-1} D_k^{'}\mathbf{Q}D_k}{C_h^{'}E(\Delta\mathbf{Y}_t\Delta\mathbf{Y}_t^{'})C_h}
\end{aligned}$$

as required. Inspecting the denominator of this expression, we see that:

$$\begin{aligned}
C_h^{'}E(\Delta\mathbf{Y}_t\Delta\mathbf{Y}_t^{'})C_h &= E(C_h^{'}\Delta\mathbf{Y}_t)^2 \\
&= Var\left((\alpha_{21}^{'}+\alpha_{22}^{'})\iota^{'}\left(\sum_{k=0}^{h-1}F^k\right)\Delta\mathbf{Y}_t\right) \\
&= Var\left((\alpha_{21}^{'}+\alpha_{22}^{'})\iota^{'}\sum_{k=0}^{h-1}E_t\Delta\mathbf{Y}_{t+k}\right) \\
&= Var\left(\sum_{k=0}^{h-1}E_t(f_{2,t+k}+u_{2,t+k})\right) \\
&= Var\left(\sum_{k=0}^{h-1}\mathbf{P}_{t+k}\right),
\end{aligned}$$

In particular, it converges to $Var\left((\alpha_{21}^{'}+\alpha_{22}^{'})\iota^{'}\left(I_{np}-F\right)^{-1}\Delta\mathbf{Y}_t\right)$ as $h\to\infty$ and therefore grows at a decreasing rate. The numerator can be expressed as:

$$\begin{aligned}
\sum_{k=0}^{h-1}D_k^{'}\mathbf{Q}D_k &= Var(\nu_{t+h}) \\
&= Var\left(\left(-(\alpha_{21}^{'}+\alpha_{22}^{'})\iota^{'}F_h^{'}+\eta^{'}(1)\Gamma^{'}(1)\right)\varepsilon_{t,h}\right) \\
&= Var\left(-(\alpha_{21}^{'}+\alpha_{22}^{'})\iota^{'}\sum_{k=0}^{h-1}\xi_{t+k}+\eta^{'}(1)(I_n-\Phi(b))^{-1}\sum_{k=1}^{h}\varepsilon_{t+k}\right).
\end{aligned}$$

To complete the proof, note that:

$$\begin{aligned}
(\alpha_{21}^{'}+\alpha_{22}^{'})\iota^{'}\xi_{t+k} &= (\alpha_{21}^{'}+\alpha_{22}^{'})\iota^{'}(\Delta\mathbf{Y}_{t+k}-E_t\Delta\mathbf{Y}_{t+k}) \\
&= (\alpha_{21}^{'}+\alpha_{22}^{'})(\Delta\mathbf{X}_{t+k}-E_t\Delta\mathbf{X}_{t+k}) \\
&\equiv \Xi_{1,t+k}.
\end{aligned}$$

By equation (A9) of Engel and West (2005) and the fact that $\eta(1) = \alpha_1 + \alpha_{21} + \alpha_{22}$, we have:

$$\begin{aligned}\eta^{'}(1)(I_n - \Phi(b))^{-1}\varepsilon_{t+k} &= \eta^{'}(1)\left(E_{t+k}\sum_{j=0}^{\infty}\Delta\mathbf{X}_{t+k+j} - E_{t+k-1}\sum_{j=0}^{\infty}\Delta\mathbf{X}_{t+k+j}\right) \\ &\equiv \Xi_{2,t+k}.\end{aligned}$$

Therefore, we have $\sum_{k=0}^{h-1} D_k^{'}\mathbf{Q}D_k = Var\left(\sum_{k=0}^{h-1}(\Xi_{1,t+k} - \Xi_{2,t+k+1})\right)$. Notice that the variance of $\Xi_{2,t+k+1}$ is the same for all k, and hence part of this numerator grows at a linear rate with h. ■

A.4 Details on Simulations

A.4.1 General Setup

As discussed in Section 4, the data vector $\Delta\mathbf{X}_t$ is set up according to different models.[26] For a given lag order p, a VAR(p) process on $\Delta\mathbf{X}_t$ can be estimated. The coefficients estimates, $\widehat{\Phi}(L)$ and error variance-covariance matrix $\widehat{\Omega}$ are obtained. This process is detailed in Appendix A.4.3.

The coefficient vectors α_1, α_{21} and α_{22} may contain coefficient estimates. For instance, in the Taylor Rule model, coefficients on cross-country output gap and inflation differences need to be estimated from the Taylor Rule relationship.

Let $Sim = 1,000,000$ and the superscript $*$ denote simulated variables. $n \times 1$ vectors of Gaussian errors ε_t^* are drawn for $t = 1, ..., Sim$. With that, pseudo-true fundamentals $\Delta\mathbf{X}_t^*$ are generated recursively using the equation:

$$(I_n - \widehat{\Phi}(L))\Delta\mathbf{X}_t^* = \widehat{\Omega}^{1/2}\varepsilon_t^*$$

for $t = 1, ..., Sim$ and $\widehat{\Omega}^{1/2}$ is obtained by Cholesky decomposition. To begin the recursion, we assume $\Delta\mathbf{X}_0^*, ..., \Delta\mathbf{X}_{-p}^* = 0$.

Next we generate z_t^*. Because all our simulation exercises falls under Case 2, using Proposition 2, we have:

$$\begin{aligned}z_t^* &= \widehat{\omega}^{'}(b)\Delta\mathbf{Y}_t^* \\ \widehat{\omega}(b) &= \left\{b\left(\alpha_{22}^{'} - \frac{1-b}{b}\alpha_{21}^{'}\right)\iota^{'} + (\alpha_1^{'} + b\alpha_{21}^{'} + b\alpha_{22}^{'})\iota^{'}\widehat{F}(b)\right\}\Delta\mathbf{Y}_t^*,\end{aligned}$$

[26] We subtract the sample means from the data so that $\Delta\mathbf{X}_t$ is mean-zero.

where $\widehat{F}(b)$ is an estimate of $F(b)$ by replacing $\Phi's$ with $\widehat{\Phi}'s$ and $\Delta\mathbf{Y}_t^* = [\Delta\mathbf{X}_t^{*'}, ..., \Delta\mathbf{X}_{t-p+1}^{*'}]'$.

Finally, following the one-horizon regression in the proof of Proposition 2, Δs_{t+1}^* for $t = 0, ..., Sim - 1$ is generated using:

$$\Delta s_{t+1}^* = \frac{1-b}{b} z_t^* - \alpha_{22}' \Delta\mathbf{X}_t^* + (\alpha_1' + b\alpha_{21}' + b\alpha_{22}')(I_n - \widehat{\Phi}(b))^{-1}\widehat{\Omega}^{1/2}\varepsilon_{t+1}^*.$$

Assuming $s_0^* = 0$, $s_t^*, t = 1, ..., Sim$ can be calculated recurisvely. We discard the first 1,000 observations of all generated variables to avoid start-up effects.

Then, using $Sim - 1000$ observations we run the long-horizon regressions (i.e. regressing $s_{t+h}^* - s_t^*$ on z_t^*) to obtain $R^2(h)$. Since Sim is already large, increases in Sim did not affect the results qualitatively.

A.4.2 Data Description

We collected quarterly data (1989Q4 to 2007Q2)[27] for 8 countries: US, Canada, Denmark, Germany, Japan, Norway, Switzerland, and UK. From the Consensus Forecast, we obtain the Uncovered Interest-rate Parity (UIP) risk premium between the US and other countries. Most remaining data are from the G10+ dataset provided by Haver Analytics. Our dataset includes:

- Uncovered Interest-rate Parity (UIP) risk premium
- Money supply (Seasonally adjusted M2 for all countries other than Japan, in which M2+CDs is used. Norway's M2 data are from International Financial Statistics (IFS) and are not seasonally adjusted. We seasonally adjusted the data using Eviews.)
- GDP (Chained real GDP for all countries. Data for Germany and Japan are calculated from nominal GDP and the GDP deflator, which we obtained from IFS.)
- CPI (In UK, we use HICP. The data for Canada and Japan are from OECD.)
- Short term interest rate (For Canada and US: 3-month treasury bill rate; for Denmark, Germany, Norway, Switzerland, and UK: 3-month interbank offer rate; for Japan: 3-month Certificate of Deposit (Gensaki) Rate. Japan's data are obtained from OECD).
- Short term interest rate targeted by the central bank (for Canada: Overnight Money Market Financing Rate (Effective); for Denmark: National Bank Discount Rate; for Germany: Base Rate; for Japan:

[27] The data for Denmark is restricted to 1990Q1 and 2007Q2 due to missing GDP data.

Overnight Call Rate (Uncollateralized); for Norway: 3-month Interbank Offer Rate; for Switzerland: 3-month Interbank Offer Rate; for United Kingdom: Base Rate; and for US: Federal Funds (effective) Rate.)

- Exchange rates (All exchange rates are in the form of units of foreign currency per US dollar.)

A.4.3 Estimating VAR(p)

A.4.3.1 Monetary Model

Fundamentals in the Monetary model include (log) money supply ($m_t, \quad m_t^*$), (log) output ($y_t, \quad y_t^*$), the (log) real exchange rate (q_t), a money demand shock ($v_t, \quad v_t^*$), and risk premium (ρ_t).[28] We use the US as the base country and denote its variables with an asterisk. For each country, real money supply ($m_t - p_t$) is regrssed on total income (y_t) and the short-term interest rate (i_t). We cannot reject the unit root hypothesis for m_t, p_t, y_t, and i_t and the null hypothesis that these variables are not cointegrated at a 5% significance level for most countries. Therefore, we proceed to take the first difference for all variables in our regression. The regression residuals are recovered as Δv_t and Δv_t^*. The real exchange rate is calculated from $q_t = s_t + p_t^* - p_t$, where s_t is the (log) nominal exchange rate, which is in the form of units of national currency per US dollar.

Under the Augmented Dickey-Fuller Unit Root test, we generally cannot reject the null hypothesis at a 95% confidence level in all countries for m_t, m_t^*, y_t, y_t^*, q_t, v_t, and v_t^*. Unit roots are strongly rejected for ρ_t in all countries.[29] Following the definition in the paper, we have:

$$\Delta \mathbf{X_t} \equiv \begin{bmatrix} \Delta m_t & \Delta m_t^* & \Delta y_t & \Delta y_t^* & \Delta q_t & \Delta v_t & \Delta v_t^* & \rho_t \end{bmatrix}. \tag{A.4.11}$$

We estimate VAR(p) coefficients for ΔX_t and the covariance matrix of residuals with $p = 1, 2, 3, 4$. The estimated coefficients and covariance matrix are used for simulations.

In an alternative setup, we regress $\Delta[(m_t - p_t) - (m_t^* - p_t^*)]$ on $\Delta(y_t - y_t^*)$ and $(i_t - i_t^*)$. The residuals are recovered as $\Delta(v_t - v_t^*)$. The matrix $\Delta \mathbf{X_t}$ contains the following variables:

$$\Delta \mathbf{X_t} = \begin{bmatrix} \Delta(m_t - m_t^*) & \Delta(y_t - y_t^*) & \Delta q_t & \Delta(v_t - v_t^*) & \rho_t \end{bmatrix}. \tag{A.4.12}$$

[28] If we assume PPP holds, the real exchange rate is zero.

[29] This result is true whether or not we include a time trend in our test, and for various lags (from 4 to 8) in a VAR(p).

A.4.3.2 Taylor Rule Model

The economic variables in the Taylor Rule model include: the (log) aggregate price (p_t), the output gap (y_t^g), CPI inflation (π_t), the monetary policy shock in the Taylor Rule (v_t), and the risk premium in the UIP condition (ρ_t). Corresponding variables in the foreign country are denoted with asterisks. As discussed in the paper, CPI is I(1) and all other variables are I(0). As a result, we define:

$$\mathbf{\Delta X_t} = \begin{bmatrix} \Delta p_t & \Delta p_t^* & y_t^g & y_t^{g*} & \pi_t & \pi_t^* & v_t & v_t^* & \rho_t \end{bmatrix}. \tag{A.4.13}$$

Note that Δp_t (Δp_t^*) and π_t (π_t^*) are collinear. We drop Δp_t and Δp_t^* from $\mathbf{\Delta X_t}$ when estimating VAR(p) coefficients and variance matrixes.

We also tried an alternative setup in which the matrix $\mathbf{\Delta X_t}$ is defined as:

$$\mathbf{\Delta X_t} = \begin{bmatrix} y_t^g - y_t^{g*} & \pi_t - \pi_t^* & v_t - v_t^* & \rho_t \end{bmatrix}. \tag{A.4.14}$$

Note that we drop $\Delta(p_t - p_t^*)$ from $\mathbf{\Delta X_t}$ because it is collinear with $\pi_t - \pi_t^*$.