Technical Appendix: Home Bias, Exchange Rate Disconnect, and Optimal Exchange Rate Policy*

Jian Wang†
Federal Reserve Bank of Dallas

March 31, 2009

Abstract

This paper examines how much the central bank should adjust the interest rate in response to real exchange rate fluctuations. The paper first demonstrates, in a two-country Dynamic Stochastic General Equilibrium (DSGE) model, that home bias in consumption is important to replicate the exchange rate volatility and exchange rate disconnect documented in the data. When home bias is high, the shock to Uncovered Interest-rate Parity (UIP) can substantially drive up exchange rate volatility while leaving the volatility of real macroeconomic variables, such as GDP, almost untouched. The model predicts that the volatility of the real exchange rate relative to that of GDP increases with the extent of home bias. This relation is supported by the data. A second-order accurate solution method is employed to find the optimal operational monetary policy rule. Our model suggests that the monetary authority should not seek to vigorously stabilize exchange rate fluctuations. In particular, when the central bank does not take a strong stance against the inflation rate, exchange rate stabilization may induce substantial welfare loss. The model does not detect welfare gain from international monetary cooperation, which extends Obstfeld and Rogoff's (2002) findings to a DSGE model.

JEL Classifications: E52, F31, F41
Keywords: Home bias, Exchange rate volatility, Exchange rate disconnect, Optimal monetary policy

*I am grateful to Charles Engel for his priceless advice and encouragement. I would like to thank Menzie Chinn, Vasco Curdiá, Kevin Grier, Bruce Hansen, Ben Keen, Robert Kollmann, Nelson Mark, Yongseok Shin, John Taylor, Ken West, and many seminar and conference participants for discussions and comments. I also want to thank the co-editor Michael Melvin and an anonymous referee for close reading and constructive criticism. Nicole Ball provided excellent research assistance. All views are those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

†Email: jian.wang@dal.frb.org Office address: Research Department, Federal Reserve Bank of Dallas, 2200 N. Pearl Street, Dallas, TX 75201 Phone: (214) 922-6471.

APPENDIX

1 Transformation of Equations

Some of our equations, for instance equation (??), are not in the format of first order difference as in (??). This can be solved by introducing two new variables. Let P_t^n be the numerator of $P_{t,t}$ and P_t^d be the denominator. By definition,

$$P_t^n \equiv \sum_{k=0}^{\infty} (\pi^{-\theta}\lambda_f)^k E_t \left[\Gamma_{t,t+k} mc_{t+k} P_{t+k}^{\theta-1} Y_{t+k}\right]$$
$$P_t^d \equiv \sum_{k=0}^{\infty} (\pi^{1-\theta}\lambda_f)^k E_t \left[\Gamma_{t,t+k} P_{t+k}^{\theta-1} Y_{t+k}\right] \tag{1}$$

From equation (1), we can write P_t^n and P_t^d into first order difference equations by using the law of iterated expectations and the pricing kennel's property that $\Gamma_{t,t+s} = \Gamma_{t,t+1}\Gamma_{t+1,t+s}$

$$P_t^n = mc_t P_t^{\theta-1} Y_t + \lambda_f \pi^{-\theta} E_t \left[\Gamma_{t,t+1} P_{t+1}^n\right] \tag{2}$$

$$P_t^d = P_t^{\theta-1} Y_t + \lambda_f \pi^{1-\theta} E_t \left[\Gamma_{t,t+1} P_{t+1}^d\right]. \tag{3}$$

By the same token, we can also convert equation (??) in the intermediate goods market into first order difference equations.

2 Stationary Economy

We make our model stationary by dividing all equations by final or intermediate goods price index. In this section, we use the final goods market as our example to explain how we do it and also list here all other equilibrium conditions defining our stationary economy.

2.1 Final Goods Sector

For home country, there are 8 aggregate level equations in the final goods market. For reader's convenience, we reproduce them here

$$mc_t = \left[\alpha P_{Ht}^{1-\gamma} + (1-\alpha)(S_t P_{Ft}^*)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{1}$$

$$P_{t,t} = \frac{P_t^n}{P_t^d} \tag{2}$$

$$P_t^n = mc_t P_t^{\theta-1} Y_t + \lambda_f \pi^{-\theta} E_t \left[\Gamma_{t,t+1} P_{t+1}^n\right] \tag{3}$$

$$P_t^d = P_t^{\theta-1} Y_t + \lambda_f \pi^{1-\theta} E_t \left[\Gamma_{t,t+1} P_{t+1}^d\right] \tag{4}$$

$$P_t^{1-\theta} = (1-\lambda_f) P_{tt}^{1-\theta} + \lambda_f (\pi P_{t-1})^{1-\theta} \tag{5}$$

$$Y_{Ht} = \alpha \left(\frac{P_{Ht}}{mc_t}\right)^{-\gamma} \left(\frac{P_t^A}{P_t}\right)^{-\theta} Y_t \tag{6}$$

$$Y_{Ft} = (1-\alpha) \left(\frac{S_t P_{Ft}}{mc_t}\right)^{-\gamma} \left(\frac{P_t^A}{P_t}\right)^{-\theta} Y_t \tag{7}$$

$$\left(P_t^A\right)^{-\theta} = (1-\lambda_f) P_{tt}^{-\theta} + \lambda_f \left(P_{t-1}^A \pi\right)^{-\theta}. \tag{8}$$

We divide each nominal variable in the above equations by the final goods price index P_t or a function of P_t. We use a hat to denote the corresponding stationary nominal variables. Let $\hat{mc}_t = \frac{mc_t}{P_t}$, $\hat{P}_{Ht} = \frac{P_{Ht}}{P_t}$, $\hat{P}_{Ft}^* = \frac{P_{Ft}^*}{P_t^*}$, $\hat{P}_t^A = \frac{P_t^A}{P_t}$, $\hat{S}_t = S_t P_t^* / P_t$, $\hat{P}_t^n = \frac{P_t^n}{P_t^\theta}$, and $\hat{P}_t^d = \frac{P_t^d}{P_t^{\theta-1}}$. After this transformation, it is easy for us to find the following equations with the stationary nominal variables

$$\hat{mc}_t = \left[\alpha \hat{P}_{Ht}^{1-\gamma} + (1-\alpha)(\hat{S}_t \hat{P}_{Ft}^*)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{9}$$

$$1 = (1-\lambda_f) \left(\frac{\hat{P}_t^n}{\hat{P}_t^d}\right)^{1-\theta} + \lambda_f \left(\frac{\pi}{\pi_t}\right)^{1-\theta} \tag{10}$$

$$\hat{P}_t^n = \theta \hat{mc}_t Y_t + \lambda_f \pi^{-\theta} E_t \left[\Gamma_{t,t+1} \hat{P}_{t+1}^n \pi_{t+1}^\theta\right] \tag{11}$$

$$\hat{P}_t^d = (\theta-1) Y_t + \lambda_f \pi^{1-\theta} E_t \left[\Gamma_{t,t+1} \hat{P}_{t+1}^d \pi_{t+1}^{\theta-1}\right] \tag{12}$$

$$Y_{Ht} = \alpha \left(\frac{\hat{P}_{Ht}}{\hat{mc}_t}\right)^{-\gamma} \left(\hat{P}_t^A\right)^{-\theta} Y_t \tag{13}$$

$$\left(\hat{P}_t^A\right)^{-\theta} = (1-\lambda_f) \left(\frac{\hat{P}_t^n}{\hat{P}_t^d}\right)^{-\theta} + \lambda_f \left(\hat{P}_{t-1}^A \frac{\pi}{\pi_t}\right)^{-\theta} \tag{14}$$

$$Y_{Ft} = (1-\alpha) \left(\frac{\hat{S}_t \hat{P}_{Ft}^*}{\hat{mc}_t}\right)^{-\gamma} \left(\hat{P}_t^A\right)^{-\theta} Y_t. \tag{15}$$

There are 7 symmetric equations in the foreign country. π_t is the inflation of final good price index (CPI). We eliminate the final goods price index from our system by introducing this new variable. In this section, we have 14 equations.

2.2 Intermediate Goods Sector

Similarly, we can transform the equations in intermediate goods market by dividing intermediate goods price index (PPI) or its function (The marginal cost function is still divided by P_t (CPI).). $\hat{mc}_t^{int} = \frac{mc_t^{int}}{P_t}$, $\hat{R}_t = \frac{R_t}{P_t}$, $\hat{W}_t = \frac{W_t}{P_t}$, $\hat{P}_{Ht}^n = \frac{P_{Ht}^n}{P_{Ht}^\theta}$, $\hat{P}_{Ht}^d = \frac{P_{Ht}^d}{P_{Ht}^{\theta-1}}$, $\hat{P}_{Ht}^A = \frac{P_{Ht}^A}{P_{Ht}}$ and $\pi_{Ht} = P_{Ht}/P_{Ht-1}$ is the inflation of PPI in home country.

The stationary economy is described by the following equilibrium conditions

$$\hat{mc}_t^{int} = A_t^{-1}\hat{R}_t^{\psi}\hat{W}_t^{1-\psi}\psi^{-\psi}(1-\psi)^{\psi-1} \tag{16}$$

$$\hat{P}_{Ht}^n = \phi\hat{mc}_t^{int}\left(Y_{Ht}+Y_{Ht}^*\right)/\hat{P}_{Ht} + \lambda_{int}\pi^{-\phi}E_t\left[\Gamma_{t,t+1}\hat{P}_{Ht+1}^n\pi_{Ht+1}^{\phi}\right] \tag{17}$$

$$\hat{P}_{Ht}^d = (\phi-1)\left(Y_{Ht}+Y_{Ht}^*\right) + \lambda_{int}\pi^{1-\phi}E_t\left[\Gamma_{t,t+1}\hat{P}_{Ht+1}^d\pi_{Ht+1}^{\phi-1}\right] \tag{18}$$

$$1 = (1-\lambda_{int})\left(\frac{\hat{P_{Ht}^n}}{\hat{P_{Ht}^d}}\right)^{1-\phi} + \lambda_{int}\left(\frac{\pi}{\pi_{Ht}}\right)^{1-\phi} \tag{19}$$

$$L_t = A_t^{-1}\left(\frac{1-\psi}{\psi}\right)^{\psi}\left(P_{Ht}^A\right)^{-\phi}\left(Y_{Ht}+Y_{Ht}^*\right) \tag{20}$$

$$\left(\hat{P}_{Ht}^A\right)^{-\phi} = (1-\lambda_{int})\left(\frac{\hat{P}_{Ht}^n}{\hat{P}_{Ht}^d}\right)^{-\phi} + \lambda_{int}\left(\hat{P}_{Ht-1}^A\frac{\pi}{\pi_{Ht}}\right)^{-\phi} \tag{21}$$

$$K_t = A_t^{-1}\left(\frac{(1-\psi)\hat{R}_t}{\psi\hat{W}_t}\right)^{\psi-1}\left(\hat{P}_{Ht}^A\right)^{-\phi}\left(Y_{Ht}+Y_{Ht}^*\right). \tag{22}$$

We introduced a new variable in this section: intermediate good price inflation (PPI). This inflation is defined equation (23)

$$\pi_{Ht} = \pi_t\hat{P}_{Ht}/\hat{P}_{Ht-1}. \tag{23}$$

There are 8 symmetric equations in the foreign country. So in this section, we have 16 equations.

2.3 Household and Monetary Rules

In this section, all nominal variables are divided by the final goods price index (CPI). The equilibrium conditions can be written in the following stationary form

$$K_{t+1} = (1-\delta)K_t + I_t \tag{24}$$

$$C_t + \hat{B}_{Ht+1} + \hat{S}_t\hat{B}_{Ft+1} + I_t + \frac{1}{2}\Phi\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + \frac{1}{2}\phi_H\hat{B}_{Ht+1}^2 + \frac{1}{2}\phi_F\left(\hat{S}_t\hat{B}_{Ft+1}\right)^2$$
$$= \hat{W}_tL_t + \hat{R}_tK_t + \frac{\hat{B}_{Ht}(1+i_{t-1})}{\pi_t} + \frac{\hat{S}_t\hat{B}_{Ft}(1+i^*_{t-1})}{\pi^*_t} + \hat{\pi}_t^{int} + \hat{\pi}_t^f \tag{25}$$

$$\rho = \hat{W}_t/C_t \tag{26}$$

$$\frac{1}{C_t}\left[1+\Phi\left(\frac{I_t}{K_t}-\delta\right)\right] - E_t\left\{\beta\frac{1}{C_{t+1}}\left(\hat{R}_{t+1} + \frac{1}{2}\Phi\left[\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 - \delta^2\right]\right)\right\} -$$
$$E_t\left\{\beta\frac{1}{C_{t+1}}(1-\delta)\left(1+\Phi\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\right)\right\} = 0 \tag{27}$$

$$1 + \phi_H\hat{B}_{Ht+1} = \beta E_t\left[\frac{C_t}{C_{t+1}}\frac{(1+i_t)}{\pi_{t+1}}\right] \tag{28}$$

$$1 + \phi_F\hat{S}_t\hat{B}_{Ft+1} = \beta E_t\left[\frac{C_t}{C_{t+1}}\frac{\hat{S}_{t+1}(1+i^*_t)}{\hat{S}_t\pi^*_{t+1}}\right] \tag{29}$$

$$\hat{B}_{Ht} + \hat{B}^*_{Ht} = 0 \tag{30}$$

$$C_t + \hat{B}_{Ht+1} + \hat{S}_t\hat{B}_{Ft+1} + I_t + \frac{1}{2}\Phi\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + \frac{1}{2}\phi_H\hat{B}_{Ht+1}^2 + \frac{1}{2}\phi_F\left(\hat{S}_t\hat{B}_{Ft+1}\right)^2 = Y_t \tag{31}$$

$$i_t = i + \Xi_\pi log\left(\frac{\pi_t}{\pi}\right) + \Xi_y log\left(\frac{Y_{Ht}+Y^*_{Ht}}{Y_H + YH^*}\right) + \Xi_s log\left(\frac{S_t}{S}\right). \tag{32}$$

Similarly, there are 9 symmetric equations in the foreign country. Putting these three sections together, we have 47 endogenous variables and 48 equations. After considering the exogenous technology shocks A_t and A^*_t, totally we have 49 variables and 50 equations. By Walras' Law, one market clearing condition is redundant.

3 Steady State

In this section, we solve for a symmetric steady state of the model. Under the symmetry, the asset holdings are zero for both countries, i.e. $\hat{B}_H = \hat{B}^*_H = \hat{B}_F = \hat{B}^*_F = 0$. Here we use the variables without time script to denote steady state values. It is pretty easy for us to find the steady state values for the following variables by simply dropping time script in the above equilibrium conditions

$$\hat{mc} = \hat{mc}^* = \frac{\theta - 1}{\theta}$$

$$\hat{P}^A = \hat{P}^{A*} = 1$$

$$\hat{P}_H^A = \hat{P}_F^{A*} = 1$$

$$\hat{R} = \hat{R}^* = 1/\beta - 1 + \delta$$

$$i = i^* = \pi/\beta - 1$$

$$\pi_H = \pi_F^* = \pi$$

$$A = A^* = 1.$$

The other variables are jointly determined by the remaining 29 equations. We use Matlab to solve the steady state numerically and find that solution is symmetric. So we can find the analytical solution by imposing symmetry from the beginning

$$\hat{mc} = \hat{mc}^* = \hat{P}_H = \hat{P}_F^* = \frac{\theta - 1}{\theta} \tag{1}$$

$$\hat{mc}^{int} = \hat{mc}^{int*} = \frac{(\phi - 1)(\theta - 1)}{\phi\theta} \tag{2}$$

$$\hat{W} = \hat{W}^* = \left(\left(\frac{1-\psi}{\psi} \right)^{1-\psi} \hat{R}^{-\psi} \psi \Theta \right)^{\frac{1}{1-\psi}} \tag{3}$$

where $\Theta = \frac{\theta\phi + 1 - \theta - \phi}{\theta\phi}$.

$$C = C^* = \hat{W}/\rho \tag{4}$$

$$Y = Y^* = \frac{C}{1 - \Theta\delta\psi/\hat{R}} \tag{5}$$

$$K = K^* = (Y - C)/\delta \tag{6}$$

$$L = L^* = \frac{1-\psi}{\psi} \frac{\hat{R}K}{\hat{W}} \tag{7}$$

$$Y_H = Y_F^* = \alpha Y \tag{8}$$

$$Y_H^* = Y_F = (1 - \alpha)Y \tag{9}$$

$$\hat{S} = 1 \tag{10}$$

$$\hat{P}^n = \hat{P}^{n*} = \frac{\theta \hat{mc} Y}{1 - \beta\lambda_f} \tag{11}$$

$$\hat{P}^d = \hat{P}^{d*} = \frac{(\theta - 1)Y}{1 - \beta\lambda_f} \tag{12}$$

$$\hat{P}_H^n = \hat{P}_F^{n*} = \frac{\phi \hat{mc}^{int} (Y_H + Y_H^*)}{(1 - \beta\lambda_{int})\hat{P}_H} \tag{13}$$

$$\hat{P}_H^d = \hat{P}_F^{d*} = \frac{(\phi - 1)\left(Y_H + Y_H^*\right)}{1 - \beta \lambda_{int}}. \tag{14}$$

This solution has been verified by the numerical solution from our Matlab programs.

4 Comparison of Inflation Targeting Regimes

In this section, we compare the CPI inflation targeting with other inflation targeting regimes. We consider 3 alternative inflation targeting regimes: (1) both countries target PPI inflation; (2) home country targets CPI while foreign country targets PPI; (3) both countries target both CPI and PPI inflations. Table (**??**) gives the comparison of CPI inflation targeting with PPI and asymmetric inflation targeting. When the central banks take strong stance against inflation rate, there is almost no difference in welfare level among these regimes. In all regimes, the welfare level is not sensitive to foreign country's policy mistake. Our results do not show strong support for the central banks to switch to PPI inflation targeting regime.

Now we turn to the case where the central banks target both CPI and PPI inflation rates

$$i_t = i + \Xi_{cpi} log\left(\frac{\pi_t^{cpi}}{\pi}\right) + \Xi_{ppi} log\left(\frac{\pi_t^{ppi}}{\pi}\right). \tag{1}$$

π_t^{cpi} and π_t^{ppi} are respectively CPI and PPI inflation rate at time t. Ξ_{cpi} and Ξ_{ppi} are weights that the central banks put on each inflation rate in their monetary policy. As before, we did a grid search over [0, 3] in steps of 0.1 for both Ξ_{cpi} and Ξ_{ppi}. Compared to the regime targeting single inflation rate, targeting both inflation rates has a larger parameter area that induces deterministic equilibrium. We find that a deterministic equilibrium exists whenever the sum of Ξ_{cpi} and Ξ_{ppi} is greater than unity.

However, the double-inflation-targeting only generates negligible welfare gains in our model. In comparison with CPI inflation targeting, the welfare gain is only about 0.014 percent.[1] Therefore in our paper, we only consider the CPI inflation targeting while discussing exchange rate policy.

[1] The maximum welfare level is -53.833 at the point Ξ_{cpi}=3 and Ξ_{ppi}=1.8.