Appendix (not for publication)

A.1 Data

The ASIP dataset covers all state-owned manufacturing firms and private manufacturing firms with sales greater than 5 million RMB (approximately 600,000 dollars at the exchange rate of 2000) between 2000 and 2007. On average, there are 120,000 firm-level observations each year. The firm-level data include some basic firm information such as firm identification number, registration type, start year, operating status and total employment. In addition, the dataset contains detailed information about each firm's balance sheet and income statement. The balance sheet data report detailed information about assets and liabilities such as total assets, fixed assets, current assets, long-run investment, total liabilities, total equities and capital. Capital information include disaggregate-level information about the ownership of capital (e.g., government collective, corporate, special districts, foreign). So we can use such information to calculate the FDI share of each firm, which is measured by the share of capital from Hong Kong, Macau, Taiwan and foreign countries.

The data of income statement include each firm's total sales, total industry production, value added, export volume, income from main product, cost from main product, financing cost, interest cost, tax, wage, employee benefit, total intermediate input, total profit, etc. The above data are used to calculate the productivity of each firm. We will describe the method of calculating firm productivity shortly.

The dataset contains the location information of the firm that enables us to find out if it is in a special economic development zone. A 4-digit Chinese industry code is also provided for each firm, which is used to match firm with sector-level financial vulnerability measures.

We obtain the following industry-level and province-level data from China Statistic Yearbook: : industry PPI and province-level variables (GDP, GDP per capital, retail sale, transportation, investment, R&D, import and export).

A.1.1 Firm Productivity

Firm productivity is calculated following Ackerberg et al. (2015) and re-scaled around industry productivity mean and divided by industry productivity standard deviation. The method of Ackerberg et al. (2015) uses the ideas in Olley and Pakes (1996) and Levinsohn and Petrin (2003) to identify firm's productivity, but does not suffer from the collinearity problems in the literature. Examples of using this method include Alfaro et al. (2013) and De Loecker and Warzynski (2012).

Consider the following production function for firm i in a given industry:

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \varepsilon_{it}, \tag{A.1.1}$$

where y_{it} is the log of output, k_{it} is the log of capital input and l_{it} is the log of labor input. These variables are observable to the econometrician. ω_{it} is the productivity shock that is observable to the firm, but unobservable to the econometrician. ε_{it} is the error term that is not predictable to the firm. OLS cannot be used to estimate equation (A.1.1) if the choice of k_{it} or l_{it} is a function of ω_{it}, which is likely to be true in reality. We follow Ackerberg et al. (2015) to solve this endogeneity issue.

First assume ω_{it} follow an exogenous first-order Markov process:

$$p(\omega_{it+1}|I_t) = p(\omega_{it+1}|\omega_t), \tag{A.1.2}$$

where I_t is firm i's information set at time t. It is further assumed that firm's intermediate input is determined after its choices of labor and capital input and the realization of ω_{it}. Suppose the demand for intermediate input takes the form of:

$$m_{it} = f_t(\omega_{it}, k_{it}, l_{it}). \tag{A.1.3}$$

It is assumed that f_t is monotonic in ω_{it}. Therefore, we can invert the input demand function to get ω_{it}:

$$\omega_{it} = f_t^{-1}(m_{it}, k_{it}, l_{it}). \tag{A.1.4}$$

Substitute equation (A.1.4) to (A.1.1), we have:

$$\begin{aligned} y_{it} &= \beta_k k_{it} + \beta_l l_{it} + f_t^{-1}(m_{it}, k_{it}, l_{it}) + \varepsilon_{it} \\ &= \Phi_t(m_{it}, k_{it}, l_{it}) + \varepsilon_{it}, \end{aligned}$$

where $\Phi_t(m_{it}, k_{it}, l_{it}) \equiv \beta_k k_{it} + \beta_l l_{it} + f_t^{-1}(m_{it}, k_{it}, l_{it})$. We employ a second-order approximation for $f_t^{-1}(m_{it}, k_{it}, l_{it})$. So the estimate of $\Phi_t(m_{it}, k_{it}, l_{it})$, $\widehat{\Phi}_t(m_{it}, k_{it}, l_{it})$, is obtained by regressing y_{it} on m_{it}, k_{it}, l_{it} and their second-order terms.[1]

Next, two moment conditions are employed to estimate β_k and β_l:

$$E\left[\xi_{it}\begin{pmatrix} k_{it} \\ l_{it} \end{pmatrix}\right] = 0, \tag{A.1.5}$$

where $\xi_{it} = \omega_t - E[\omega_t|\omega_{t-1}]$ is the innovation in ω_t. These two moment conditions are from the assumption that capital and labor inputs are chosen before the realization of ω_t.

To be specific, for given $\widehat{\beta}_k$ and $\widehat{\beta}_l$, we have:

$$\widehat{\omega}_{it} = \widehat{\Phi}_t(m_{it}, k_{it}, l_{it}) - \widehat{\beta}_k k_{it} - \widehat{\beta}_l l_{it}. \tag{A.1.6}$$

Then $\widehat{\xi}_{it}$ is obtained with an third-order approximation by regressing $\widehat{\omega}_{it}$ on $\widehat{\omega}_{it-1}$, $\widehat{\omega}^2_{it-1}$ and $\widehat{\omega}^3_{it-1}$. In the estimation, $\widehat{\beta}_k$ and $\widehat{\beta}_l$ are selected to minimize the sample analogue to the moment conditions in equation (A.1.5):

$$\min_{\widehat{\beta}_k, \widehat{\beta}_l} \Lambda = \frac{1}{T}\frac{1}{N}\sum_{t=1}^{T}\sum_{i=1}^{N}\widehat{\xi}_{it}(\widehat{\beta}_k, \widehat{\beta}_l)\begin{pmatrix} k_{it} \\ l_{it} \end{pmatrix}, \tag{A.1.7}$$

where T is the number of sample periods and N is the number of firms in the industry.

[1] Cross terms of these variables are also included in the regression.

In our exercise, we first group firms according to China's 2-digit industry code. For each industry, we follow the above procedure to estimate firm's productivity during the period 2000-2007 ($T = 8$). In this way, we allow β_k and β_l to vary across different industries, but remain constant over time.

In our estimation, k_{it} is measured by fixed capital reported in firm's balance sheet, l_{it} is measure by the total number of employees and m_{it} is measured by intermediate input reported in firm's income statement. Both fixed capital and intermediate input are deflated by industry-level PPI obtained from China Statistic Yearbook.

Given the estimated $\widehat{\beta}_k$ and $\widehat{\beta}_l$ from equation (A.1.7), we can calculate firm i's productivity in year t, $\widehat{\omega}_{it}$, from equation (A.1.6). Then $\widehat{\omega}_{it}$ is normalized around the industrial mean:

$$\widetilde{\omega}_{it} = \frac{\widehat{\omega}_{it} - \mu_t}{\sigma_t}, \tag{A.1.8}$$

where μ_t is the industrial mean of $\widehat{\omega}_{it}$ and σ_t is the standard deviation of $\widehat{\omega}_{it}$. $\widetilde{\omega}_{it}$ is our final measure of firm i's productivity in all our empirical exercises.

A.1.2 Financial vulnerability

We employ five measures for financial vulnerability at the sector level, following Manova et al. (2015). These five measures are described in Table 1 and are calculated from data on all publicly traded U.S.-based firms.[2] The use of the U.S. data ensures that the financial vulnerability measures are not endogenously determined by China's level of financial development. Indeed, these measures are intended to capture features inherent to the nature of the manufacturing process, which remain the same across countries and are beyond the control of individual firms. Consistent with this argument, the measures display more cross-sector variations than cross-firm variations within a sector. Each financial vulnerability variable

[2]The raw data on U.S. firms are obtained from Compustat's annual industrial files.

is measured by the median among all firms in the sector and are available for 3-digit ISIC sectors. We will describe later how to match these 3-digit ISIC data with the 4-digit Chinese industry code in our dataset.

The first three measures use firm's dependence on external finance in a sector as proxy for the sector's liquidity constraint. The first measure is the share of capital expenditure that is not financed by operation cash flow, which we refer to as external finance dependence. The other two measures are the share of R&D in total sales and the share of inventory to sales, which we refer to as the inventory ratio and R&D ratio. Capital expenditure, R&D investment and inventory are important up-front costs and may reflect a firm's liquidity constraint. While companies in all industries may have to pay fixed costs and face liquidity constraints, the relative importance of such costs varies systematically across sectors. The above three measure can hopefully captures the systematical differences across sectors.

The fourth measure considers other sources of external finance that are in the form of trade credit. If a firm has access to buyer or seller trade credit, it is less dependent on the formal financial market and hence less financially constrained. This financial vulnerability variable is measured by the ratio of the change in account payable to the change in total asset.

The last measure of financial vulnerability, asset tangibility, captures firm's ability to raise external finance. Tangible assets can usually serve as collateral for external finance. Therefore, firms with a higher share of tangible assets (defined as the ratio of net plant, property and equipment to total book value assets) are less financially constrained.

Following Manova et al. (2015) and other studies in the literature, we obtain the external finance, inventory ratio, R&D ratio and asset tangibility from Kroszner et al. (2007), who follow the methodology of Rajan and Zingales (1998) and Claessens and Laeven (2003). They are averages over the 1980-1999 period for the median U.S. firms in each sector. Trade credit measure is obtained from Fisman and Love (2003), who calculate from the same data for 1980-1989.

The five measures are not highly correlated indicating that they capture conceptionally different dimensions of financial vulnerability. Following Manova et al. (2015), we calculate the first principal component (FPC) of the five indicators and use it as our preferred proxy for sector's financial vulnerability. Manova et al. (2015) argue that FPC provides a cleaner index of financial vulnerability than each individual measure because the individual measures might be correlated with industrial characteristics unrelated to financial frictions. The FPC index has a positive loading on external finance, the inventory ratio, and the R&D ratio, but a negative loading on asset tangibility and trade credit. This is the consistent with the intuitions we discussed above. In the end, FPC accounts for 45.9% of variance for all five measures.

Table A.1: The Elasticity of Productivity w.r.t.FDI for new firms (age≤ 2) in quantile regressions

	Low financial vulnerability			High financial vulnerability		
Quantile (%)	Coef.	s.e.	Psudo-R^2	Coef.	s.e.	Psudo-R^2
5	$-0{\cdot}043$	0·040	0·185	$-0{\cdot}181^{***}$	0·023	0·100
10	$-0{\cdot}007$	26·000	0·197	$-0{\cdot}122^{***}$	0·023	0·100
15	0·029	0·022	0·199	$-0{\cdot}076^{***}$	0·026	0·099
20	0·015	0·018	0·195	$-0{\cdot}043^{**}$	0·017	0·098
25	0·005	0·021	0·190	$-0{\cdot}012$	0·018	0·098
50	$0{\cdot}083^{***}$	0·022	0·168	$0{\cdot}086^{***}$	0·009	0·093
75	$0{\cdot}149^{***}$	0·020	0·149	0·110	0·011	0·090

Note: The financial vulnerability is measured by the first principle component (FPC). The low and high financial vulnerability refers to the bottom and top 25% of FPC, respectively. New firms are defined as the firms whose age equals two years or less. *, ** and *** denote the statistical significance at the 10%, 5% and 1% levels respectively.

Table A.2: The Elasticity of Productivity w.r.t.FDI for new firms (age$\leq$ 4) in quantile regressions

	Low financial vulnerability			High financial vulnerability		
Quantile (%)	Coef.	s.e.	Psudo-R^2	Coef.	s.e.	Psudo-R^2
5	$-0{\cdot}002$	0·031	0·185	$-0{\cdot}130^{***}$	0·031	0·098
10	$0{\cdot}050^{**}$	0·024	0·198	$-0{\cdot}062^{***}$	0·019	0·102
15	$0{\cdot}051^{**}$	0·021	0·201	$-0{\cdot}028^{**}$	0·014	0·103
20	$0{\cdot}071^{***}$	0·018	0·199	$-0{\cdot}008$	0·011	0·103
25	$0{\cdot}071^{***}$	0·016	0·195	0·014	0·009	0·103
50	$0{\cdot}120^{***}$	0·013	0·177	$0{\cdot}092^{***}$	0·011	0·099
75	$0{\cdot}157^{***}$	0·017	0·157	$0{\cdot}124^{***}$	0·010	0·096

Note: The financial vulnerability is measured by the first principle component (FPC). The low and high financial vulnerability refers to the bottom and top 25% of FPC, respectively. New firms are defined as the firms whose age equals two years or less. *, ** and *** denote the statistical significance at the 10%, 5% and 1% levels respectively.

A.2 The Two-Country Model of FDI under Financial Frictions

Households:

$$\max \quad V = C = \Phi C_0^{\theta_0} {C_1}^{\theta}$$
$$s.t. \quad PC = WL + T$$

where $\Phi = \theta_0^{-\theta_0}\theta^{-\theta}$ and $\theta_0 + \theta = 1$. C_0 denotes the consumption on the homogeneous good which is produced in the perfectly competitive industry with constant return to scale technology. When θ_0 is zero, then our framework will be a general equilibrium, otherwise, it is a partial equilibrium where real exchange rate is unity. Home efficiency conditions are

$$
\begin{aligned}
C_1 &= \left(\frac{P_1}{P}\right)^{-1} \theta C \\
P &= (P_0)^{\theta_0} P_1{}^{\theta} \\
PC &= P_0 C_0 + P_1 C_1 = WL + T
\end{aligned}
$$

Symmetrically, Foreign efficiency conditions are:

$$
\begin{aligned}
C_1^* &= \left(\frac{P_1^*}{P^*}\right)^{-1} \theta C^* \\
P^* &= (P_0^*)^{\theta_0} P_1^{*\theta} \\
P^* C^* &= P_0^* C_0^* + P_1^* C_1^* = W^* L^* + T^*
\end{aligned}
$$

The Industry for the Homogeneous Good: The industry for the homogenous good is perfectly competitive and uses CRTS technology: it requires one unit of labor to produce one unit of the good. Producers do local currency pricing.

Home country:

$$
\begin{aligned}
\max \quad & P_0 Y_0 + \epsilon P_0^* Y_0^X - W L_0 \\
s.t. \quad & Y_0 + Y_0^X = L_0
\end{aligned}
$$

where ϵ is the nominal exchange rate: units of Home currency per one unit of Foreign currency.

Foreign country:

$$
\begin{aligned}
\max \quad & P_0^* Y_0^* + \frac{1}{\epsilon} P_0 Y_0^{X*} - W^* L_0^* \\
s.t. \quad & Y_0^* + Y_0^{X*} = L_0^*
\end{aligned}
$$

Demand and equilibrium conditions are given by:

$$
\begin{aligned}
C_0 &= Y_0 + Y_0^{X*} \\
C_0^* &= Y_0^* + Y_0^X \\
C_0 + C_0^* &= L_0 + L_0^* \\
W &= P_0 = \epsilon P_0^* \\
W^* &= P_0^* = \frac{1}{\epsilon} P_0
\end{aligned}
$$

We take the homogeneous good as a numéraire, and $W = \epsilon W^* = P_0 = \epsilon P_0^*$ holds if the weight on the homogeneous good is not zero: $\theta_0 > 0$. This implies Home(Foreign) labor is also the numéraire. If the world were a currency union, then the nominal exchange rate would be unity: $\epsilon = 1$. We define two kinds of real exchange rate: one is the labor real exchange rate Q_L as units of home labor in exchange for one unit of foreign labor $Q_L \equiv \epsilon \frac{W^*}{W}$ and the other is the consumption real exchange rate Q as units of home consumption basket in exchange for one unit of foreign consumption basket $Q \equiv \epsilon \frac{P^*}{P}$. Therefore, $Q = Q_L \frac{\frac{W}{P}}{\frac{W^*}{P^*}}$ or $Q_L = Q \frac{\frac{P}{W}}{\frac{P^*}{W^*}}$ hold.

Demand on differentiated goods in Home country:

The model features consumption home bias. Households in Home prefer domestically-produced goods C^H to imported goods C^F, which is captured by the parameter ν.

$$
C_1 = \frac{\left(C^H\right)^{\nu} \left(C^F\right)^{1-\nu}}{(\nu)^{\nu} (1-\nu)^{1-\nu}}
$$

Then the efficiency conditions are

$$\begin{aligned}
C^H &= \left(\frac{P^H}{P_1}\right)^{-1} \nu C_1 = \left(\frac{P^H}{P}\right)^{-1} \nu\theta C \\
C^F &= \left(\frac{P^F}{P_1}\right)^{-1} (1-\nu) C_1 = \left(\frac{P^F}{P}\right)^{-1} (1-\nu)\theta C \\
P_1 &= \left(P^H\right)^{\nu} \left(P^F\right)^{1-\nu} \\
P_1 C_1 &= P^H C^H + P^F C^F
\end{aligned}$$

where we have $\frac{P_1}{P} C_1 = \frac{P^H}{P} C^H + \frac{P^F}{P} C^F = \nu\theta C + (1-\nu)\theta C = \theta C$.

The composite consumption of domestic products in Home, C^H, is comprised of those products made by domestic firms and by FDI firms. We can find Hicksian demand by solving

$$\begin{aligned}
\min \quad & \int_{\omega\in\Omega} p^D(\omega) y^D(\omega) d\omega + \int_{\omega^*\in\Omega^I} p^I(\omega^*) y^I(\omega^*) d\omega^* \\
s.t. \quad & \left(\int_{\omega\in\Omega} \left[y^D(\omega)\right]^{\rho} d\omega + \int_{\omega^*\in\Omega^I} \left[y^I(\omega^*)\right]^{\rho} d\omega^*\right)^{\frac{1}{\rho}} \geq C^H
\end{aligned}$$

Then efficiency conditions are

$$\begin{aligned}
\left\{P^H\right\}^{1-\sigma} &= \int_{\omega\in\Omega} \left(p^D(\omega)\right)^{1-\sigma} d\omega + \int_{\omega^*\in\Omega^I} \left(p^I(\omega^*)\right)^{1-\sigma} d\omega^* \\
y^D(\omega) &= \left(\frac{p^D(\omega)}{P^H}\right)^{-\sigma} C^H = \left(\frac{p^D(\omega)}{P}\right)^{-\sigma} \left(\frac{P^H}{P}\right)^{\sigma-1} \nu\theta C \\
y^I(\omega^*) &= \left(\frac{p^I(\omega^*)}{P^H}\right)^{-\sigma} C^H = \left(\frac{p^I(\omega^*)}{P}\right)^{-\sigma} \left(\frac{P^H}{P}\right)^{\sigma-1} \nu\theta C \\
P^H C^H &= \int_{\omega\in\Omega} p^D(\omega) y^D(\omega) d\omega + \int_{\omega^*\in\Omega^I} p^I(\omega^*) y^I(\omega^*) d\omega^*
\end{aligned}$$

where we define $\sigma \equiv \frac{1}{1-\rho}$, $1-\sigma = \frac{-\rho}{1-\rho}$, and $\rho = \frac{\sigma-1}{\sigma}$ with $\sigma > 1$ and $0 < \rho < 1$. The composite consumption of products imported from Foreign under producer-currency pricing is given by

$$\begin{aligned}
\min \quad & \int_{\omega^*\in\Omega^*} \epsilon p^{D*}(\omega^*) y^{D,X*}(\omega^*) d\omega^* \\
s.t. \quad & \left(\int_{\omega^*\in\Omega^*} \left[y^{D,X*}(\omega^*)\right]^{\rho} d\omega^*\right)^{\frac{1}{\rho}} \geq C^F
\end{aligned}$$

Efficiency conditions are

$$
\begin{aligned}
\left\{P^F\right\}^{1-\sigma} &= \int_{\omega^*\in\Omega^*} \left(\epsilon p^{D*}(\omega^*)\right)^{1-\sigma} d\omega^* \\
y^{D,X*}(\omega^*) &= \left(\frac{\epsilon p^{D*}(\omega^*)}{P^F}\right)^{-\sigma} C^F = \left(\frac{\epsilon p^{D*}(\omega^*)}{P}\right)^{-\sigma} \left(\frac{P^F}{P}\right)^{\sigma-1} (1-\nu)\theta C \\
P^F C^F &= \int_{\omega^*\in\Omega^*} \epsilon p^{D*}(\omega^*) y^{D,X*}(\omega^*) d\omega^*
\end{aligned}
$$

where we define $\sigma \equiv \frac{1}{1-\rho}$, $1-\sigma = \frac{-\rho}{1-\rho}$, and $\rho = \frac{\sigma-1}{\sigma}$ with $\sigma > 1$ and $0 < \rho < 1$.

Demand on differentiated goods in Foreign country: Likewise, households in Foreign are also more inclined to consume domestic goods C^{F*} than imported goods C^{H*}, which is captured by the parameter ν^*.

$$
C_1^* = \frac{\left(C^{F*}\right)^{\nu^*} \left(C^{H*}\right)^{1-\nu^*}}{\left(\nu^*\right)^{\nu^*} \left(1-\nu^*\right)^{1-\nu^*}}
$$

Then efficiency conditions are:

$$
\begin{aligned}
C^{F*} &= \left(\frac{P^{F*}}{P_1^*}\right)^{-1} \nu^* C_1^* = \left(\frac{P^{F*}}{P^*}\right)^{-1} \nu^* \theta C^* \\
C^{H*} &= \left(\frac{P^{H*}}{P_1^*}\right)^{-1} (1-\nu^*) C_1^* = \left(\frac{P^{H*}}{P^*}\right)^{-1} (1-\nu^*)\theta C^* \\
P_1^* &= \left(P^{F*}\right)^{\nu^*} \left(P^{H*}\right)^{1-\nu^*} \\
P_1^* C_1^* &= P^{F*} C^{F*} + P^{H*} C^{H*}
\end{aligned}
$$

where we have $\frac{P_1^*}{P^*} C_1^* = \frac{P^{F*}}{P^*} C^{F*} + \frac{P^{H*}}{P^*} C^{H*} = \nu^* \theta C^* + (1-\nu^*)\theta C^* = \theta C^*$.

The composite consumption of domestic products in Foreign, C^{F*}, is comprised of those products made by Foreign domestic firms. We can find Hicksian demand by solving

$$
\begin{aligned}
\min \quad & \int_{\omega^*\in\Omega^*} p^{D*}(\omega^*) y^{D*}(\omega^*) d\omega^* \\
s.t. \quad & \left(\int_{\omega^*\in\Omega^*} \left[y^{D*}(\omega^*)\right]^{\rho} d\omega^*\right)^{\frac{1}{\rho}} \geq C^{F*}
\end{aligned}
$$

Efficiency conditions are:

$$\begin{aligned}
{P^{F*}}^{1-\sigma} &= \int_{\omega^*\in\Omega^*} \left(p^{D*}(\omega^*)\right)^{1-\sigma} d\omega^* \\
y^{D*}(\omega^*) &= \left(\frac{p^{D*}(\omega^*)}{P^{F*}}\right)^{-\sigma} C^{F*} = \left(\frac{p^{D*}(\omega^*)}{P^*}\right)^{-\sigma} \left(\frac{P^{F*}}{P^*}\right)^{\sigma-1} \nu^*\theta C^* \\
P^{F*}C^{F*} &= \int_{\omega^*\in\Omega^*} p^{D*}(\omega^*)y^{D*}(\omega^*)d\omega^*
\end{aligned}$$

where we define $\sigma \equiv \frac{1}{1-\rho}$, $1-\sigma = \frac{-\rho}{1-\rho}$, and $\rho = \frac{\sigma-1}{\sigma}$ with $\sigma > 1$ and $0 < \rho < 1$.

The composite consumption of products imported from Home under producer-currency pricing is also defined by the standard CES aggregator and Hicksian demand can be derived by solving

$$\begin{aligned}
\min \quad & \int_{\omega\in\Omega} \frac{1}{\epsilon} p^D(\omega)y^{D,X}(\omega)d\omega + \int_{\omega^*\in\Omega^I} \frac{1}{\epsilon} p^I(\omega^*)y^{I,X}(\omega^*)d\omega^* \\
s.t. \quad & \left(\int_{\omega\in\Omega} \left[y^{D,X}(\omega)\right]^\rho d\omega + \int_{\omega^*\in\Omega^I} \left[y^{I,X}(\omega^*)\right]^\rho d\omega^*\right)^{\frac{1}{\rho}} \geq C^{H*}
\end{aligned}$$

Therefore, efficiency conditions are

$$\begin{aligned}
{P^{H*}}^{1-\sigma} &= \int_{\omega\in\Omega} \left(\frac{p^D(\omega)}{\epsilon}\right)^{1-\sigma} d\omega + \int_{\omega^*\in\Omega^I} \left(\frac{p^I(\omega^*)}{\epsilon}\right)^{1-\sigma} d\omega^* \\
y^{D,X}(\omega) &= \left(\frac{p^D(\omega)}{\epsilon P^{H*}}\right)^{-\sigma} C^{H*} = \left(\frac{p^D(\omega)}{\epsilon P^*}\right)^{-\sigma} \left(\frac{P^{H*}}{P^*}\right)^{\sigma-1} (1-\nu^*)\theta C^* \\
y^{I,X}(\omega^*) &= \left(\frac{p^I(\omega^*)}{\epsilon P^{H*}}\right)^{-\sigma} C^{H*} = \left(\frac{p^I(\omega^*)}{\epsilon P^*}\right)^{-\sigma} \left(\frac{P^{H*}}{P^*}\right)^{\sigma-1} (1-\nu^*)\theta C^* \\
P^{H*}C^{H*} &= \int_{\omega\in\Omega} \frac{p^D(\omega)}{\epsilon} y^{D,X}(\omega)d\omega + \int_{\omega^*\in\Omega^I} \frac{p^I(\omega^*)}{\epsilon} y^{I,X}(\omega^*)d\omega^*
\end{aligned}$$

where we define $\sigma \equiv \frac{1}{1-\rho}$, $1-\sigma = \frac{-\rho}{1-\rho}$, and $\rho = \frac{\sigma-1}{\sigma}$ with $\sigma > 1$ and $0 < \rho < 1$.

Domestic Firms under imperfect financial markets in Home:

$$\begin{aligned}
\pi^D(z) &= \max \ \tau_C^D \left[\begin{array}{ll} \tau_V^D p^D(z)\left(y^D(z)+y^{D,X}(z)\right) & -\tau_L^D W l^D(z) - f^D W + \zeta f^D W \\ & -\lambda x(z) - (1-\lambda)\chi F^D W \end{array} \right] \\
s.t. \quad & \tau_V^D p^D(z)\left(y^D(z)+y^{D,X}(z)\right) - \tau_L^D W l^D(z) - f^D W + \zeta f^D W \geq x(z) \\
& \lambda x(z) + (1-\lambda)\chi F^D W \geq \zeta f^D W \\
& l^D(z) = \frac{y^D(z)+y^{D,X}(z)}{z} \\
& y^D(z) = \left(\frac{p^D(z)}{W}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} \\
& y^{D,X}(z) = \left(\frac{p^D(z)}{W}\right)^{-\sigma} {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}
\end{aligned}$$

Solving the maximization problem delivers equilibrium conditions:

$$\begin{aligned}
\frac{p^D(z)}{W} &= \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)\frac{1}{z} \\
l^D(z) &= z^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \frac{\frac{r^D(z)}{W}}{\tau_V^D\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)} = \frac{\rho}{\tau_L^D}\frac{r^D(z)}{W} \\
\frac{r^D(z)}{W} &\equiv \tau_V^D\frac{p^D(z)}{W}\left(y^D(z)+y^{D,X}(z)\right) = z^{\sigma-1}\tau_V^D\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \sigma\left[\frac{1}{\tau_C^D}\frac{\pi^D(z)}{W} + f^D\right] \\
\frac{\pi^D(z)}{W} &= \tau_C^D\left[\tau_V^D\frac{p^D(z)}{W}\left(y^D(z)+y^{D,X}(z)\right) - \tau_L^D l^D(z) - f^D\right] = \tau_C^D\left[\frac{\tau_V^D}{\sigma}\frac{p^D(z)}{W}\left(y^D(z)+y^{D,X}(z)\right) - f^D\right] \\
&= \tau_C^D\left[\frac{1}{\sigma}\frac{r^D(z)}{W} - f^D\right] = \tau_C^D\left[\frac{\tau_L^D}{\sigma-1}l^D(z) - f^D\right] \\
\frac{\xi^D(z)}{W} &= \tau_C^D\frac{r^D(z)}{W} - \frac{\pi^D(z)}{W} = \tau_C^D\left[\frac{\sigma-1}{\sigma}\frac{r^D(z)}{W} + f^D\right]
\end{aligned}$$

where $\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] = \left(\frac{W}{P}\right)^{-\sigma}\left[\left(\frac{P^H}{P}\right)^{\sigma-1}\nu\theta C + Q^{\sigma}\left(\frac{P^{H*}}{P^*}\right)^{\sigma-1}(1-\nu^*)\theta C^*\right]$. From the firm's participation constraint, we can get the cutoff productivity Z^D:

$$\begin{aligned}
\frac{r^D(Z^D)}{W} - \tau_L^D l^D(Z^D) &= \frac{1}{\sigma}\frac{r^D(Z^D)}{W} \\
&= (Z^D)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \frac{x(z)}{W} + f^D - \zeta f^D \\
&= \left[f^D + \left(\frac{1}{\lambda} - 1\right)(\zeta f^D - \chi F^D)\right]
\end{aligned}$$

That is,

$$\begin{aligned}
\frac{\pi^D(Z^D)}{W} &= \tau_C^D\left[\frac{1}{\sigma}\frac{r^D(Z^D)}{W} - f^D\right] \\
&= \tau_C^D\left(\frac{1}{\lambda} - 1\right)(\zeta f^D - \chi F^D)
\end{aligned}$$

Note that we have nonzero ex-ante profit for the marginal firm of productivity Z^D: $\frac{\pi^D(Z^D)}{W} > 0$. Instead, after the defaulting shock with probability $(1-\lambda)$ is realized, the ex-post profit of the firm becomes zero if the firm turns out not to default.

Foreign firms under no financial frictions in Foreign:

When Foreign firms establish FDI subsidiaries, their productivity in the host country is exogenously reduced by a factor of $\alpha \in (0, 1]$: $g = \alpha z$, where z is drawn from the distribution $G^*(z)$. Aggregate profit across all Foreign firms in terms of Foreign currency is given by

$$\int_{\omega^* \in \Omega^*} \pi^{D*}(z) d\omega^* + \int_{\omega^* \in \Omega^I} \frac{1}{\epsilon}\pi^I(\alpha z) d\omega^*$$

(1) Domestic Sales and Exporting of Foreign firms

$$
\begin{aligned}
\pi^{D*}(z) &= \max \ \tau_C^{D*}\left[\tau_V^{D*}p^{D*}(z)\left(y^{D*}(z)+y^{D,X*}(z)\right)-\tau_L^{D*}W^*l^{D*}(z)-f^{D*}W^*\right] \\
s.t. \quad & l^{D*}(z)=\frac{y^{D*}(z)+y^{D,X*}(z)}{z} \\
& y^{D*}(z)=\left(\frac{p^{D*}(z)}{W^*}\right)^{-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*} \\
& y^{D,X*}(z)=\left(\frac{p^{D*}(z)}{W^*}\right)^{-\sigma}Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}
\end{aligned}
$$

Note that $\sigma > 1$ implies the good z is a substitute among intra-industry goods. Therefore, if the overall price level $\frac{P^{F*}}{P^*}$ increases, this means other intra-industry differentiated goods become more expensive and so the demand on the good z increases. Then equilibrium conditions are:

$$
\begin{aligned}
\frac{p^{D*}(z)}{W^*} &= \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)\frac{1}{z} \\
l^{D*}(z) &= z^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma}\left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}+Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] \\
&= \frac{\frac{r^{D*}(z)}{W^*}}{\tau_V^{D*}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)}=\frac{\rho}{\tau_L^{D*}}\frac{r^{D*}(z)}{W^*} \\
\frac{r^{D*}(z)}{W^*} &\equiv \tau_V^{D*}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)+y^{D,X*}(z)\right) \\
&= z^{\sigma-1}\tau_V^{D*}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}+Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] \\
&= \sigma\left[\frac{1}{\tau_C^{D*}}\frac{\pi^{D*}(z)}{W^*}+f^{D*}\right] \\
\frac{\pi^{D*}(z)}{W^*} &\equiv \tau_C^{D*}\left[\tau_V^{D*}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)+y^{D,X*}(z)\right)-\tau_L^{D*}l^{D*}(z)-f^{D*}\right] \\
&= \tau_C^{D*}\left[\frac{\tau_V^{D*}}{\sigma}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)+y^{D,X*}(z)\right)-f^{D*}\right] \\
&= \tau_C^{D*}\left[\frac{1}{\sigma}\frac{r^{D*}(z)}{W^*}-f^{D*}\right]=\tau_C^{D*}\left[\frac{\tau_L^{D*}}{\sigma-1}l^{D*}(z)-f^{D*}\right] \\
\frac{\xi^{D*}(z)}{W^*} &= \tau_C^{D*}\frac{r^{D*}(z)}{W^*}-\frac{\pi^{D*}(z)}{W^*}=\tau_C^{D*}\left[\frac{\sigma-1}{\sigma}\frac{r^{D*}(z)}{W^*}+f^{D*}\right]
\end{aligned}
$$

where

$$\begin{aligned}
&\left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}+{Q_L}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right]\\
=\;&\left(\frac{W^*}{P^*}\right)^{-\sigma}\left[\left(\frac{P^{F*}}{P^*}\right)^{\sigma-1}\nu^*\theta C^*+Q^{-\sigma}\left(\frac{P^F}{P}\right)^{\sigma-1}(1-\nu)\theta C\right].
\end{aligned}$$

From the firm's zero profit condition, we can get the cutoff productivity Z^{D*}:

$$\begin{aligned}
\frac{r^{D*}(Z^{D*})}{W^*}-\tau_L^{D*}l^{D*}(Z^{D*}) &= \frac{1}{\sigma}\frac{r^{D*}(Z^{D*})}{W^*}\\
&= (Z^{D*})^{\sigma-1}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}+{Q_L}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right]\\
&= f^{D*}
\end{aligned}$$

That is,

$$\frac{\pi^{D*}(Z^{D*})}{W^*} = \tau_C^{D*}\left[\frac{1}{\sigma}\frac{r^{D*}(Z^{D*})}{W^*}-f^{D*}\right]=0$$

(2) FDI Subsidiaries of Foreign firms

When Foreign firms establish FDI subsidiaries, their productivity is exogenously reduced by a factor of $\alpha\in(0,1]$: $g=\alpha z$, where z is drawn from the distribution $G^*(z)$.

$$\begin{aligned}
\pi^I(g) \;=\;& \max\;\tau_C^I\left[\tau_V^I p^I(g)\left(y^I(g)+y^{I,X}(g)\right)-\tau_L^I W l^I(g)-f^I W\right]\\
s.t.\quad & l^I(g)=\frac{y^I(g)+y^{I,X}(g)}{g}\\
& y^I(g)=\left(\frac{p^I(g)}{W}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}\\
& y^{I,X}(g)=\left(\frac{p^I(g)}{W}\right)^{-\sigma}{Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}
\end{aligned}$$

Solving the maximization problem leads to equilibrium conditions:

$$\begin{aligned}
\frac{p^I(g)}{W} &= \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)\frac{1}{g} \\
l^I(g) &= g^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+{Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \frac{\frac{r^I(g)}{W}}{\tau_V^I\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)} = \frac{\rho}{\tau_L^I}\frac{r^I(g)}{W} \\
\frac{r^I(g)}{W} &\equiv \tau_V^I\frac{p^I(g)}{W}\left(y^I(g)+y^{I,X}(g)\right) = g^{\sigma-1}\tau_V^I\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+{Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \sigma\left[\frac{1}{\tau_C^I}\frac{\pi^I(g)}{W}+f^I\right] \\
\frac{\pi^I(g)}{W} &\equiv \tau_C^I\left[\tau_V^I\frac{p^I(g)}{W}\left(y^I(g)+y^{I,X}(g)\right)-\tau_L^I l^I(g)-f^I\right] = \tau_C^I\left[\frac{\tau_V^I}{\sigma}\frac{p^I(g)}{W}\left(y^I(g)+y^{I,X}(g)\right)-f^I\right] \\
&= \tau_C^I\left[\frac{1}{\sigma}\frac{r^I(g)}{W}-f^I\right] = \tau_C^I\left[\frac{\tau_L^I}{\sigma-1}l^I(g)-f^I\right] \\
\frac{\xi^I(g)}{W} &= \tau_C^I\frac{r^I(g)}{W}-\frac{\pi^I(g)}{W} = \tau_C^I\left[\frac{\sigma-1}{\sigma}\frac{r^I(g)}{W}+f^I\right]
\end{aligned}$$

where

$$\begin{aligned}
&\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+{Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
= &\left(\frac{W}{P}\right)^{-\sigma}\left[\left(\frac{P^H}{P}\right)^{\sigma-1}\nu\theta C+Q^{\sigma}\left(\frac{P^{H*}}{P^*}\right)^{\sigma-1}(1-\nu^*)\theta C^*\right].
\end{aligned}$$

From the firm's zero profit condition, we can obtain the cutoff productivity Z^I:

$$\begin{aligned}
\frac{r^I(\alpha Z^I)}{W}-\tau_L^I l^I(\alpha Z^I) &= \frac{1}{\sigma}\frac{r^I(\alpha Z^I)}{W} \\
&= \left(\alpha Z^I\right)^{\sigma-1}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+{Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= f^I
\end{aligned}$$

That is,

$$\frac{\pi^I\left(\alpha Z^I\right)}{W} = \tau_C^I\left[\frac{1}{\sigma}\frac{r^I\left(\alpha Z^I\right)}{W}-f^I\right] = 0$$

Productivity Distribution:

Assume productivity of Home firms and Foreign firms follow Pareto distribution, given by:

$$G(z) = 1 - (z_{min})^{\eta} z^{-\eta}, \qquad G^*(z) = 1 - (z^*_{min})^{\eta^*} z^{-\eta^*}.$$

Define $J(z)$ and $J^*(z)$ as:

$$J(z) \equiv \int_z^{\infty} a^{\sigma-1} dG(a) = \underbrace{\frac{\eta (z_{min})^{\eta}}{\eta - \sigma + 1}}_{\equiv \widetilde{\eta}} \{z\}^{-\eta+\sigma-1}, \qquad J^*(z) \equiv \int_z^{\infty} a^{\sigma-1} dG^*(a) = \underbrace{\frac{\eta^* (z^*_{min})^{\eta^*}}{\eta^* - \sigma + 1}}_{\equiv \widetilde{\eta}^*} \{z\}^{-\eta^*+\sigma-1},$$

where $\eta > \sigma - 1$ and $\eta^* > \sigma - 1$ are required. In addition, we can compute:

$$\int_z^{\infty} a^{-1} dG(a) = \tfrac{\eta (z_{min})^{\eta}}{\eta+1} \{z\}^{-\eta-1}, \qquad \int_z^{\infty} a^{\sigma} dG(a) = \tfrac{\eta (z_{min})^{\eta}}{\eta-\sigma} \{z\}^{-\eta+\sigma},$$
$$\int_z^{\infty} a^{-1} dG^*(a) = \tfrac{\eta^* (z^*_{min})^{\eta^*}}{\eta^*+1} \{z\}^{-\eta^*-1}, \qquad \int_z^{\infty} a^{\sigma} dG^*(a) = \tfrac{\eta^* (z^*_{min})^{\eta^*}}{\eta^*-\sigma} \{z\}^{-\eta^*+\sigma},$$

where $\eta > \sigma$ and $\eta^* > \sigma$ are required. Define average productivity $\widetilde{Z}^D$, $\widetilde{Z}^I$, and $\widetilde{Z}^{D*}$ using cutoff productivity Z^D, Z^I, and Z^{D*}:

$$\begin{aligned}
\widetilde{Z}^D &\equiv \left[\int_{Z^D}^{\infty} z^{\sigma-1} \frac{dG(z)}{1 - G(Z^D)}\right]^{\frac{1}{\sigma-1}} = \left[\frac{J(Z^D)}{1 - G(Z^D)}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta}{\eta - \sigma + 1}\right]^{\frac{1}{\sigma-1}} Z^D, \\
\widetilde{Z}^I &\equiv \left[\int_{Z^I}^{\infty} (\alpha z)^{\sigma-1} \frac{dG^*(z)}{1 - G^*(Z^I)}\right]^{\frac{1}{\sigma-1}} = \alpha \left[\frac{J^*(Z^I)}{1 - G^*(Z^I)}\right]^{\frac{1}{\sigma-1}} = \alpha \left[\frac{\eta^*}{\eta^* - \sigma + 1}\right]^{\frac{1}{\sigma-1}} Z^I, \\
\widetilde{Z}^{D*} &\equiv \left[\int_{Z^{D*}}^{\infty} z^{\sigma-1} \frac{dG^*(z)}{1 - G^*(Z^{D*})}\right]^{\frac{1}{\sigma-1}} = \left[\frac{J^*(Z^{D*})}{1 - G^*(Z^{D*})}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta^*}{\eta^* - \sigma + 1}\right]^{\frac{1}{\sigma-1}} Z^{D*}.
\end{aligned}$$

That is,

$$\begin{aligned}
\frac{J(Z^D)}{1 - G(Z^D)} &= \left(\widetilde{Z}^D\right)^{\sigma-1} \text{ and } \left(\frac{\widetilde{Z}^D}{Z^D}\right)^{\sigma-1} = \int_{Z^D}^{\infty} \left(\frac{z}{Z^D}\right)^{\sigma-1} \frac{dG(z)}{1 - G(Z^D)} = \frac{\eta}{\eta - \sigma + 1}, \\
\frac{J^*(Z^I)}{1 - G^*(Z^I)} &= \left(\frac{\widetilde{Z}^I}{\alpha}\right)^{\sigma-1} \text{ and } \left(\frac{\widetilde{Z}^I}{Z^I}\right)^{\sigma-1} = \int_{Z^I}^{\infty} \left(\frac{\alpha z}{Z^I}\right)^{\sigma-1} \frac{dG^*(z)}{1 - G^*(Z^I)} = \frac{\eta^*}{\eta^* - \sigma + 1} \alpha^{\sigma-1}, \\
\frac{J^*(Z^{D*})}{1 - G^*(Z^{D*})} &= \left(\widetilde{Z}^{D*}\right)^{\sigma-1} \text{ and } \left(\frac{\widetilde{Z}^{D*}}{Z^{D*}}\right)^{\sigma-1} = \int_{Z^{D*}}^{\infty} \left(\frac{z}{Z^{D*}}\right)^{\sigma-1} \frac{dG^*(z)}{1 - G^*(Z^{D*})} = \frac{\eta^*}{\eta^* - \sigma + 1},
\end{aligned}$$

where

$$\int_{Z^I}^{\infty}(\alpha z)^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^I)}=\frac{\alpha^{\sigma-1}J^*(Z^I)}{1-G^*(Z^I)}=\left(\widetilde{Z}^I\right)^{\sigma-1}.$$

Therefore, even if the productivity of FDI subsidiaries is reduced by a factor α when it starts its business in the host country, it does not change the ex-post conditional probability density, $\frac{dG^*(z)}{1-G^*(Z^I)}$.

Free Entry Condition:

Incumbents might exit with exogenous probability δ. The expected life-time operating profit of a potential entrant should equal the entry costs. Home local firms equate their ex-ante expected profit to sunk entry costs.

$$\begin{aligned}\frac{W}{P}F^D &= \left(1-G(Z^D)\right)\begin{bmatrix} & \int_{Z^D}^{\infty}\frac{\pi^D(z)}{P}\frac{dG(z)}{(1-G(Z^D))} \\ +(1-\delta) & \begin{bmatrix} \int_{Z^D}^{\infty}\frac{\pi^D(z)}{P}\frac{dG(z)}{(1-G(Z^D))} \\ + \quad \cdots \end{bmatrix}\end{bmatrix} \\ &= \left(1-G(Z^D)\right)\sum_{t=0}^{\infty}(1-\delta)^t\int_{Z^D}^{\infty}\frac{\pi^D(z)}{P}\frac{dG(z)}{(1-G(Z^D))}=\left(\frac{1-G(Z^D)}{\delta}\right)\int_{Z^D}^{\infty}\frac{\pi^D(z)}{P}\frac{dG(z)}{(1-G(Z^D))}\end{aligned}$$

That is,

$$\begin{aligned}&F^D \\ =\ &\left(\frac{1-G(Z^D)}{\delta}\right)\int_{Z^D}^{\infty}\frac{\pi^D(z)}{W}\frac{dG(z)}{(1-G(Z^D))} \\ =\ &\left(\frac{1-G(Z^D)}{\delta}\right)\tau_C^D\left\{\frac{J(Z^D)}{1-G(Z^D)}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]-f^D\right\} \\ =\ &\left(\frac{1-G(Z^D)}{\delta}\right)\tau_C^D\left\{\left(\widetilde{Z}^D\right)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]-f^D\right\} \\ =\ &\left(\frac{1-G(Z^D)}{\delta}\right)\frac{\pi^D\left(\widetilde{Z}^D\right)}{W}\end{aligned}$$

Therefore,

$$\frac{\pi^D(\widetilde{Z}^D)}{W} = \frac{\delta F^D}{1-G(Z^D)}$$

Likewise, Foreign firms equate their ex-ante expected profit to sunk entry costs.

$$\frac{W^*}{P^*}F^{D*} = \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\int_{Z^{D*}}^{\infty}\frac{\pi^{D*}(z)}{P^*}\frac{dG^*(z)}{1-G^*(Z^{D*})} + \left(\frac{1-G^*(Z^I)}{Q\cdot\delta}\right)\int_{Z^I}^{\infty}\frac{\pi^I(\alpha z)}{P}\frac{dG^*(z)}{1-G^*(Z^I)}$$

That is,

$$\begin{aligned}
& F^{D*} \\
= & \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\int_{Z^{D*}}^{\infty}\frac{\pi^{D*}(z)}{W^*}\frac{dG^*(z)}{1-G^*(Z^{D*})} + \left(\frac{1-G^*(Z^I)}{Q_L\cdot\delta}\right)\int_{Z^I}^{\infty}\frac{\pi^I(\alpha z)}{W}\frac{dG^*(z)}{1-G^*(Z^I)} \\
= & \begin{array}{l} \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}\left\{\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*} + Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] - f^{D*}\right\} \\ + \left(\frac{1-G^*(Z^I)}{Q_L\cdot\delta}\right)\tau_C^I\left\{\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] - f^I\right\} \end{array} \\
= & \begin{array}{l} \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}\left\{\left(\widetilde{Z}^{D*}\right)^{\sigma-1}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*} + Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] - f^{D*}\right\} \\ + \left(\frac{1-G^*(Z^I)}{Q_L\cdot\delta}\right)\tau_C^I\left\{\left(\widetilde{Z}^I\right)^{\sigma-1}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] - f^I\right\} \end{array} \\
= & \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} + \left(\frac{1-G^*(Z^I)}{Q_L\cdot\delta}\right)\frac{\pi^I(\widetilde{Z}^I)}{W}
\end{aligned}$$

Therefore,

$$\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} + \frac{1}{Q_L}\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\pi^I(\widetilde{Z}^I)}{W} = \frac{\delta F^{D*}}{1-G^*(Z^{D*})}$$

Free Entry Condition and Zero-Profit Cutoff Productivity:

In Home country, by combining zero-profit cutoff productivity condition and free entry condition, given by:

$$\begin{aligned}
\frac{1}{\sigma}\frac{r^D(Z^D)}{W} &= f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right), \\
F^D &= \left(\frac{1-G(Z^D)}{\delta}\right)\frac{\pi^D\left(\widetilde{Z}^D\right)}{W},
\end{aligned}$$

we can specify the cutoff productivity for Home local firms, Z^D:

$$\begin{aligned}
& F^D \\
= \;& \left(\frac{1-G(Z^D)}{\delta}\right)\tau_C^D\left[f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right]\left[\frac{\frac{r^D(\widetilde{Z}^D)}{W}}{\frac{r^D(Z^D)}{W}}-\frac{f^D}{f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)}\right] \\
= \;& \left(\frac{1-G(Z^D)}{\delta}\right)\tau_C^D\left[f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right]\left[\left(\frac{\widetilde{Z}^D}{Z^D}\right)^{\sigma-1}-\frac{f^D}{f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)}\right] \\
= \;& \left(\frac{1-G(Z^D)}{\delta}\right)\tau_C^D\left[f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right]\left[\int_{Z^D}^{\infty}\left(\frac{z}{Z^D}\right)^{\sigma-1}\frac{dG(z)}{1-G(Z^D)}-\frac{f^D}{f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)}\right] \\
= \;& \left(\frac{1-G(Z^D)}{\delta}\right)\tau_C^D\left[f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right]\left[\frac{\eta}{\eta-\sigma+1}-\frac{f^D}{f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)}\right]
\end{aligned}$$

$$\therefore (Z^D)^{\eta} = \left(\frac{\tau_C^D}{\delta}\right)\left(\frac{f^D}{F^D}\right)\left(\frac{(\sigma-1)\,(z_{min})^{\eta}}{\eta-\sigma+1}\right)\left[1+\frac{\eta}{\sigma-1}\left(\frac{1}{\lambda}-1\right)\left(\zeta-\chi\frac{F^D}{f^D}\right)\right].$$

In Home country, by combining two zero-profit cutoff productivity conditions, given by:

$$\begin{aligned}
\frac{1}{\sigma}\frac{r^D(Z^D)}{W} &= \left[f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right] \\
\frac{1}{\sigma}\frac{r^I(\alpha Z^I)}{W} &= f^I
\end{aligned}$$

we can solve for the cutoff productivity for FDI firms, Z^I:

$$\begin{aligned}
\frac{\frac{r^D(Z^D)}{W}}{\frac{r^I(\alpha Z^I)}{W}} &= \frac{(Z^D)^{\sigma-1}\tau_V^D\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]}{(\alpha Z^I)^{\sigma-1}\tau_V^I\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]} \\
&= \frac{f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)}{f^I} \\
\therefore \alpha Z^I &= Z^D\left(\frac{f^I}{\left[f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right]}\right)^{\frac{1}{\sigma-1}}\left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}}\frac{\tau_L^I}{\tau_L^D} \\
&= z_{min}\left(\frac{\tau_C^D}{\delta}\right)^{\frac{1}{\eta}}\left(\frac{\eta}{F^D(\eta-\sigma+1)}\right)^{\frac{1}{\eta}}\frac{\left[\frac{f^D(\sigma-1)}{\eta}+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right]^{\frac{1}{\eta}}}{\left[f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right]^{\frac{1}{\sigma-1}}}(f^I)^{\frac{1}{\sigma-1}}\left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}}\frac{\tau_L^I}{\tau_L^D}
\end{aligned}$$

Similarly, in Foreign country, by combining three equilibrium conditions which are given as,

$$
\begin{aligned}
\frac{1}{\sigma}\frac{r^{D*}(Z^{D*})}{W^*} &= f^{D*}, \\
\frac{1}{\sigma}\frac{r^{I}(\alpha Z^{I})}{W} &= f^{I}, \\
F^{D*} &= \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} + \left(\frac{1-G^*(Z^{I})}{Q_L\cdot\delta}\right)\frac{\pi^{I}(\widetilde{Z}^{I})}{W},
\end{aligned}
$$

we can derive the cutoff productivity of Foreign local firms, Z^{D*}:

$$
\begin{aligned}
& F^{D*} \\
= & \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}f^{D*}\left[\frac{\frac{r^{D*}(\widetilde{Z}^{D*})}{W^*}}{\frac{r^{D*}(Z^{D*})}{W^*}}-1\right] + \left(\frac{1-G^*(Z^{I})}{Q_L\cdot\delta}\right)\tau_C^{I}f^{I}\left[\frac{\frac{r^{I}(\widetilde{Z}^{I})}{W}}{\frac{r^{I}(\alpha Z^{I})}{W}}-1\right] \\
= & \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}f^{D*}\left[\left(\frac{\widetilde{Z}^{D*}}{Z^{D*}}\right)^{\sigma-1}-1\right] + \left(\frac{1-G^*(Z^{I})}{Q_L\cdot\delta}\right)\tau_C^{I}f^{I}\left[\left(\frac{\widetilde{Z}^{I}}{\alpha Z^{I}}\right)^{\sigma-1}-1\right] \\
= & \quad \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}f^{D*}\left[\int_{Z^{D*}}^{\infty}\left(\frac{z}{Z^{D*}}\right)^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^{D*})}-1\right] \\
& + \left(\frac{1-G^*(Z^{I})}{Q_L\cdot\delta}\right)\tau_C^{I}f^{I}\left[\int_{Z^{I}}^{\infty}\left(\frac{z}{Z^{I}}\right)^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^{I})}-1\right] \\
= & \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}f^{D*}\left[\frac{\eta^*}{\eta^*-\sigma+1}-1\right] + \left(\frac{1-G^*(Z^{I})}{Q_L\cdot\delta}\right)\tau_C^{I}f^{I}\left[\frac{\eta^*}{\eta^*-\sigma+1}-1\right] \\
= & \left(\frac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}f^{D*}\left[\frac{\sigma-1}{\eta^*-\sigma+1}\right] + \left(\frac{1-G^*(Z^{I})}{Q_L\cdot\delta}\right)\tau_C^{I}f^{I}\left[\frac{\sigma-1}{\eta^*-\sigma+1}\right]
\end{aligned}
$$

$$
\therefore \left(Z^{D*}\right)^{\eta^*} = \frac{\tau_C^{D*}f^{D*}\frac{(\sigma-1)\left(z^*_{min}\right)^{\eta^*}}{\eta^*-\sigma+1}}{\delta F^{D*}-\left(\frac{1}{Q_L}\right)\tau_C^{I}f^{I}\frac{(\sigma-1)\left(z^*_{min}\right)^{\eta^*}}{\eta^*-\sigma+1}\left(Z^{I}\right)^{-\eta^*}} = \frac{\left(\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*}\right)\left(z^*_{min}\right)^{\eta^*}}{\delta F^{D*}-\left(\frac{1}{Q_L}\right)\left(\frac{\pi^{I}(\widetilde{Z}^{I})}{W}\right)\left(z^*_{min}\right)^{\eta^*}\left(Z^{I}\right)^{-\eta^*}}
$$

Price Index: For Home country, we obtain:

$$
\begin{aligned}
\left(\frac{P_1}{W}\right) &= \left(\frac{P^H}{W}\right)^{\nu}\left(\frac{P^F}{W}\right)^{(1-\nu)} \\
\left(\frac{P^H}{W}\right)^{1-\sigma} &= \int_{\omega\in\Omega}\left(\frac{p^D(\omega)}{W}\right)^{1-\sigma} d\omega + \int_{\omega^*\in\Omega^I}\left(\frac{p^I(\omega^*)}{W}\right)^{1-\sigma} d\omega^* \\
&= M\int_{Z^D}^{\infty}\left(\frac{p^D(z)}{W}\right)^{1-\sigma}\frac{dG(z)}{1-G(Z^D)} + M^*\int_{Z^I}^{\infty}\left(\frac{p^I(\alpha z)}{W}\right)^{1-\sigma}\frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&= M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} + M^*\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} \\
&= M\left(\widetilde{Z}^D\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} + \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)M^*\left(\widetilde{Z}^I\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} \\
&= M\left(\frac{p^D(\widetilde{Z}^D)}{W}\right)^{1-\sigma} + M^I\left(\frac{p^I(\widetilde{Z}^I)}{W}\right)^{1-\sigma} \\
\left(\frac{P^F}{W}\right)^{1-\sigma} &= \int_{\omega^*\in\Omega^*}\left(\frac{\epsilon p^{D*}(\omega^*)}{W}\right)^{1-\sigma} d\omega^* = M^*\int_{Z^{D*}}^{\infty}\left(\frac{Q_L p^{D*}(z)}{W^*}\right)^{1-\sigma}\frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&= M^*\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{Q_L}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} = M^*\left(\widetilde{Z}^{D*}\right)^{\sigma-1}\left(\frac{Q_L}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} \\
&= M^*\left(Q_L\frac{p^{D*}(\widetilde{Z}^{D*})}{W^*}\right)^{1-\sigma}
\end{aligned}
$$

where $M^I \equiv \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) M^*$. For Foreign country, we obtain:

$$\begin{aligned}
\left(\frac{P_1^*}{W^*}\right) &= \left(\frac{P^{F*}}{W^*}\right)^{\nu^*} \left(\frac{P^{H*}}{W^*}\right)^{(1-\nu^*)} \\
\left(\frac{P^{F*}}{W^*}\right)^{1-\sigma} &= \int_{\omega^*\in\Omega^*} \left(\frac{p^{D*}(\omega^*)}{W^*}\right)^{1-\sigma} d\omega^* = M^* \int_{Z^{D*}}^{\infty} \left(\frac{p^{D*}(z)}{W^*}\right)^{1-\sigma} \frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&= M^* \frac{J^*(Z^{D*})}{1-G^*(Z^{D*})} \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} = M^* \left(\widetilde{Z}^{D*}\right)^{\sigma-1} \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} \\
&= M^* \left(\frac{p^{D*}(\widetilde{Z}^{D*})}{W^*}\right)^{1-\sigma} \\
\left(\frac{P^{H*}}{W^*}\right)^{1-\sigma} &= \int_{\omega\in\Omega} \left(\frac{p^D(\omega)}{\epsilon W^*}\right)^{1-\sigma} d\omega + \int_{\omega^*\in\Omega^I} \left(\frac{p^I(\omega^*)}{\epsilon W^*}\right)^{1-\sigma} d\omega^* \\
&= M \int_{Z^D}^{\infty} \left(\frac{p^D(z)}{WQ_L}\right)^{1-\sigma} \frac{dG(z)}{1-G(Z^D)} + M^* \int_{Z^I}^{\infty} \left(\frac{p^I(\alpha z)}{WQ_L}\right)^{1-\sigma} \frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&= M \frac{J(Z^D)}{1-G(Z^D)} \left(\frac{1}{Q_L}\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} + \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) M^* \alpha^{\sigma-1} \frac{J^*(Z^I)}{1-G^*(Z^I)} \left(\frac{1}{Q_L}\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} \\
&= M \left(\widetilde{Z}^D\right)^{\sigma-1} \left(\frac{1}{Q_L}\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} + M^I \left(\widetilde{Z}^I\right)^{\sigma-1} \left(\frac{1}{Q_L}\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} \\
&= M \left(\frac{1}{Q_L}\frac{p^D(\widetilde{Z}^D)}{W}\right)^{1-\sigma} + M^I \left(\frac{1}{Q_L}\frac{p^I(\widetilde{Z}^I)}{W}\right)^{1-\sigma}
\end{aligned}$$

Therefore, observe that

$$\left(\frac{P^F}{W}\right) = Q_L \left(\frac{P^{F*}}{W^*}\right) \quad \text{and} \quad \left(\frac{P^{H*}}{W^*}\right) = \left(\frac{1}{Q_L}\right)\left(\frac{P^H}{W}\right)$$

The Evolution of the Mass of Firms:

$$\begin{aligned}
M &= \left(1-G(Z^D)\right) M^E + (1-\delta)M \\
M^* &= \left(1-G^*(Z^{D*})\right) M^{E*} + (1-\delta)M^* \\
M^I &\equiv \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) M^* = \left(\frac{Z^I}{Z^{D*}}\right)^{-\eta^*} M^*
\end{aligned}$$

Incumbents in both countries exit with probability δ next period. Entrants exit if their productivity is so low that they cannot cover fixed overhead costs. Notice that M^E and M^{E*} represent the mass of potential entrants who pay fixed entry costs in Home and Foreign, respectively. M^I denotes the mass of Foreign firms which establish subsidiary business in the FDI host country. All Foreign firms of mass M^* serve both Home and Foreign markets through their local sales and export sales. Among them, the portion $M^I = \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) M^*$ of Foreign firms establish FDI subsidiaries in the Home country thanks to their productivity advantages. Foreign firms of mass M^I have two establishments: local headquarters in Foreign and FDI subsidiaries in Home. Both of these have the same productivity level. Their headquarters and subsidiaries serve Home and Foreign markets through local sales and export sales. That is, Foreign firms with productivity higher than Z^I manage headquarter business in Foreign and it serves Foreign and Home markets through local sales and export sales. In addition, they operate FDI affiliates in Home and it also engages in local sales and export sales. The idea here is that Foreign headquarters and Home FDI subsidiaries produce different products.

Market Demand:

Using $\left(\frac{P^F}{W}\right) = Q_L \left(\frac{P^{F*}}{W^*}\right)$ and $\left(\frac{P^{H*}}{W^*}\right) = \left(\frac{1}{Q_L}\right)\left(\frac{P^H}{W}\right)$, we derive market demand for Home firms and Foreign firms as

$$
\begin{aligned}
A &\equiv \left(\frac{W}{P}\right)^{-\sigma}\left[\left(\frac{P^H}{P}\right)^{\sigma-1}\nu\theta C + Q^{\sigma}\left(\frac{P^{H*}}{P^*}\right)^{\sigma-1}(1-\nu^*)\theta C^*\right] \\
&= \left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
A^* &\equiv \left(\frac{W^*}{P^*}\right)^{-\sigma}\left[\left(\frac{P^{F*}}{P^*}\right)^{\sigma-1}\nu^*\theta C^* + Q^{-\sigma}\left(\frac{P^F}{P}\right)^{\sigma-1}(1-\nu)\theta C\right] \\
&= \left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*} + {Q_L}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] \\
&= \left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\nu^*\theta\frac{P^*C^*}{W^*} + \frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right]
\end{aligned}
$$

Labor Market Clearing Condition:

By using $M^I = \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) M^*$, we can find labor market clearing conditions for Home and Foreign:

$$
\begin{aligned}
L - L_0 &= \left(\begin{array}{ll} & M^E F^D \\ + & M \int_{Z^D}^{\infty} \left(l^D(z) + f^D\right) \frac{dG(z)}{1-G(Z^D)} \\ + & M^* \int_{Z^I}^{\infty} \left(l^I(\alpha z) + f^I\right) \frac{dG^*(z)}{1-G^*(Z^{D*})} \end{array}\right) \\
&= \left(\begin{array}{ll} & M\left(\frac{\delta F^D}{1-G(Z^D)} + f^D\right) + M^I f^I \\ + & M \frac{J(Z^D)}{1-G(Z^D)} \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma} \left[\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta\frac{PC}{W} + Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\ + & M^I \alpha^{\sigma-1} \frac{J^*(Z^I)}{1-G^*(Z^I)} \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma} \left[\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta\frac{PC}{W} + Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \end{array}\right) \\
&= M\left(\frac{\delta F^D}{1-G(Z^D)}\right) + M f^D + M l^D(\widetilde{Z}^D) + M^I f^I + M^I l^I(\widetilde{Z}^I)
\end{aligned}
$$

$$
\begin{aligned}
L^* - L_0^* &= \left(\begin{array}{ll} & M^{E*} F^{D*} \\ + & M^* \int_{Z^{D*}}^{\infty} \left(l^{D*}(z) + f^{D*}\right) \frac{dG^*(z)}{1-G^*(Z^{D*})} \end{array}\right) \\
&= \left(\begin{array}{ll} & M^*\left(\frac{\delta F^{D*}}{1-G^*(Z^{D*})} + f^{D*}\right) \\ + & M^* \frac{J^*(Z^{D*})}{1-G^*(Z^{D*})} \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} \left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta\frac{P^*C^*}{W^*} + Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] \end{array}\right) \\
&= M^*\left(\frac{\delta F^{D*}}{1-G^*(Z^{D*})}\right) + M^* f^{D*} + M^* l^{D*}(\widetilde{Z}^{D*})
\end{aligned}
$$

Labor Market Clearing Condition and Free Entry Condition for Home: Combine Home labor market clearing condition with Home free entry condition to get:

$$
\begin{aligned}
L - L_0 &= M\left(\frac{\delta F^D}{1-G(Z^D)}\right) + M f^D + M^I f^I + M l^D\left(\widetilde{Z}^D\right) + M^I l^I\left(\widetilde{Z}^I\right) \\
&= M\frac{\pi^D(\widetilde{Z}^D)}{W} + M f^D + M^I f^I + M l^D\left(\widetilde{Z}^D\right) + M^I l^I\left(\widetilde{Z}^I\right) \\
&= M\tau_C^D\left[\frac{\tau_L^D}{\sigma-1} l^D(\widetilde{Z}^D) - f^D\right] + M f^D + M^I f^I + M l^D\left(\widetilde{Z}^D\right) + M^I l^I\left(\widetilde{Z}^I\right) \\
&= M\left[\left(\frac{\tau_C^D\tau_L^D}{\sigma-1} + 1\right) l^D(\widetilde{Z}^D) + \left(1-\tau_C^D\right) f^D\right] + M^I\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]
\end{aligned}
$$

Therefore, the mass of Home firms can be determined by:

$$M = \frac{(L-L_0) - M^I\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]}{\left(\frac{\delta F^D}{1-G(Z^D)}\right) + f^D + l^D\left(\widetilde{Z}^D\right)} = \frac{(L-L_0) - M^I\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]}{\left(\frac{\tau_C^D\tau_L^D}{\sigma-1}+1\right) l^D\left(\widetilde{Z}^D\right) + \left(1-\tau_C^D\right) f^D}$$

Labor Market Clearing Condition and Free Entry Condition for Foreign: Combine Foreign labor market clearing condition with Foreign free entry condition to get:

$$\begin{aligned}
& L^* - L_0^* \\
= \;& M^*\left(\frac{\delta F^{D*}}{1-G^*(Z^{D*})}\right) + M^* f^{D*} + M^* l^{D*}(\widetilde{Z}^{D*}) \\
= \;& \left(\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} + \frac{1}{Q_L}\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\pi^I(\widetilde{Z}^I)}{W}\right) + M^* f^{D*} + M^* l^{D*}(\widetilde{Z}^{D*}) \\
= \;& M^*\left\{\tau_C^{D*}\left[\frac{\tau_L^{D*}}{\sigma-1} l^{D*}(\widetilde{Z}^{D*}) - f^{D*}\right] + \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\tau_C^I}{Q_L}\left[\frac{\tau_L^I}{\sigma-1} l^I(\widetilde{Z}^I) - f^I\right] + f^{D*} + l^{D*}(\widetilde{Z}^{D*})\right\} \\
= \;& M^*\left\{\left(\frac{\tau_C^{D*}\tau_L^{D*}}{\sigma-1}+1\right) l^{D*}(\widetilde{Z}^{D*}) + \left(1-\tau_C^{D*}\right) f^{D*} + \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\tau_C^I}{Q_L}\left[\frac{\tau_L^I}{\sigma-1} l^I(\widetilde{Z}^I) - f^I\right]\right\}
\end{aligned}$$

Therefore, the mass of Foreign firms can be determined by:

$$\begin{aligned}
M^* &= \frac{L^* - L_0^*}{\left(\frac{\delta F^{D*}}{1-G^*(Z^{D*})}\right) + f^{D*} + l^{D*}(\widetilde{Z}^{D*})} \\
&= \frac{L^* - L_0^*}{\left\{\left(\frac{\tau_C^{D*}\tau_L^{D*}}{\sigma-1}+1\right) l^{D*}(\widetilde{Z}^{D*}) + \left(1-\tau_C^{D*}\right) f^{D*} + \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\tau_C^I}{Q_L}\left[\frac{\tau_L^I}{\sigma-1} l^I(\widetilde{Z}^I) - f^I\right]\right\}}
\end{aligned}$$

Aggregate Prices, Outputs, and Sales of Firms:

$$\begin{aligned}
\int_{Z^D}^{\infty} \frac{p^D(z)}{W}\frac{dG(z)}{1-G(Z^D)} &= \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)\int_{Z^D}^{\infty}\frac{1}{z}\frac{dG(z)}{1-G(Z^D)} &&= \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)\frac{\eta}{\eta+1}\left\{Z^D\right\}^{-1}, \\
\int_{Z^I}^{\infty} \frac{p^I(\alpha z)}{W}\frac{dG^*(z)}{1-G^*(Z^I)} &= \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)\int_{Z^I}^{\infty}\frac{1}{\alpha z}\frac{dG^*(z)}{1-G^*(Z^I)} &&= \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\frac{1}{\alpha}\right)\frac{\eta^*}{\eta^*+1}\left\{Z^I\right\}^{-1}, \\
\int_{Z^{D*}}^{\infty} \frac{p^{D*}(z)}{W^*}\frac{dG^*(z)}{1-G^*(Z^{D*})} &= \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)\int_{Z^{D*}}^{\infty}\frac{1}{z}\frac{dG^*(z)}{1-G^*(Z^{D*})} &&= \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)\frac{\eta^*}{\eta^*+1}\left\{Z^{D*}\right\}^{-1}.
\end{aligned}$$

$$
\begin{aligned}
&\textstyle\int_{Z^D}^{\infty} \left(y^D(z) + y^{D,X}(z)\right) \frac{dG(z)}{1-G(Z^D)} \\
&= \textstyle\int_{Z^D}^{\infty} \left(\frac{p^D(z)}{W}\right)^{-\sigma} \frac{dG(z)}{1-G(Z^D)} \left(\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta\frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta\frac{P^*C^*}{W^*}\right), \\
&= \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma} \textstyle\int_{Z^D}^{\infty} z^{\sigma} \frac{dG(z)}{1-G(Z^D)} \left(\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta\frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta\frac{P^*C^*}{W^*}\right), \\
&= \left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma} \frac{\eta}{\eta-\sigma} \left\{Z^D\right\}^{\sigma} \left(\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta\frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta\frac{P^*C^*}{W^*}\right),
\end{aligned}
$$

$$
\begin{aligned}
&\textstyle\int_{Z^I}^{\infty} \left(y^I(\alpha z) + y^{I,X}(\alpha z)\right) \frac{dG^*(z)}{1-G^*(Z^I)} \\
&= \textstyle\int_{Z^I}^{\infty} \left(\frac{p^I(\alpha z)}{W}\right)^{-\sigma} \frac{dG^*(z)}{1-G^*(Z^I)} \left(\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta\frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta\frac{P^*C^*}{W^*}\right), \\
&= \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\frac{1}{\alpha}\right)^{-\sigma} \textstyle\int_{Z^I}^{\infty} z^{\sigma} \frac{dG^*(z)}{1-G^*(Z^I)} \left(\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta\frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta\frac{P^*C^*}{W^*}\right), \\
&= \left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma} \frac{\eta^*}{\eta^*-\sigma} \left\{\alpha Z^I\right\}^{\sigma} \left(\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta\frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta\frac{P^*C^*}{W^*}\right),
\end{aligned}
$$

$$
\begin{aligned}
&\textstyle\int_{Z^{D*}}^{\infty} \left(y^{D*}(z) + y^{D,X*}(z)\right) \frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&= \textstyle\int_{Z^{D*}}^{\infty} \left(\frac{p^{D*}(z)}{W^*}\right)^{-\sigma} \frac{dG^*(z)}{1-G^*(Z^{D*})} \left(\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta\frac{P^*C^*}{W^*} + Q_L{}^{-\sigma} \left(\frac{P^F}{W}\right)^{\sigma-1} (1-\nu)\theta\frac{PC}{W}\right), \\
&= \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} \textstyle\int_{Z^{D*}}^{\infty} z^{\sigma} \frac{dG^*(z)}{1-G^*(Z^{D*})} \left(\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta\frac{P^*C^*}{W^*} + Q_L{}^{-\sigma} \left(\frac{P^F}{W}\right)^{\sigma-1} (1-\nu)\theta\frac{PC}{W}\right), \\
&= \left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} \frac{\eta^*}{\eta^*-\sigma} \left\{Z^{D*}\right\}^{\sigma} \left(\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta\frac{P^*C^*}{W^*} + Q_L{}^{-\sigma} \left(\frac{P^F}{W}\right)^{\sigma-1} (1-\nu)\theta\frac{PC}{W}\right).
\end{aligned}
$$

$$\int_{Z^D}^{\infty} \frac{p^D(z)}{W} \left(y^D(z) + y^{D,X}(z)\right) \frac{dG(z)}{1-G(Z^D)}$$
$$= \int_{Z^D}^{\infty} \left(\frac{p^D(z)}{W}\right)^{1-\sigma} \frac{dG(z)}{1-G(Z^D)} \left(\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta \frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \frac{P^*C^*}{W^*}\right),$$
$$= \left(\frac{1}{\rho} \frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} \int_{Z^D}^{\infty} z^{\sigma-1} \frac{dG(z)}{1-G(Z^D)} \left(\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta \frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \frac{P^*C^*}{W^*}\right),$$
$$= \left(\frac{1}{\rho} \frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} \left(\widetilde{Z}^D\right)^{\sigma-1} \left(\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta \frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \frac{P^*C^*}{W^*}\right),$$

$$\int_{Z^I}^{\infty} \frac{p^I(\alpha z)}{W} \left(y^I(\alpha z) + y^{I,X}(\alpha z)\right) \frac{dG^*(z)}{1-G^*(Z^I)}$$
$$= \int_{Z^I}^{\infty} \left(\frac{p^I(\alpha z)}{W}\right)^{1-\sigma} \frac{dG^*(z)}{1-G^*(Z^I)} \left(\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta \frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \frac{P^*C^*}{W^*}\right),$$
$$= \left(\frac{1}{\rho} \frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} \int_{Z^I}^{\infty} (\alpha z)^{\sigma-1} \frac{dG^*(z)}{1-G^*(Z^I)} \left(\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta \frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \frac{P^*C^*}{W^*}\right),$$
$$= \left(\frac{1}{\rho} \frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} \left(\widetilde{Z}^I\right)^{\sigma-1} \left(\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta \frac{PC}{W} + Q_L{}^{\sigma} \left(\frac{P^{H*}}{W^*}\right)^{\sigma-1} (1-\nu^*)\theta \frac{P^*C^*}{W^*}\right),$$

$$\int_{Z^{D*}}^{\infty} \frac{p^{D*}(z)}{W^*} \left(y^{D*}(z) + y^{D,X*}(z)\right) \frac{dG^*(z)}{1-G^*(Z^{D*})}$$
$$= \int_{Z^{D*}}^{\infty} \left(\frac{p^{D*}(z)}{W^*}\right)^{1-\sigma} \frac{dG^*(z)}{1-G^*(Z^{D*})} \left(\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta \frac{P^*C^*}{W^*} + Q_L{}^{-\sigma} \left(\frac{P^F}{W}\right)^{\sigma-1} (1-\nu)\theta \frac{PC}{W}\right),$$
$$= \left(\frac{1}{\rho} \frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} \int_{Z^{D*}}^{\infty} z^{\sigma-1} \frac{dG^*(z)}{1-G^*(Z^{D*})} \left(\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta \frac{P^*C^*}{W^*} + Q_L{}^{-\sigma} \left(\frac{P^F}{W}\right)^{\sigma-1} (1-\nu)\theta \frac{PC}{W}\right),$$
$$= \left(\frac{1}{\rho} \frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} \left(\widetilde{Z}^{D*}\right)^{\sigma-1} \left(\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta \frac{P^*C^*}{W^*} + Q_L{}^{-\sigma} \left(\frac{P^F}{W}\right)^{\sigma-1} (1-\nu)\theta \frac{PC}{W}\right).$$

Government Budget Balance: Home government budget balance implies

$$\begin{aligned}
T &= M \int_{Z^D}^{\infty} \left(\tau_L^D - 1\right) W l^D(z) \frac{dG(z)}{1-G(Z^D)} \\
&+ M \int_{Z^D}^{\infty} \left(1-\tau_V^D\right) p^D(z) \left(y^D(z) + y^{D,X}(z)\right) \frac{dG(z)}{1-G(Z^D)} \\
&+ M \int_{Z^D}^{\infty} \left(1-\tau_C^D\right) \left\{\tau_V^D p^D(z) \left(y^D(z) + y^{D,X}(z)\right) - \tau_L^D W l^D(z) - f^D W\right\} \frac{dG(z)}{1-G(Z^D)} \\
&+ M^I \int_{Z^I}^{\infty} \left(\tau_L^I - 1\right) W l^I(\alpha z) \frac{dG^*(z)}{1-G^*(Z^I)} \\
&+ M^I \int_{Z^I}^{\infty} \left(1-\tau_V^I\right) p^I(\alpha z) \left(y^I(\alpha z) + y^{I,X}(\alpha z)\right) \frac{dG^*(z)}{1-G^*(Z^I)} \\
&+ M^I \int_{Z^I}^{\infty} \left(1-\tau_C^I\right) \left\{\tau_V^I p^I(\alpha z) \left(y^I(\alpha z) + y^{I,X}(\alpha z)\right) - \tau_L^I W l^I(\alpha z) - f^I W\right\} \frac{dG^*(z)}{1-G^*(Z^I)}
\end{aligned}$$

That is,

$$
\begin{aligned}
\frac{T}{W} &= M\left(\tau_L^D - 1\right) l^D(\widetilde{Z}^D) \\
&+ M\left(1-\tau_V^D\right)\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^D(z)+y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)} \\
&+ M\left(1-\tau_C^D\right)\left\{\tau_V^D\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^D(z)+y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)} - \tau_L^D l^D(\widetilde{Z}^D) - f^D\right\} \\
&+ M^I\left(\tau_L^I - 1\right) l^I(\widetilde{Z}^I) \\
&+ M^I\left(1-\tau_V^I\right)\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^I(\alpha z)+y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)} \\
&+ M^I\left(1-\tau_C^I\right)\left\{\tau_V^I\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^I(\alpha z)+y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)} - \tau_L^I l^I(\widetilde{Z}^I) - f^I\right\}
\end{aligned}
$$

Foreign government budget balance implies

$$
\begin{aligned}
T^* &= M^*\int_{Z^{D*}}^{\infty}\left(\tau_L^{D*}-1\right) W^* l^{D*}(z)\frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&+ M^*\int_{Z^{D*}}^{\infty}\left(1-\tau_V^{D*}\right) p^{D*}(z)\left(y^{D*}(z)+y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&+ M^*\int_{Z^{D*}}^{\infty}\left(1-\tau_C^{D*}\right)\left\{\tau_V^{D*}p^{D*}(z)\left(y^{D*}(z)+y^{D,X*}(z)\right) - \tau_L^{D*}W^* l^{D*}(z) - f^{D*}W^*\right\}\frac{dG^*(z)}{1-G^*(Z^{D*})}
\end{aligned}
$$

That is,

$$
\begin{aligned}
\frac{T^*}{W^*} &= M^*\left(\tau_L^{D*}-1\right) l^{D*}(\widetilde{Z}^{D*}) \\
&+ M^*\left(1-\tau_V^{D*}\right)\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)+y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})} \\
&+ M^*\left(1-\tau_C^{D*}\right)\left\{\tau_V^{D*}\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)+y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})} - \tau_L^{D*} l^{D*}(\widetilde{Z}^{D*}) - f^{D*}\right\}
\end{aligned}
$$

The Resource Constraint:

Home household budget constraint can be written as

$$\begin{aligned}
\frac{P}{W}C &= L + \frac{T}{W} = \frac{P_0}{W}C_0 + \frac{P^H}{W}C^H + \frac{P^F}{W}C^F \\
&= \begin{array}{l} \frac{P_0}{W}C_0 + M\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^D(z)\right)\frac{dG(z)}{1-G(Z^D)} \\ +\ M^I\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^I(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)} + M^*Q_L\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})}\end{array}
\end{aligned}$$

Use household budget constraint, labor market clearing condition, free entry condition, and government budget balance to obtain Home resource constraint:

$$\begin{aligned}
&\left(\frac{P_0}{W}C_0 - L_0\right) + M^*Q_L\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})} + M^I\frac{\pi^I(\widetilde{Z}^I)}{W} \\
=\ & M\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)} + M^I\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)}
\end{aligned}$$

Similarly, Foreign household budget constraint can be written as

$$\begin{aligned}
\frac{P^*}{W^*}C^* &= L^* + \frac{T^*}{W^*} = \frac{P_0^*}{W^*}C_0^* + \frac{P^{F*}}{W^*}C^{F*} + \frac{P^{H*}}{W^*}C^{H*} \\
&= \begin{array}{l} \frac{P_0^*}{W^*}C_0^* + M^*\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})} \\ +\ M\frac{1}{Q_L}\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)} + M^I\frac{1}{Q_L}\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)}\end{array}
\end{aligned}$$

Use household budget constraint, labor market clearing condition, free entry condition, and government budget balance to obtain Foreign resource constraint:

$$\begin{aligned}
&\left(Q_L L_0^* - Q_L\frac{P_0^*}{W^*}C_0^*\right) + M^*Q_L\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})} + M^I\frac{\pi^I(\widetilde{Z}^I)}{W} \\
=\ & M\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)} + M^I\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)}
\end{aligned}$$

Under the partial equilibrium with $W = \epsilon W^* = P_0 = \epsilon P_0^*$ and $Q_L = 1$, these two Home and Foreign resource constraints are identical since $Q_L L_0^* - Q_L\frac{P_0^*}{W^*}C_0^* = \frac{P_0}{W}C_0 - L_0$ holds. This is also true in the general equilibrium with zero weight on the homogeneous good: $\theta_0 = 0$ and $C_0 = C_0^* = L_0 = L_0^* = P_0 = P_0^* = 0$ (note that $0^0 = 1$).

A.2.1 Aggregate and Average Measures over Firms and Welfare Decomposition

Define

$$
\begin{aligned}
M^{DI} &\equiv M + M^{I} \\
\widetilde{Z}^{DI} &\equiv \left[\frac{1}{M^{DI}}\left(M\left(\frac{\tau_L^D}{\tau_V^D}\frac{1}{\widetilde{Z}^D}\right)^{1-\sigma} + M^{I}\left(\frac{\tau_L^I}{\tau_V^I}\frac{1}{\widetilde{Z}^I}\right)^{1-\sigma}\right)\right]^{\frac{1}{\sigma-1}} \\
\log\left(M^{HF}\right) &\equiv \nu\log\left(M^{DI}\right) + (1-\nu)\log\left(M^{*}\right) \\
\log\left(\frac{1}{\widetilde{Z}^{HF}}\right) &\equiv \nu\log\left(\frac{1}{\widetilde{Z}^{DI}}\right) + (1-\nu)\log\left(Q_L\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right) \\
\log\left(M^{HF*}\right) &\equiv \nu^{*}\log\left(M^{*}\right) + (1-\nu^{*})\log\left(M^{DI}\right) \\
\log\left(\frac{1}{\widetilde{Z}^{HF*}}\right) &\equiv \nu^{*}\log\left(\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right) + (1-\nu^{*})\log\left(\frac{1}{Q_L}\frac{1}{\widetilde{Z}^{DI}}\right)
\end{aligned}
$$

where $\rho \equiv \frac{\sigma-1}{\sigma}$.

Welfare Decomposition for Home consumption:

We can decompose Home consumption index C^H of domestically-produced goods by:

$$\begin{aligned}
C^H &= \left(M\left[y^D\left(\widetilde{Z}^D\right)\right]^\rho + M^I\left[y^I\left(\widetilde{Z}^I\right)\right]^\rho\right)^{\frac{1}{\rho}} = \left(\int_{\omega\in\Omega}\left[y^D(\omega)\right]^\rho d\omega + \int_{\omega^*\in\Omega^I}\left[y^I(\omega^*)\right]^\rho d\omega^*\right)^{\frac{1}{\rho}} \\
&= \left(\int_{Z^D}^\infty\left(\frac{p^D(z)}{P}\right)^{1-\sigma}\frac{MdG(z)}{(1-G(Z^D))} + \int_{Z^I}^\infty\left(\frac{p^I(\alpha z)}{P}\right)^{1-\sigma}\frac{M^I dG^*(z)}{(1-G^*(Z^I))}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{P^H}{P}\right)^{\sigma-1}\nu\theta C \\
&= \left(\int_{Z^D}^\infty\left(\frac{p^D(z)}{W}\right)^{1-\sigma}\frac{MdG(z)}{(1-G(Z^D))} + \int_{Z^I}^\infty\left(\frac{p^I(\alpha z)}{W}\right)^{1-\sigma}\frac{M^I dG^*(z)}{(1-G^*(Z^I))}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{W}{P}\right)^{-\sigma}\left(\frac{P^H}{P}\right)^{\sigma-1}\nu\theta C \\
&= \left(\int_{Z^D}^\infty\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\frac{1}{z}\right)^{1-\sigma}\frac{MdG(z)}{(1-G(Z^D))} + \int_{Z^I}^\infty\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\frac{1}{\alpha z}\right)^{1-\sigma}\frac{M^I dG^*(z)}{(1-G^*(Z^I))}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} \\
&= \left(M\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left(\widetilde{Z}^D\right)^{\sigma-1} + M^I\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left(\widetilde{Z}^I\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} \\
&= \left(M\left[\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\frac{1}{\widetilde{Z}^D}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}\right]^{\frac{\sigma-1}{\sigma}} + M^I\left[\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\frac{1}{\widetilde{Z}^I}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}\right]^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(M\left[y^D\left(\widetilde{Z}^D\right)\right]^\rho + M^I\left[y^I\left(\widetilde{Z}^I\right)\right]^\rho\right)^{\frac{1}{\rho}} \\
&= \left(M\left(\frac{\tau_L^D}{\tau_V^D}\frac{1}{\widetilde{Z}^D}\right)^{1-\sigma} + M^I\left(\frac{\tau_L^I}{\tau_V^I}\frac{1}{\widetilde{Z}^I}\right)^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{1}{\rho}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} \\
&= \left(M^{DI}\right)^{\frac{1}{\rho}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} = \left(M^{DI}\right)^{\frac{1}{\rho}-1}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-1}\nu\theta\frac{PC}{W} \\
&= \underbrace{\left(M^{DI}\right)^{\frac{1}{\sigma-1}}}_{\text{Variety Effect}}\underbrace{\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-1}}_{\text{Productivity Effect}}\underbrace{\nu\theta\left(L+\frac{T}{W}\right)}_{\text{Income Effect}}
\end{aligned}$$

where variety effect increases the welfare due to the love of variety (extensive margin); productivity effect lowers the marginal cost and the overall price; inccome effect raises output per each variety due to the increase in total demand (intensive margin). Note that the price index for Home domestic composite consumption can be solved as:

$$\begin{aligned}
\left(\frac{P^H}{W}\right)^{1-\sigma} &= M\left(\frac{p^D(\widetilde{Z}^D)}{W}\right)^{1-\sigma} + M^I\left(\frac{p^I(\widetilde{Z}^I)}{W}\right)^{1-\sigma} \\
&= \left(M\left(\frac{\tau_L^D}{\tau_V^D}\frac{1}{\widetilde{Z}^D}\right)^{1-\sigma} + M^I\left(\frac{\tau_L^I}{\tau_V^I}\frac{1}{\widetilde{Z}^I}\right)^{1-\sigma}\right)\left(\frac{1}{\rho}\right)^{1-\sigma} \\
&= M^{DI}\left(\widetilde{Z}^{DI}\right)^{\sigma-1}\left(\frac{1}{\rho}\right)^{1-\sigma} \\
&= M^{DI}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{1-\sigma}
\end{aligned}$$

Likewise, we can decompose Home consumption index C^F on imported goods:

$$\begin{aligned}
C^F &= \left(\int_{\omega^*\in\Omega^*}\left[y^{D,X*}(\omega^*)\right]^\rho d\omega^*\right)^{\frac{1}{\rho}} \\
&= \left[M^*\int_{Z^{D*}}^{\infty}\left(\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{z}\right)^{-\sigma} Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right)^{\frac{\sigma-1}{\sigma}}\frac{dG^*(z)}{1-G^*(Z^{D*})}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[M^*\int_{Z^{D*}}^{\infty} z^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^{D*})}\left(\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[M^*(\widetilde{Z}^{D*})^{\sigma-1}\left(\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[M^*\left(\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-\sigma} Q_L{}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
&= (M^*)^{\frac{1}{\rho}}\, y^{D,X*}(\widetilde{Z}^{D*}) \\
&= (M^*)^{\frac{1}{\rho}}\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W} \\
&= \underbrace{(M^*)^{\frac{1}{\sigma-1}}}_{\text{Variety Effect}}\underbrace{\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-1}}_{\text{Productivity Effect}}\underbrace{(1-\nu)\theta\left(L+\frac{T}{W}\right)}_{\text{Income Effect}}
\end{aligned}$$

Its price index can be found to be:

$$\begin{aligned}
\left(\frac{P^F}{W}\right)^{1-\sigma} &= \int_{\omega^*\in\Omega^*}\left(Q_L\frac{p^{D*}(\omega^*)}{W^*}\right)^{1-\sigma}d\omega^* = M^*\left(Q_L\frac{p^{D*}(\widetilde{Z}^{D*})}{W^*}\right)^{1-\sigma} = M^*\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{1-\sigma} \\
\frac{P^F}{W}C^F &= \int_{\omega^*\in\Omega^*}Q_L\frac{p^{D*}(\omega^*)}{W^*}y^{D,X*}(\omega^*)d\omega^* = M^*\left(Q_L\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})}\right) \\
&= (1-\nu)\theta\frac{PC}{W}
\end{aligned}$$

Therefore, the composite consumption of differentiated goods in Home is given by:

$$\begin{aligned}
C_1 &= \frac{\left(C^H\right)^{\nu}\left(C^F\right)^{1-\nu}}{(\nu)^{\nu}(1-\nu)^{1-\nu}} \\
&= \frac{\left(\left(M^{DI}\right)^{\frac{1}{\rho}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}\right)^{\nu}\left((M^*)^{\frac{1}{\rho}}\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right)^{1-\nu}}{(\nu)^{\nu}(1-\nu)^{1-\nu}} \\
&= \left(\left(M^{DI}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\theta\frac{PC}{W}\right)^{\nu}\left((M^*)^{\frac{\sigma}{\sigma-1}}\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}\theta\frac{PC}{W}\right)^{1-\nu} \\
&= \left(\left(M^{DI}\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-1}\theta\frac{PC}{W}\right)^{\nu}\left((M^*)^{\frac{1}{\sigma-1}}\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-1}\theta\frac{PC}{W}\right)^{1-\nu} \\
&= \left[\left(M^{DI}\right)^{\nu}(M^*)^{1-\nu}\right]^{\frac{1}{\sigma-1}}\left[\frac{1}{\rho}\left(\frac{1}{\widetilde{Z}^{DI}}\right)^{\nu}\left(Q_L\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{1-\nu}\right]^{-1}\theta\frac{PC}{W} \\
&= \underbrace{\left[M^{HF}\right]^{\frac{1}{\sigma-1}}}_{\text{Variety Effect: Extensive Margin}}\underbrace{\left[\frac{1}{\rho}\left(\frac{1}{\widetilde{Z}^{HF}}\right)\right]^{-1}}_{\text{Productivity Effect}}\underbrace{\theta\left(L+\frac{T}{W}\right)}_{\text{Income Effect: Intensive Margin}}
\end{aligned}$$

where variety effect increases the welfare since consumers have more varieties to consume and their utilities feature the love of variety (extensive margin); productivity effect reduces the marginal cost and the aggregate price decreases; income effect raises output per each variety due to the rise in total demand (intensive margin). The price index in Home is given by:

$$
\begin{aligned}
\frac{P_1}{W} &= \left(\frac{P^H}{W}\right)^{\nu}\left(\frac{P^F}{W}\right)^{1-\nu} \\
&= \left[\left(M^{DI}\right)^{\frac{1}{1-\sigma}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)\right]^{\nu}\left[\left(M^{*}\right)^{\frac{1}{1-\sigma}}\left(Q_L\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)\right]^{1-\nu} \\
&= \left[\left(M^{DI}\right)^{\nu}\left(M^{*}\right)^{1-\nu}\right]^{\frac{1}{1-\sigma}}\frac{1}{\rho}\left(\frac{1}{\widetilde{Z}^{DI}}\right)^{\nu}\left(Q_L\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{1-\nu} \\
&= \underbrace{\left[\frac{1}{M^{HF}}\right]^{\frac{1}{\sigma-1}}}_{\text{Competition Effect}}\ \underbrace{\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{HF}}\right)}_{\text{Productivity Effect}}
\end{aligned}
$$

where $V = C = \Phi\left(C_0\right)^{\theta_0}\left(C_1\right)^{\theta_1}$ and $P = \left(P_0\right)^{\theta_0}\left(P_1\right)^{\theta}$.

Welfare Decomposition for Foreign consumption:

Symmetrically, we can decompose Foreign consumption index C^{F*} of locally-produced goods by:

$$
\begin{aligned}
C^{F*} &= \left(\int_{\omega^*\in\Omega^*}\left[y^{D*}(\omega^*)\right]^{\rho}d\omega^*\right)^{\frac{1}{\rho}} \\
&= \left[M^*\int_{Z^{D*}}^{\infty}\left(\left(\frac{p^{D*}(z)}{W^*}\right)^{-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}\right)^{\frac{\sigma-1}{\sigma}}\frac{dG^*(z)}{1-G^*(Z^{D*})}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[M^*\int_{Z^{D*}}^{\infty}\left(\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{z}\right)^{-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}\right)^{\frac{\sigma-1}{\sigma}}\frac{dG^*(z)}{1-G^*(Z^{D*})}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[M^*\left(\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left(M^*\right)^{\frac{1}{\rho}}y^{D*}(\widetilde{Z}^{D*}) \\
&= \left(M^*\right)^{\frac{1}{\rho}}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*} \\
&= \left(M^*\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-1}\nu^*\theta\left(L^*+\frac{T^*}{W^*}\right)
\end{aligned}
$$

The price index for Foreign domestic composite consumption can be solved as:

$$\begin{aligned}
\left(\frac{P^{F*}}{W^*}\right)^{1-\sigma} &= \int_{\omega^*\in\Omega^*}\left(\frac{p^{D*}(\omega^*)}{W^*}\right)^{1-\sigma} d\omega^* = M^*\left(\frac{p^{D*}(\widetilde{Z}^{D*})}{W^*}\right)^{1-\sigma} = M^*\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{1-\sigma} \\
\frac{P^{F*}}{W^*}C^{F*} &= \int_{\omega^*\in\Omega^*}\frac{p^{D*}(\omega^*)}{W^*}y^{D*}(\omega^*)d\omega^* = M^*\left(\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})}\right) = \nu^*\theta\frac{P^*C^*}{W^*}
\end{aligned}$$

Likewise, we can decompose Foreign consumption index C^{H*} on imported goods:

$$\begin{aligned}
C^{H*} &= \left(M\left[y^{D,X}(\widetilde{Z}^D)\right]^{\rho} + M^I\left[y^{I,X}(\widetilde{Z}^I)\right]^{\rho}\right)^{\frac{1}{\rho}} \\
&= \left(\int_{\omega\in\Omega}\left[y^{D,X}(\omega)\right]^{\rho}d\omega + \int_{\omega^*\in\Omega^I}\left[y^{I,X}(\omega^*)\right]^{\rho}d\omega^*\right)^{\frac{1}{\rho}} \\
&= \begin{pmatrix} \int_{Z^D}^{\infty}\left(\frac{p^D(z)}{W}\right)^{1-\sigma}\frac{MdG(z)}{(1-G(Z^D))} \\ +\ \int_{Z^I}^{\infty}\left(\frac{p^I(\alpha z)}{W}\right)^{1-\sigma}\frac{M^IdG^*(z)}{(1-G^*(Z^I))}\end{pmatrix}^{\frac{\sigma}{\sigma-1}} {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*} \\
&= \left(M\left(\frac{p^D(\widetilde{Z}^D)}{W}\right)^{1-\sigma} + M^I\left(\frac{p^I(\widetilde{Z}^I)}{W}\right)^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}} {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*} \\
&= \left(M^{DI}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}} {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*} \\
&= \left(M^{DI}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\frac{1}{Q_L}\right)^{-\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*} \\
&= \left(M^{DI}\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\frac{1}{Q_L}\right)^{-1}(1-\nu^*)\theta\left(L^*+\frac{T^*}{W^*}\right)
\end{aligned}$$

Its price index can be found to be:

$$\begin{aligned}
\left(\frac{P^{H*}}{W^*}\right)^{1-\sigma} &= \int_{\omega\in\Omega}\left(\frac{1}{Q_L}\frac{p^D(\omega)}{W}\right)^{1-\sigma}d\omega + \int_{\omega^*\in\Omega^I}\left(\frac{1}{Q_L}\frac{p^I(\omega^*)}{W}\right)^{1-\sigma}d\omega^* \\
&= M\left(\frac{1}{Q_L}\frac{p^D(\widetilde{Z}^D)}{W}\right)^{1-\sigma} + M^I\left(\frac{1}{Q_L}\frac{p^I(\widetilde{Z}^I)}{W}\right)^{1-\sigma} \\
&= M^{DI}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\frac{1}{Q_L}\right)^{1-\sigma}
\end{aligned}$$

Therefore, the composite consumption in Foreign is given by:

$$
\begin{aligned}
C_1^* &= \frac{\left(C^{F*}\right)^{\nu^*}\left(C^{H*}\right)^{1-\nu^*}}{\left(\nu^*\right)^{\nu^*}\left(1-\nu^*\right)^{1-\nu^*}} \\
&= \frac{\left(\left(M^*\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-1}\nu^*\theta\left(L^*+\frac{T^*}{W^*}\right)\right)^{\nu^*}\left(\left(M^{DI}\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\frac{1}{Q_L}\right)^{-1}(1-\nu^*)\theta\left(L^*+\frac{T^*}{W^*}\right)\right)^{1-\nu^*}}{\left(\nu^*\right)^{\nu^*}\left(1-\nu^*\right)^{1-\nu^*}} \\
&= \left(\left(M^*\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{-1}\theta\left(L^*+\frac{T^*}{W^*}\right)\right)^{\nu^*}\left(\left(M^{DI}\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{Q_L}\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)^{-1}\theta\left(L^*+\frac{T^*}{W^*}\right)\right)^{1-\nu^*} \\
&= \left[\left(M^*\right)^{\nu^*}\left(M^{DI}\right)^{1-\nu^*}\right]^{\frac{1}{\sigma-1}}\left[\frac{1}{\rho}\left(\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{\nu^*}\left(\frac{1}{Q_L}\frac{1}{\widetilde{Z}^{DI}}\right)^{1-\nu^*}\right]^{-1}\theta\left(L^*+\frac{T^*}{W^*}\right) \\
&= \underbrace{\left[M^{HF*}\right]^{\frac{1}{\sigma-1}}}_{\text{Variety Effect: Extensive Margin}}\underbrace{\left[\frac{1}{\rho}\left(\frac{1}{\widetilde{Z}^{HF*}}\right)\right]^{-1}}_{\text{Productivity Effect}}\underbrace{\theta\left(L^*+\frac{T^*}{W^*}\right)}_{\text{Income Effect: Intensive Margin}}
\end{aligned}
$$

where variety effect increases the welfare since consumers have more varieties to consume and their utilities feature the love of variety (extensive margin); productivity effect reduces the marginal cost and the aggregate price decreases; income effect raises output per each variety due to the rise in total demand (intensive margin). The price index in Foreign is derived as:

$$
\begin{aligned}
\frac{P_1^*}{W^*} &= \left(\frac{P^{F*}}{W^*}\right)^{\nu^*}\left(\frac{P^{H*}}{W^*}\right)^{1-\nu^*} \\
&= \left[\left(M^*\right)^{\frac{1}{1-\sigma}}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)\right]^{\nu^*}\left[\left(M^{DI}\right)^{\frac{1}{1-\sigma}}\left(\frac{1}{Q_L}\frac{1}{\rho}\frac{1}{\widetilde{Z}^{DI}}\right)\right]^{1-\nu^*} \\
&= \left[\left(M^*\right)^{\nu^*}\left(M^{DI}\right)^{1-\nu^*}\right]^{\frac{1}{1-\sigma}}\frac{1}{\rho}\left(\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right)^{\nu^*}\left(\frac{1}{Q_L}\frac{1}{\widetilde{Z}^{DI}}\right)^{1-\nu^*} \\
&= \underbrace{\left[\frac{1}{M^{HF*}}\right]^{\frac{1}{\sigma-1}}}_{\text{Competition Effect}}\underbrace{\left(\frac{1}{\rho}\frac{1}{\widetilde{Z}^{HF*}}\right)}_{\text{Productivity Effect}}
\end{aligned}
$$

where $V^* = C^* = \Phi\left(C_0^*\right)^{\theta_0}\left(C_1^*\right)^{\theta_1}$ and $P^* = \left(P_0^*\right)^{\theta_0}\left(P_1^*\right)^{\theta}$.

Aggregate Measures across Firms:

Aggregate labor demand, revenue, profits, and costs are given by

$$\begin{aligned}
L^D &= \int_{Z^D}^{\infty} l^D(z)\frac{MdG(z)}{(1-G(Z^D))} = M\cdot l^D(\widetilde{Z}^D) = M\cdot\frac{1}{\tau_L^D}\frac{\sigma-1}{\sigma}\frac{r^D(\widetilde{Z}^D)}{W}\\
\frac{R^D}{W} &= \int_{Z^D}^{\infty} \tau_C^D\frac{r^D(z)}{W}\frac{MdG(z)}{(1-G(Z^D))} = M\cdot\tau_C^D\frac{r^D(\widetilde{Z}^D)}{W}\\
\frac{\Pi^D}{W} &= \int_{Z^D}^{\infty} \frac{\pi^D(z)}{W}\frac{MdG(z)}{(1-G(Z^D))} = M\cdot\frac{\pi^D(\widetilde{Z}^D)}{W} = M\cdot\tau_C^D\left[\frac{1}{\sigma}\frac{r^D(\widetilde{Z}^D)}{W} - f^D\right]\\
\frac{\Xi^D}{W} &= \int_{Z^D}^{\infty} \frac{\xi^D(z)}{W}\frac{MdG(z)}{(1-G(Z^D))} = M\cdot\frac{\xi^D(\widetilde{Z}^D)}{W} = M\cdot\tau_C^D\left[\frac{\sigma-1}{\sigma}\frac{r^D(\widetilde{Z}^D)}{W} + f^D\right]
\end{aligned}$$

$$\begin{aligned}
L^I &= \int_{Z^I}^{\infty} l^I(z)\frac{M^IdG^*(z)}{(1-G^*(Z^I))} = M^I\cdot l^I(\widetilde{Z}^I) = M^I\cdot\frac{1}{\tau_L^I}\frac{\sigma-1}{\sigma}\frac{r^I(\widetilde{Z}^I)}{W}\\
\frac{R^I}{W} &= \int_{Z^I}^{\infty} \tau_C^I\frac{r^I(z)}{W}\frac{M^IdG^*(z)}{(1-G^*(Z^I))} = M^I\cdot\tau_C^I\frac{r^I(\widetilde{Z}^I)}{W}\\
\frac{\Pi^I}{W} &= \int_{Z^I}^{\infty} \frac{\pi^I(z)}{W}\frac{M^IdG^*(z)}{(1-G^*(Z^I))} = M^I\cdot\frac{\pi^I(\widetilde{Z}^I)}{W} = M^I\cdot\tau_C^I\left[\frac{1}{\sigma}\frac{r^I(\widetilde{Z}^I)}{W} - f^I\right]\\
\frac{\Xi^I}{W} &= \int_{Z^I}^{\infty} \frac{\xi^I(z)}{W}\frac{M^IdG^*(z)}{(1-G^*(Z^I))} = M^I\cdot\frac{\xi^I(\widetilde{Z}^I)}{W} = M^I\cdot\tau_C^I\left[\frac{\sigma-1}{\sigma}\frac{r^I(\widetilde{Z}^I)}{W} + f^I\right]
\end{aligned}$$

$$\begin{aligned}
L^{D*} &= \int_{Z^{D*}}^{\infty} l^{D*}(z)\frac{M^*dG^*(z)}{(1-G^*(Z^{D*}))} = M^*\cdot l^{D*}(\widetilde{Z}^{D*}) = M^*\cdot\frac{1}{\tau_L^{D*}}\frac{\sigma-1}{\sigma}\frac{r^{D*}(\widetilde{Z}^{D*})}{W^*}\\
\frac{R^{D*}}{W^*} &= \int_{Z^{D*}}^{\infty} \tau_C^{D*}\frac{r^{D*}(z)}{W^*}\frac{M^*dG^*(z)}{(1-G^*(Z^{D*}))} = M^*\cdot\tau_C^{D*}\frac{r^{D*}(\widetilde{Z}^{D*})}{W^*}\\
\frac{\Pi^{D*}}{W^*} &= \int_{Z^{D*}}^{\infty} \frac{\pi^{D*}(z)}{W^*}\frac{M^*dG^*(z)}{(1-G^*(Z^{D*}))} = M^*\cdot\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} = M^*\cdot\tau_C^{D*}\left[\frac{1}{\sigma}\frac{r^{D*}(\widetilde{Z}^{D*})}{W^*} - f^{D*}\right]\\
\frac{\Xi^{D*}}{W^*} &= \int_{Z^{D*}}^{\infty} \frac{\xi^{D*}(z)}{W^*}\frac{M^*dG^*(z)}{(1-G^*(Z^{D*}))} = M^*\cdot\frac{\xi^{D*}(\widetilde{Z}^{D*})}{W^*} = M^*\cdot\tau_C^{D*}\left[\frac{\sigma-1}{\sigma}\frac{r^{D*}(\widetilde{Z}^{D*})}{W^*} + f^{D*}\right]
\end{aligned}$$

Market Share of Home local firms and Home FDI firms among Home firms, excluding Foreign local firms are defined as:

$$MS^{H,D} = \frac{\frac{R^D}{W}}{\frac{R^D}{W}+\frac{R^I}{W}} \quad \text{and} \quad MS^{H,I} = \frac{\frac{R^I}{W}}{\frac{R^D}{W}+\frac{R^I}{W}}$$

Market Share of Home local firms, Home FDI firms, and Foreign local firms in two countries are defined as:

$$
\begin{aligned}
MS^D &= \frac{\frac{R^D}{W}}{\frac{R^D}{W} + \frac{R^I}{W} + Q_L \frac{R^{D^*}}{W^*}} \\
MS^I &= \frac{\frac{R^I}{W}}{\frac{R^D}{W} + \frac{R^I}{W} + Q_L \frac{R^{D^*}}{W^*}} \\
MS^{D*} &= \frac{Q_L \frac{R^{D^*}}{W^*}}{\frac{R^D}{W} + \frac{R^I}{W} + Q_L \frac{R^{D^*}}{W^*}}
\end{aligned}
$$

A.2.2 Equilibrium Conditions and Proof of Propositions

A.2.2.1 Total Equilibrium Conditions

Recall we assume there is only one differentiated-good sector. Due to the presence of the homogeneous-good sector, we get $W = \epsilon W^* = P_0 = \epsilon P_0^*$ and $Q_L = \frac{\epsilon W^*}{W} = 1$. Since the labor real exchange rate Q_L is already pinned down, the resource constraint (A.2.25) needs not to be included in the equilibrium system. Instead of using labor market clearing conditions, we use budget constraints of households to find the equilibrium allocation: $\frac{PC}{W} = L + \frac{T}{W}$ and $\frac{P^*C^*}{W^*} = L^* + \frac{T^*}{W^*}$. We solve for eighteen endogenous variables: Z^D, Z^I, Z^{D*}, M, M^*, M^I, M^E, M^{E*}, $\frac{P^H}{W}$, $\frac{P^{H*}}{W^*}$, $\frac{P^{F*}}{W^*}$, $\frac{P^F}{W}$, $\frac{P}{W}$, $\frac{P^*}{W^*}$, C, C^*, $\frac{T}{W}$, $\frac{T^*}{W^*}$. When we take the lump-sum transfer to be chosen exogenously, then one of wedges among $\left[\tau_C^D, \tau_V^D, \tau_L^D, \tau_C^I, \tau_V^I, \tau_L^I\right]$ will be endogenously determined through Home government budget balance (A.2.26).

Definitions & Substitutes:

$$
\begin{aligned}
\sigma &= \frac{1}{1-\rho} \text{ and } \rho = \frac{\sigma-1}{\sigma} \\
A &\equiv \left[\left(\frac{P^H}{W}\right)^{\sigma-1} \nu\theta\frac{PC}{W} + Q_L{}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
A^* &\equiv \left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \nu^*\theta\frac{P^*C^*}{W^*} + Q_L{}^{-\sigma}\left(\frac{P^{F}}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] \\
&= \left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\nu^*\theta\frac{P^*C^*}{W^*} + \frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right]
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{\eta} &\equiv \frac{\eta\left(z_{min}\right)^{\eta}}{\eta-\sigma+1} \\
\widetilde{\eta}^* &\equiv \frac{\eta^*\left(z^*_{min}\right)^{\eta^*}}{\eta^*-\sigma+1} \\
1-G(z) &= (z_{min})^{\eta} z^{-\eta} \\
1-G^*(z) &= (z^*_{min})^{\eta^*} z^{-\eta^*} \\
J(z) &= \widetilde{\eta}\left\{z\right\}^{-\eta+\sigma-1} \\
J^*(z) &= \widetilde{\eta}^*\left\{z\right\}^{-\eta^*+\sigma-1} \\
\frac{J(z)}{1-G(z)} &= \frac{\eta}{\eta-\sigma+1}\left(z\right)^{\sigma-1} \\
\frac{J^*(z)}{1-G^*(z)} &= \frac{\eta^*}{\eta^*-\sigma+1}\left(z\right)^{\sigma-1}
\end{aligned}
$$

$$
\begin{aligned}
M^{DI} &\equiv M + M^I \\
\widetilde{Z}^{DI} &\equiv \left[\frac{1}{M^{DI}}\left(M\left(\frac{\tau_L^D}{\tau_V^D}\frac{1}{\widetilde{Z}^D}\right)^{1-\sigma} + M^I\left(\frac{\tau_L^I}{\tau_V^I}\frac{1}{\widetilde{Z}^I}\right)^{1-\sigma}\right)\right]^{\frac{1}{\sigma-1}} \\
\log\left(M^{HF}\right) &\equiv \nu\log\left(M^{DI}\right) + (1-\nu)\log\left(M^*\right) \\
\log\left(\frac{1}{\widetilde{Z}^{HF}}\right) &\equiv \nu\log\left(\frac{1}{\widetilde{Z}^{DI}}\right) + (1-\nu)\log\left(Q_L\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right) \\
\log\left(M^{HF*}\right) &\equiv \nu^*\log\left(M^*\right) + (1-\nu^*)\log\left(M^{DI}\right) \\
\log\left(\frac{1}{\widetilde{Z}^{HF*}}\right) &\equiv \nu^*\log\left(\frac{\tau_L^{D*}}{\tau_V^{D*}}\frac{1}{\widetilde{Z}^{D*}}\right) + (1-\nu^*)\log\left(\frac{1}{Q_L}\frac{1}{\widetilde{Z}^{DI}}\right)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{Z}^D &\equiv \left[\int_{Z^D}^{\infty} z^{\sigma-1}\frac{dG(z)}{1-G(Z^D)}\right]^{\frac{1}{\sigma-1}} = \left[\frac{J(Z^D)}{1-G(Z^D)}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta}{\eta-\sigma+1}\right]^{\frac{1}{\sigma-1}} Z^D \\
\widetilde{Z}^I &\equiv \left[\int_{Z^I}^{\infty} (\alpha z)^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^I)}\right]^{\frac{1}{\sigma-1}} = \alpha\left[\frac{J^*(Z^I)}{1-G^*(Z^I)}\right]^{\frac{1}{\sigma-1}} = \alpha\left[\frac{\eta^*}{\eta^*-\sigma+1}\right]^{\frac{1}{\sigma-1}} Z^I \\
\widetilde{Z}^{D*} &\equiv \left[\int_{Z^{D*}}^{\infty} z^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^{D*})}\right]^{\frac{1}{\sigma-1}} = \left[\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta^*}{\eta^*-\sigma+1}\right]^{\frac{1}{\sigma-1}} Z^{D*} \\
\frac{J(Z^D)}{1-G(Z^D)} &= \left(\widetilde{Z}^D\right)^{\sigma-1} \text{ and } \left(\frac{\widetilde{Z}^D}{Z^D}\right)^{\sigma-1} = \int_{Z^D}^{\infty}\left(\frac{z}{Z^D}\right)^{\sigma-1}\frac{dG(z)}{1-G(Z^D)} = \frac{\eta}{\eta-\sigma+1} \\
\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)} &= \left(\widetilde{Z}^I\right)^{\sigma-1} \text{ and } \left(\frac{\widetilde{Z}^I}{\alpha Z^I}\right)^{\sigma-1} = \int_{Z^I}^{\infty}\left(\frac{z}{Z^I}\right)^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^I)} = \frac{\eta^*}{\eta^*-\sigma+1} \\
\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})} &= \left(\widetilde{Z}^{D*}\right)^{\sigma-1} \text{ and } \left(\frac{\widetilde{Z}^{D*}}{Z^{D*}}\right)^{\sigma-1} = \int_{Z^{D*}}^{\infty}\left(\frac{z}{Z^{D*}}\right)^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^{D*})} = \frac{\eta^*}{\eta^*-\sigma+1}
\end{aligned}
$$

Zero Profit Cutoff Productivity Conditions:

$$
\begin{aligned}
&\left(Z^D\right)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
= \; &\left[f^D + \left(\tfrac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]
\end{aligned}
\tag{A.2.9}
$$

$$(\alpha Z^I)^{\sigma-1}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W}+{Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]=f^I \tag{A.2.10}$$

$$(Z^{D*})^{\sigma-1}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*}+{Q_L}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right]=f^{D*} \tag{A.2.11}$$

Free Entry for Home firms:

$$\begin{aligned} & F^D \\ = & \left(\tfrac{1-G(Z^D)}{\delta}\right)\tau_C^D\left\{\tfrac{J(Z^D)}{1-G(Z^D)}\tfrac{\tau_V^D}{\sigma}\left(\tfrac{1}{\rho}\tfrac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left[\begin{array}{cc} & \left(\tfrac{P^H}{W}\right)^{\sigma-1}\nu\theta\tfrac{PC}{W} \\ + & {Q_L}^{\sigma}\left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\tfrac{P^*C^*}{W^*}\end{array}\right]-f^D\right\} \end{aligned} \tag{A.2.12}$$

Free Entry for Foreign firms:

$$\begin{aligned} & F^{D*} \\ = & \begin{array}{l} \left(\tfrac{1-G^*(Z^{D*})}{\delta}\right)\tau_C^{D*}\left\{\tfrac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\tfrac{\tau_V^{D*}}{\sigma}\left(\tfrac{1}{\rho}\tfrac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left[\begin{array}{cc} & \left(\tfrac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\tfrac{P^*C^*}{W^*} \\ + & {Q_L}^{-\sigma}\left(\tfrac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\tfrac{PC}{W}\end{array}\right]-f^{D*}\right\} \\ +\left(\tfrac{1-G^*(Z^I)}{Q_L\cdot\delta}\right)\tau_C^I\left\{\alpha^{\sigma-1}\tfrac{J^*(Z^I)}{1-G^*(Z^I)}\tfrac{\tau_V^I}{\sigma}\left(\tfrac{1}{\rho}\tfrac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left[\begin{array}{cc} & \left(\tfrac{P^H}{W}\right)^{\sigma-1}\nu\theta\tfrac{PC}{W} \\ + & {Q_L}^{\sigma}\left(\tfrac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\tfrac{P^*C^*}{W^*}\end{array}\right]-f^I\right\} \end{array} \end{aligned} \tag{A.2.13}$$

Price Index for Home:

$$\left(\frac{P^H}{W}\right)^{1-\sigma} = M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} + M^I\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} \tag{A.2.14}$$

$$\left(\frac{P^F}{W}\right) = Q_L\left(\frac{P^{F*}}{W^*}\right) \tag{A.2.15}$$

Price Index for Foreign:

$$\left(\frac{P^{F*}}{W^*}\right)^{1-\sigma} = M^*\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} \tag{A.2.16}$$

$$\left(\frac{P^{H*}}{W^*}\right) = \left(\frac{1}{Q_L}\right)\left(\frac{P^H}{W}\right) \tag{A.2.17}$$

The Evolution of the Mass of Firms:

$$M^E = \frac{\delta M}{(1-G(Z^D))} \tag{A.2.18}$$

$$M^{E*} = \frac{\delta M^*}{(1-G^*(Z^{D*}))} \tag{A.2.19}$$

$$M^I = \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)M^* = \left(\frac{Z^{D*}}{Z^I}\right)^{\eta^*}M^* \tag{A.2.20}$$

$$\frac{M^I}{1-G^*(Z^I)} = \frac{M^*}{1-G^*(Z^{D*})}$$

Aggregate Price Index:

$$\left(\frac{P}{W}\right)^{1-\sigma} = \left(\frac{P_0}{W}\right)^{\theta_0(1-\sigma)}\left(\frac{P^H}{W}\right)^{(1-\sigma)\nu\theta}\left(Q_L\frac{P^{F*}}{W^*}\right)^{(1-\sigma)(1-\nu)\theta} \tag{A.2.21}$$

$$\left(\frac{P^*}{W^*}\right)^{1-\sigma} = \left(\frac{P_0^*}{W^*}\right)^{\theta_0(1-\sigma)}\left(\frac{P^{F*}}{W^*}\right)^{(1-\sigma)\nu^*\theta}\left(\frac{1}{Q_L}\frac{P^H}{W}\right)^{(1-\sigma)(1-\nu^*)\theta} \tag{A.2.22}$$

where $W = P_0$, $W^* = P_0^*$, and $\theta_0 + \theta = 1$ hold.

Labor Market Clearing Condition in Home:

$$L - L_0 = \left(\begin{array}{cl} & M\left(\frac{\delta F^D}{1-G(Z^D)} + f^D\right) + M^I f^I \\ + & M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\ + & M^I\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma}\left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \end{array} \right) \tag{A.2.23}$$

Labor Market Clearing Condition in Foreign:

$$L^* - L_0^* = \left(\begin{array}{cl} & M^*\left(\frac{\delta F^{D*}}{1-G^*(Z^{D*})} + f^{D*}\right) \\ + & M^*\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma}\left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*} + {Q_L}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] \end{array} \right) \tag{A.2.24}$$

The Resource Constraint:

$$\begin{aligned} & \left(\frac{P_0}{W}C_0 - L_0\right) + M^* Q_L \frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right] \quad \text{(A.2.25)} \\ & + M^I\tau_C^I\left\{\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] - f^I\right\} \\ = \; & M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\ & + M^I\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \end{aligned}$$

Home Government Budget Balance:

$$
\begin{aligned}
\frac{T}{W} &= M\left(\tau_L^D-1\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W}+Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&+ M\left(1-\tau_V^D\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W}+Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&+ M\left(1-\tau_C^D\right)\left\{\frac{J(Z^D)}{1-G(Z^D)}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W}+Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]-f^D\right\} \\
&+ M^I\left(\tau_L^I-1\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W}+Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&+ M^I\left(1-\tau_V^I\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W}+Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&+ M^I\left(1-\tau_C^I\right)\left\{\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W}+Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right]-f^I\right\}
\end{aligned}
\tag{A.2.26}
$$

Foreign Government Budget Balance:

$$
\begin{aligned}
\frac{T^*}{W^*} &= M^*\left(\tau_L^{D*}-1\right)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\nu^*\theta\frac{P^*C^*}{W^*}+\frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right] \\
&+ M^*\left(1-\tau_V^{D*}\right)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\nu^*\theta\frac{P^*C^*}{W^*}+\frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right] \\
&+ M^*\left(1-\tau_C^{D*}\right)\left\{\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\nu^*\theta\frac{P^*C^*}{W^*}+\frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right]-f^{D*}\right\}
\end{aligned}
\tag{A.2.27}
$$

A.2.2.2 Proof of Propositions

By combining equations for cutoff productivity (A.2.9) and (A.2.10) with equations for Home free entry condition (A.2.12), we can pin down Z^D and Z^I,

$$Z^D = z_{min} \left(\frac{\tau_C^D}{\delta}\right)^{\frac{1}{\eta}} \left(\frac{\eta}{F^D(\eta - \sigma + 1)}\right)^{\frac{1}{\eta}} \left[\frac{f^D(\sigma - 1)}{\eta} + \left(\frac{1}{\lambda} - 1\right)\left(\zeta f^D - \chi F^D\right)\right]^{\frac{1}{\eta}} \tag{A.2.28}$$

$$\alpha Z^I = Z^D \left(\frac{f^I}{\left[f^D + \left(\frac{1}{\lambda} - 1\right)\left(\zeta f^D - \chi F^D\right)\right]}\right)^{\frac{1}{\sigma-1}} \left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}} \frac{\tau_L^I}{\tau_L^D} \tag{A.2.29}$$

$$= z_{min} \left(\frac{\tau_C^D}{\delta}\right)^{\frac{1}{\eta}} \left(\frac{\eta}{F^D(\eta - \sigma + 1)}\right)^{\frac{1}{\eta}} \frac{\left[\frac{f^D(\sigma-1)}{\eta} + \left(\frac{1}{\lambda} - 1\right)\left(\zeta f^D - \chi F^D\right)\right]^{\frac{1}{\eta}}}{\left[f^D + \left(\frac{1}{\lambda} - 1\right)\left(\zeta f^D - \chi F^D\right)\right]^{\frac{1}{\sigma-1}}} \left(f^I\right)^{\frac{1}{\sigma-1}} \left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}} \frac{\tau_L^I}{\tau_L^D}$$

Proposition 1. *If the term* $\left(\frac{f^I}{\left[f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]}\right)^{\frac{1}{\sigma-1}} \left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}} \frac{\tau_L^I}{\tau_L^D}$ *is less than one, the cutoff productivity of FDI firms is lower than that of local firms:* $\alpha Z^I < Z^D$.

Proof. This proposition follows from the equation (A.2.29). □

Proposition 2. *Suppose there are no wedges from taxes:* $\tau_V^D = \tau_V^I = \tau_L^D = \tau_L^I = 1$ *in Home. In the case that there is no financial friction (*$\lambda = 1$*), the cutoff productivity of FDI firms is not lower than that of domestic firms:* $\alpha Z^I \geq Z^D$.

Proof. This follows from the equation (A.2.29) with the assumption of fixed production costs: $f^I \geq f^D$. □

Proposition 3. *Suppose there are no wedges from taxes:* $\tau_V^D = \tau_V^I = \tau_L^D = \tau_L^I = 1$ *in Home. In the case that local firms and FDI firms have the same fixed production costs:* $f^I = f^D$*, the cutoff productivity of FDI firms is lower than that of domestic firms:* $\alpha Z^I < Z^D$ *under financial frictions,* $\lambda < 1$.

Proof. This follows from the equation (A.2.29) with the assumption of $\sigma > 1$, $k > 0$, $0 < \lambda < 1$, and $\zeta f^D - \chi F^D > 0$. □

A.2.2.3 Characterization of the equilibrium system

We restrict our model to have only one sector for the differentiated goods. The model is tractable enough to allow for the closed-form equilibrium allocation. Since there are tradable homogeneous goods which are produced in all coutries, we have $W = \epsilon W^* = P_0 = \epsilon P_0^*$ and $Q_L = \frac{\epsilon W^*}{W} = 1$. Firstly, by combining equations (A.2.9), (A.2.10), (A.2.11), (A.2.12), and (A.2.13), we can pin down Z^D, Z^I, and Z^{D*}:

$$\begin{aligned}
Z^D &= \left\{ \left(\frac{\tau_C^D}{\delta}\right) \left(\frac{f^D}{F^D}\right) \left(\frac{(\sigma-1)(z_{min})^\eta}{\eta-\sigma+1}\right) \left[1 + \frac{\eta}{\sigma-1}\left(\frac{1}{\lambda}-1\right)\left(\zeta - \chi\frac{F^D}{f^D}\right)\right] \right\}^{\frac{1}{\eta}} \\
\alpha Z^I &= Z^D \left(\frac{f^I}{\left[f^D + \left(\frac{1}{\lambda}-1\right)(\zeta f^D - \chi F^D)\right]} \right)^{\frac{1}{\sigma-1}} \left(\frac{\tau_V^D}{\tau_V^I}\right)^{\frac{\sigma}{\sigma-1}} \frac{\tau_L^I}{\tau_L^D} \\
Z^{D*} &= \left\{ \frac{\tau_C^{D*} f^{D*} \frac{(\sigma-1)\left(z^*_{min}\right)^{\eta^*}}{\eta^*-\sigma+1}}{\delta F^{D*} - \left(\frac{1}{Q_L}\right) \tau_C^I f^I \frac{(\sigma-1)\left(z^*_{min}\right)^{\eta^*}}{\eta^*-\sigma+1} (Z^I)^{-\eta^*}} \right\}^{\frac{1}{\eta^*}}
\end{aligned}$$

Therefore, we can find the market demand A and A^* from equations (A.2.9), (A.2.10), and (A.2.11):

$$\begin{aligned}
A &= \left(\frac{P^H}{W}\right)^{\sigma-1} \left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] = \frac{\left[f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]}{(Z^D)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}} \\
A^* &= \left(\frac{P^{F*}}{W^*}\right)^{\sigma-1} \left[\nu^*\theta\frac{P^*C^*}{W^*} + \frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right] = \frac{f^{D*}}{(Z^{D*})^{\sigma-1}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}}
\end{aligned}$$

where we have $\frac{\left[f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]}{(Z^D)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}} = \frac{f^I}{(\alpha Z^I)^{\sigma-1}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}}$.

Combining these with price indexes (A.2.14) and (A.2.16), we can obtain:

$$\begin{aligned}
\frac{1}{A^*}\left[\nu^*\theta\frac{P^*C^*}{W^*}+\frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right] &= \left(\frac{P^{F*}}{W^*}\right)^{1-\sigma} \\
&= M^*\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} \\
\frac{1}{A}\left[\nu\theta\frac{PC}{W}+Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] &= \left(\frac{P^H}{W}\right)^{1-\sigma} \\
&= \begin{array}{c} M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} \\ +\quad M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\end{array}
\end{aligned}$$

That is,

$$\begin{aligned}
\left[\frac{(1-\nu)\theta}{Q_L}\frac{PC}{W}+\nu^*\theta\frac{P^*C^*}{W^*}\right] &= M^*\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^* \\
\left[\nu\theta\frac{PC}{W}+Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] &= \begin{array}{c} M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A \\ +\quad M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A\end{array}
\end{aligned}$$

where we can solve for $\frac{PC}{W}$ and $\frac{P^*C^*}{W^*}$ by using:

$$\begin{pmatrix} \frac{(1-\nu)\theta}{Q_L} & \nu^*\theta \\ \nu\theta & Q_L(1-\nu^*)\theta \end{pmatrix}^{-1} = \frac{1}{\theta(\nu+\nu^*-1)}\begin{pmatrix} -Q_L(1-\nu^*) & \nu^* \\ \nu & -\frac{(1-\nu)}{Q_L} \end{pmatrix}$$

Therefore, we obtain:

$$\frac{PC}{W} = \frac{1}{\theta(\nu+\nu^*-1)}\begin{pmatrix} (M)\,(\nu^*)\,\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A \\ +\quad (M^*)\,(-Q_L(1-\nu^*))\,\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^* \\ +\quad (M^*)\,(\nu^*)\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A \end{pmatrix}$$

$$\frac{P^*C^*}{W^*} = \frac{1}{\theta(\nu+\nu^*-1)}\begin{pmatrix} (M)\left(\frac{-(1-\nu)}{Q_L}\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A \\ +\quad (M^*)\,(\nu)\,\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^* \\ +\quad (M^*)\left(\frac{-(1-\nu)}{Q_L}\right)\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A \end{pmatrix}$$

Note that Home and Foreign budget constraints are given by $\frac{PC}{W} = L + \frac{T}{W}$ and $\frac{P^*C^*}{W^*} = L^* + \frac{T^*}{W^*}$. By substituting out for $\frac{T}{W}$ and $\frac{T*}{W*}$, we can rewrite government budget balance (A.2.26) and (A.2.27) as:

$$
\begin{aligned}
\frac{PC}{W} - L \quad &= \quad M\left(\tau_L^D - 1\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma} A \\
&+ \quad M\left(1 - \tau_V^D\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} A \\
&+ \quad M\left(1 - \tau_C^D\right)\left\{\frac{J(Z^D)}{1-G(Z^D)}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} A - f^D\right\} \\
&+ \quad M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(\tau_L^I - 1\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma} A \\
&+ \quad M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(1 - \tau_V^I\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} A \\
&+ \quad M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(1 - \tau_C^I\right)\left\{\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma} A - f^I\right\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{P^*C^*}{W^*} - L^* \quad &= \quad M^*\left(\tau_L^{D*} - 1\right)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} A^* \\
&+ \quad M^*\left(1 - \tau_V^{D*}\right)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} A^* \\
&+ \quad M^*\left(1 - \tau_C^{D*}\right)\left\{\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma} A^* - f^{D*}\right\}
\end{aligned}
$$

By feeding equations for $\frac{PC}{W}$ and $\frac{P^*C^*}{W^*}$ into government budget balance, we obtain two linear simultaneous equations for M and M^*:

$$\frac{1}{\theta(\nu+\nu^*-1)}\left(\begin{array}{cl} & (M)\,(\nu^*)\,\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A \\ + & (M^*)\,(-Q_L(1-\nu^*))\,\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^* \\ + & (M^*)\,(\nu^*)\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A \end{array}\right)-L$$

$$= M\left[\begin{array}{cl} & \left(\tau_L^D-1\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma}A \\ + & \left(1-\tau_V^D\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A \\ + & \left(1-\tau_C^D\right)\left\{\frac{J(Z^D)}{1-G(Z^D)}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A-f^D\right\} \end{array}\right]$$

$$+M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left[\begin{array}{cl} & \left(\tau_L^I-1\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma}A \\ + & \left(1-\tau_V^I\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A \\ + & \left(1-\tau_C^I\right)\left\{\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A-f^I\right\} \end{array}\right]$$

and

$$\frac{1}{\theta(\nu+\nu^*-1)}\left(\begin{array}{cl} & (M)\left(\frac{-(1-\nu)}{Q_L}\right)\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A \\ + & (M^*)\,(\nu)\,\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^* \\ + & (M^*)\left(\frac{-(1-\nu)}{Q_L}\right)\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A \end{array}\right)-L^*$$

$$= M^*\left[\begin{array}{cl} & \left(\tau_L^{D*}-1\right)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma}A^* \\ + & \left(1-\tau_V^{D*}\right)\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^* \\ + & \left(1-\tau_C^{D*}\right)\left\{\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^*-f^{D*}\right\} \end{array}\right]$$

Define:

$$\Theta^D \equiv \frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}A,\quad \Theta^I \equiv \frac{\alpha^{\sigma-1}J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}A,\quad \Theta^{D*} \equiv \frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}A^*$$

Then we can find M and M^* by solving:

$$L = M\left[\begin{array}{cl} + & \frac{(\nu^*)\Theta^D}{\theta(\nu+\nu^*-1)} \\ - & \left(\tau_L^D-1\right)\Theta^D\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-1} \\ - & \left(1-\tau_V^D\right)\Theta^D \\ - & \left(1-\tau_C^D\right)\left\{\frac{\tau_V^D}{\sigma}\Theta^D - f^D\right\} \end{array}\right] + M^*\left[\begin{array}{cl} + & \frac{(-Q_L(1-\nu^*))\Theta^{D*}}{\theta(\nu+\nu^*-1)} \\ + & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{(\nu^*)\Theta^I}{\theta(\nu+\nu^*-1)} \\ - & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(\tau_L^I-1\right)\Theta^I\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-1} \\ - & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(1-\tau_V^I\right)\Theta^I \\ - & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(1-\tau_C^I\right)\left\{\frac{\tau_V^I}{\sigma}\Theta^I - f^I\right\} \end{array}\right]$$

$$L^* = M\left[\begin{array}{cl} + & \frac{\left(\frac{-(1-\nu)}{Q_L}\right)\Theta^D}{\theta(\nu+\nu^*-1)} \end{array}\right] + M^*\left[\begin{array}{cl} + & \frac{(\nu)\Theta^{D*}}{\theta(\nu+\nu^*-1)} \\ + & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\left(\frac{-(1-\nu)}{Q_L}\right)\Theta^I}{\theta(\nu+\nu^*-1)} \\ - & \left(\tau_L^{D*}-1\right)\Theta^{D*}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-1} \\ - & \left(1-\tau_V^{D*}\right)\Theta^{D*} \\ - & \left(1-\tau_C^{D*}\right)\left\{\frac{\tau_V^{D*}}{\sigma}\Theta^{D*} - f^{D*}\right\} \end{array}\right]$$

which corresponds to:

$$\begin{bmatrix} \left(\begin{array}{cl} + & \frac{(\nu^*)\Theta^D}{\theta(\nu+\nu^*-1)} \\ - & \left(\tau_L^D-1\right)\Theta^D\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-1} \\ - & \left(1-\tau_V^D\right)\Theta^D \\ - & \left(1-\tau_C^D\right)\left\{\frac{\tau_V^D}{\sigma}\Theta^D - f^D\right\} \end{array}\right) & \left(\begin{array}{cl} + & \frac{(-Q_L(1-\nu^*))\Theta^{D*}}{\theta(\nu+\nu^*-1)} \\ + & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{(\nu^*)\Theta^I}{\theta(\nu+\nu^*-1)} \\ - & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(\tau_L^I-1\right)\Theta^I\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-1} \\ - & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(1-\tau_V^I\right)\Theta^I \\ - & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\left(1-\tau_C^I\right)\left\{\frac{\tau_V^I}{\sigma}\Theta^I - f^I\right\} \end{array}\right) \\ \left(\begin{array}{cl} + & \frac{\left(\frac{-(1-\nu)}{Q_L}\right)\Theta^D}{\theta(\nu+\nu^*-1)} \end{array}\right) & \left(\begin{array}{cl} + & \frac{(\nu)\Theta^{D*}}{\theta(\nu+\nu^*-1)} \\ + & \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)\frac{\left(\frac{-(1-\nu)}{Q_L}\right)\Theta^I}{\theta(\nu+\nu^*-1)} \\ - & \left(\tau_L^{D*}-1\right)\Theta^{D*}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-1} \\ - & \left(1-\tau_V^{D*}\right)\Theta^{D*} \\ - & \left(1-\tau_C^{D*}\right)\left\{\frac{\tau_V^{D*}}{\sigma}\Theta^{D*} - f^{D*}\right\} \end{array}\right) \end{bmatrix} \begin{pmatrix} M \\ M^* \end{pmatrix} = \begin{pmatrix} L \\ L^* \end{pmatrix}$$

where $M^I = \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)M^* = \left(\frac{Z^{D*}}{Z^I}\right)^{\eta^*}M^*$, $M^E = \frac{\delta M}{(1-G(Z^D))}$, and $M^{E*} = \frac{\delta M^*}{(1-G^*(Z^{D*}))}$.

Therefore, we have found Z^D, Z^I, Z^{D*}, A, A^*, M, M^*, M^I, M^E, and M^{E*}. We can derive the rest of endogenous variables as follows.

$$\left(\frac{P^H}{W}\right) = \left\{M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma} + M^I\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}\right\}^{\frac{1}{1-\sigma}},$$

$$\left(\frac{P^{F*}}{W^*}\right) = (M^*)^{\frac{1}{1-\sigma}}\left(\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\right)^{\frac{1}{1-\sigma}}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right),$$

where $\left(\frac{P^F}{W}\right) = Q_L\left(\frac{P^{F*}}{W^*}\right)$ and $\left(\frac{P^{H*}}{W^*}\right) = \left(\frac{1}{Q_L}\right)\left(\frac{P^H}{W}\right)$.

$$\left(\frac{P}{W}\right) = \left(\frac{P_0}{W}\right)^{\theta_0}\left(\frac{P^H}{W}\right)^{\nu\theta}\left(Q_L\frac{P^{F*}}{W^*}\right)^{(1-\nu)\theta}$$

$$\left(\frac{P^*}{W^*}\right) = \left(\frac{P_0^*}{W^*}\right)^{\theta_0}\left(\frac{P^{F*}}{W^*}\right)^{\nu^*\theta}\left(\frac{1}{Q_L}\frac{P^H}{W}\right)^{(1-\nu^*)\theta}$$

where $W = P_0$, $W^* = P_0^*$, and $\theta_0 + \theta = 1$ hold. Then we have:

$$\frac{PC}{W} = \left(\begin{array}{cl} & (M)\,\frac{(\nu^*)\Theta^D}{\theta(\nu+\nu^*-1)} \\ + & (M^*)\,\frac{(-Q_L(1-\nu^*))\Theta^{D*}}{\theta(\nu+\nu^*-1)} \\ + & \left(M^I\right)\frac{(\nu^*)\Theta^I}{\theta(\nu+\nu^*-1)} \end{array}\right), \quad \text{and} \quad \frac{P^*C^*}{W^*} = \left(\begin{array}{cl} & (M)\,\frac{\left(\frac{-(1-\nu)}{Q_L}\right)\Theta^D}{\theta(\nu+\nu^*-1)} \\ + & (M^*)\,\frac{(\nu)\Theta^{D*}}{\theta(\nu+\nu^*-1)} \\ + & \left(M^I\right)\frac{\left(\frac{-(1-\nu)}{Q_L}\right)\Theta^I}{\theta(\nu+\nu^*-1)} \end{array}\right),$$

$\frac{T}{W} = \frac{PC}{W} - L$, $\frac{T^*}{W^*} = \frac{P^*C^*}{W^*} - L^*$, $C = \frac{\left(\frac{PC}{W}\right)}{\left(\frac{P}{W}\right)}$, $C^* = \frac{\left(\frac{P^*C^*}{W^*}\right)}{\left(\frac{P^*}{W^*}\right)}$, $C_0 = \left(\frac{P_0}{W}\right)^{-1}\theta_0\left(\frac{PC}{W}\right)$, $C_0^* = \left(\frac{P_0^*}{W^*}\right)^{-1}\theta_0\left(\frac{P^*C^*}{W^*}\right)$,

$$L_0 = L - \left(\begin{array}{cl} & M\left(\frac{\delta F^D}{1-G(Z^D)} + f^D\right) + M^I f^I \\ + & M\frac{J(Z^D)}{1-G(Z^D)}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma} A \\ + & M^I\alpha^{\sigma-1}\frac{J^*(Z^I)}{1-G^*(Z^I)}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma} A \end{array}\right)$$

$$L_0^* = L^* - \left(\begin{array}{cl} & M^*\left(\frac{\delta F^{D*}}{1-G^*(Z^{D*})} + f^{D*}\right) \\ + & M^*\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} A^* \end{array}\right)$$

where $W = P_0$ and $W^* = P_0^*$.

A.2.3 Average Measures under Pareto distribution

If we assume that productivity of firms follows the Pareto distribution, then it makes average measures constant. Note that we have derived the market demand as:

$$
\begin{aligned}
A &\equiv \left[\left(\frac{P^H}{W}\right)^{\sigma-1}\nu\theta\frac{PC}{W} + {Q_L}^{\sigma}\left(\frac{P^{H*}}{W^*}\right)^{\sigma-1}(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] = \left(\frac{P^H}{W}\right)^{\sigma-1}\left[\nu\theta\frac{PC}{W} + Q_L(1-\nu^*)\theta\frac{P^*C^*}{W^*}\right] \\
&= \frac{\left[f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right]}{(Z^D)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}} = \frac{f^I}{(\alpha Z^I)^{\sigma-1}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}} \\
A^* &\equiv \left[\left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\nu^*\theta\frac{P^*C^*}{W^*} + {Q_L}^{-\sigma}\left(\frac{P^F}{W}\right)^{\sigma-1}(1-\nu)\theta\frac{PC}{W}\right] = \left(\frac{P^{F*}}{W^*}\right)^{\sigma-1}\left[\nu^*\theta\frac{P^*C^*}{W^*} + \frac{1}{Q_L}(1-\nu)\theta\frac{PC}{W}\right] \\
&= \frac{f^{D*}}{(Z^{D*})^{\sigma-1}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}}
\end{aligned}
$$

Also, average productivity is linear in cutoff productivity:

$$
\begin{aligned}
\widetilde{Z}^D &= \left[\int_{Z^D}^{\infty} z^{\sigma-1}\frac{dG(z)}{1-G(Z^D)}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta}{\eta-\sigma+1}\right]^{\frac{1}{\sigma-1}} Z^D \\
\widetilde{Z}^I &= \left[\int_{Z^I}^{\infty} (\alpha z)^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^I)}\right]^{\frac{1}{\sigma-1}} = \alpha\left[\frac{\eta^*}{\eta^*-\sigma+1}\right]^{\frac{1}{\sigma-1}} Z^I \\
\widetilde{Z}^{D*} &= \left[\int_{Z^{D*}}^{\infty} z^{\sigma-1}\frac{dG^*(z)}{1-G^*(Z^{D*})}\right]^{\frac{1}{\sigma-1}} = \left[\frac{\eta^*}{\eta^*-\sigma+1}\right]^{\frac{1}{\sigma-1}} Z^{D*}
\end{aligned}
$$

Then we can characterize average labor as follows:

$$
\begin{aligned}
l^D(\widetilde{Z}^D) &= \left(\widetilde{Z}^D\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma} A = \left(\frac{\eta}{\eta-\sigma+1}\right)\left(Z^D\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{-\sigma}\frac{\left(f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right)}{\left(Z^D\right)^{\sigma-1}\frac{\tau_V^D}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^D}{\tau_V^D}\right)^{1-\sigma}} \\
&= \left(f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right)\left(\frac{\sigma-1}{\tau_L^D}\right)\left(\frac{\eta}{\eta-\sigma+1}\right) \\
l^I\left(\widetilde{Z}^I\right) &= \left(\widetilde{Z}^I\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma} A = \alpha^{\sigma-1}\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)\left(Z^I\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{-\sigma}\left(\frac{f^I}{\left(\alpha Z^I\right)^{\sigma-1}\frac{\tau_V^I}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^I}{\tau_V^I}\right)^{1-\sigma}}\right) \\
&= f^I\left(\frac{\sigma-1}{\tau_L^I}\right)\left(\frac{\eta^*}{\eta^*-\sigma+1}\right) \\
l^{D*}\left(\widetilde{Z}^{D*}\right) &= \left(\widetilde{Z}^{D*}\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma} A^* = \left(\frac{\eta^*}{\eta^*-\sigma+1}\right)\left(Z^{D*}\right)^{\sigma-1}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{-\sigma}\left(\frac{f^{D*}}{\left(Z^{D*}\right)^{\sigma-1}\frac{\tau_V^{D*}}{\sigma}\left(\frac{1}{\rho}\frac{\tau_L^{D*}}{\tau_V^{D*}}\right)^{1-\sigma}}\right) \\
&= f^{D*}\left(\frac{\sigma-1}{\tau_L^{D*}}\right)\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)
\end{aligned}
$$

And average revenues are given by:

$$
\begin{aligned}
\tau_C^D\frac{r^D(\widetilde{Z}^D)}{W} &= \tau_C^D\left(\frac{\tau_L^D}{\rho}\right) l^D(\widetilde{Z}^D) = \tau_C^D\sigma\left(f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right)\left(\frac{\eta}{\eta-\sigma+1}\right) \\
\tau_C^I\frac{r^I(\widetilde{Z}^I)}{W} &= \tau_C^I\left(\frac{\tau_L^I}{\rho}\right) l^I(\widetilde{Z}^I) = \tau_C^I\sigma f^I\left(\frac{\eta^*}{\eta^*-\sigma+1}\right) \\
\tau_C^{D*}\frac{r^{D*}(\widetilde{Z}^{D*})}{W^*} &= \tau_C^{D*}\left(\frac{\tau_L^{D*}}{\rho}\right) l^{D*}(\widetilde{Z}^{D*}) = \tau_C^{D*}\sigma f^{D*}\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)
\end{aligned}
$$

And average profits are given by:

$$
\begin{aligned}
\frac{\pi^D(\widetilde{Z}^D)}{W} &= \tau_C^D\left[\frac{\tau_L^D}{\sigma-1} l^D(\widetilde{Z}^D) - f^D\right] = \tau_C^D\left[\left(f^D+\left(\frac{1}{\lambda}-1\right)\left(\zeta f^D-\chi F^D\right)\right)\left(\frac{\eta}{\eta-\sigma+1}\right) - f^D\right] \\
\frac{\pi^I(\widetilde{Z}^I)}{W} &= \tau_C^I\left[\frac{\tau_L^I}{\sigma-1} l^I(\widetilde{Z}^I) - f^I\right] = \tau_C^I\left[f^I\left(\frac{\eta^*}{\eta^*-\sigma+1}\right) - f^I\right] \\
\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} &= \tau_C^{D*}\left[\frac{\tau_L^{D*}}{\sigma-1} l^{D*}(\widetilde{Z}^{D*}) - f^{D*}\right] = \tau_C^{D*}\left[f^{D*}\left(\frac{\eta^*}{\eta^*-\sigma+1}\right) - f^{D*}\right]
\end{aligned}
$$

And average costs are given by:

$$\frac{\xi^D(\widetilde{Z}^D)}{W} = \tau_C^D\left[\frac{\sigma-1}{\sigma}\frac{r^D(\widetilde{Z}^D)}{W}+f^D\right] = \tau_C^D\left[(\sigma-1)\left(f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)\right)\left(\frac{\eta}{\eta-\sigma+1}\right)+f^D\right]$$

$$\frac{\xi^I(\widetilde{Z}^I)}{W} = \tau_C^I\left[\frac{\sigma-1}{\sigma}\frac{r^I(\widetilde{Z}^I)}{W}+f^I\right] = \tau_C^I\left[(\sigma-1)f^I\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)+f^I\right]$$

$$\frac{\xi^{D*}(\widetilde{Z}^{D*})}{W^*} = \tau_C^{D*}\left[\frac{\sigma-1}{\sigma}\frac{r^{D*}(\widetilde{Z}^{D*})}{W^*}+f^{D*}\right] = \tau_C^{D*}\left[(\sigma-1)f^{D*}\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)+f^{D*}\right]$$

A.3 Empirical Target Moments in Calibration

This section presents the details of empirical target moments used in our model calibration. We take our model to the Chinese data in 2000. We define a FDI firm as a firm of which capital is occupied by foreigners by more than 10%. In calibrating the ratio of fixed production costs between FDI and local firms, we take the empirical tangible asset ratio between these firms averaged over eight years, 1.115. We assume the asset depreciation is the same across all firms.

Table A.3: Empirical Tangible Asset Ratio for Calibration

	Foreign Capital $\geq$ 10%	Foreign Capital $\geq$ 25%
Year	$\frac{\text{FDI fixed assets per FDI firm}}{\text{Local fixed assets per local firm}}$	$\frac{\text{FDI fixed assets per FDI firm}}{\text{Local fixed assets per local firm}}$
2000	1.288	1.217
2001	1.185	1.110
2002	1.074	1.010
2003	1.020	0.975
2004	1.072	1.024
2005	1.065	0.994
2006	1.111	1.057
2007	1.103	1.059
Average	1.115	1.056

All data are from manufacturing sectors and from Chinese firm data.

Model Ratios in Aggregate: For calibration, the model moments are defined as follows:

(1) Ratio of the value of exports to the value of imports in aggregate

$$\frac{\left[\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D,X}(\omega)d\omega+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I,X}(\omega^*)d\omega^*\right]}{\left[\int_{\omega^*\in\Omega^*}Q_L\frac{P^{D*}(\omega^*)}{W^*}y^{D,X*}(\omega^*)d\omega^*\right]}$$
$$=\quad\frac{M\int_{Z^D}^{\infty}\frac{p^D(z)}{W}\left(y^{D,X}(z)\right)\frac{dG(z)}{1-G(Z^D)}+M^I\int_{Z^I}^{\infty}\frac{p^I(\alpha z)}{W}\left(y^{I,X}(\alpha z)\right)\frac{dG^*(z)}{1-G^*(Z^I)}}{Q_LM^*\int_{Z^{D*}}^{\infty}\frac{p^{D*}(z)}{W^*}\left(y^{D,X*}(z)\right)\frac{dG^*(z)}{1-G^*(Z^{D*})}}$$

(2) Ratio of the value of exports to the value of domestic sales in aggregate

$$\frac{\left[\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D,X}(\omega)d\omega+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I,X}(\omega^*)d\omega^*\right]}{\left[\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D}(\omega)d\omega+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I}(\omega^*)d\omega^*\right]}$$

(3) Ratio of the value of imports to the value of total products in aggregate

$$\frac{\left[\int_{\omega^*\in\Omega^*}Q_L\frac{P^{D*}(\omega^*)}{W^*}y^{D,X*}(\omega^*)d\omega^*\right]}{\left[\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D}(\omega)d\omega+\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D,X}(\omega)d\omega+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I}(\omega^*)d\omega^*+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I,X}(\omega^*)d\omega^*\right]}$$

(4) Ratio of the value of FDI-firm products to the value of total products in aggregate

$$\frac{\left[\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I}(\omega^*)d\omega^*+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I,X}(\omega^*)d\omega^*\right]}{\left[\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D}(\omega)d\omega+\int_{\omega\in\Omega}\frac{P^D(\omega)}{W}y^{D,X}(\omega)d\omega+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I}(\omega^*)d\omega^*+\int_{\omega^*\in\Omega^I}\frac{P^I(\omega^*)}{W}y^{I,X}(\omega^*)d\omega^*\right]}$$

A.4 More Details on Counterfactuals

A.4.1 Removing tax benefits of FDI firms

Welfare gains from the tax reform exhibit a humped shape. In the second row of Figure A.1, we plotted Home consumption C on the left[3] and its percent changes relative to the benchmark level under $\tau_V^I = 0.85$ on the right. To see why welfare gains are not monotone, observe the third row of Figure A.1. The effect from the varieties raises consumption for all tax rates in the experiment, however, the effect from aggregate productivity exhibits a humped shape: it increases up to 31% of tax on FDI and then decreases. Adverse effect from decreasing productivity becomes more dominant as the tax on FDI rises further beyond 31% and eventually consumption declines when the tax rate exceeds 33%.

The gains from product varieties are positive under the tax reform. Due to the tax-cut on local firms, more and more Home domestic incumbents operate their businesses while more and more FDI firms exit. Their net effect is the gain in total varieties. However, aggregate productivity also shows a humped-shape pattern like consumption. In the net effect, the welfare initially increases until it reaches the maximum at around 33% of tax on revenues of FDI firms, and then it monotonically decreases.

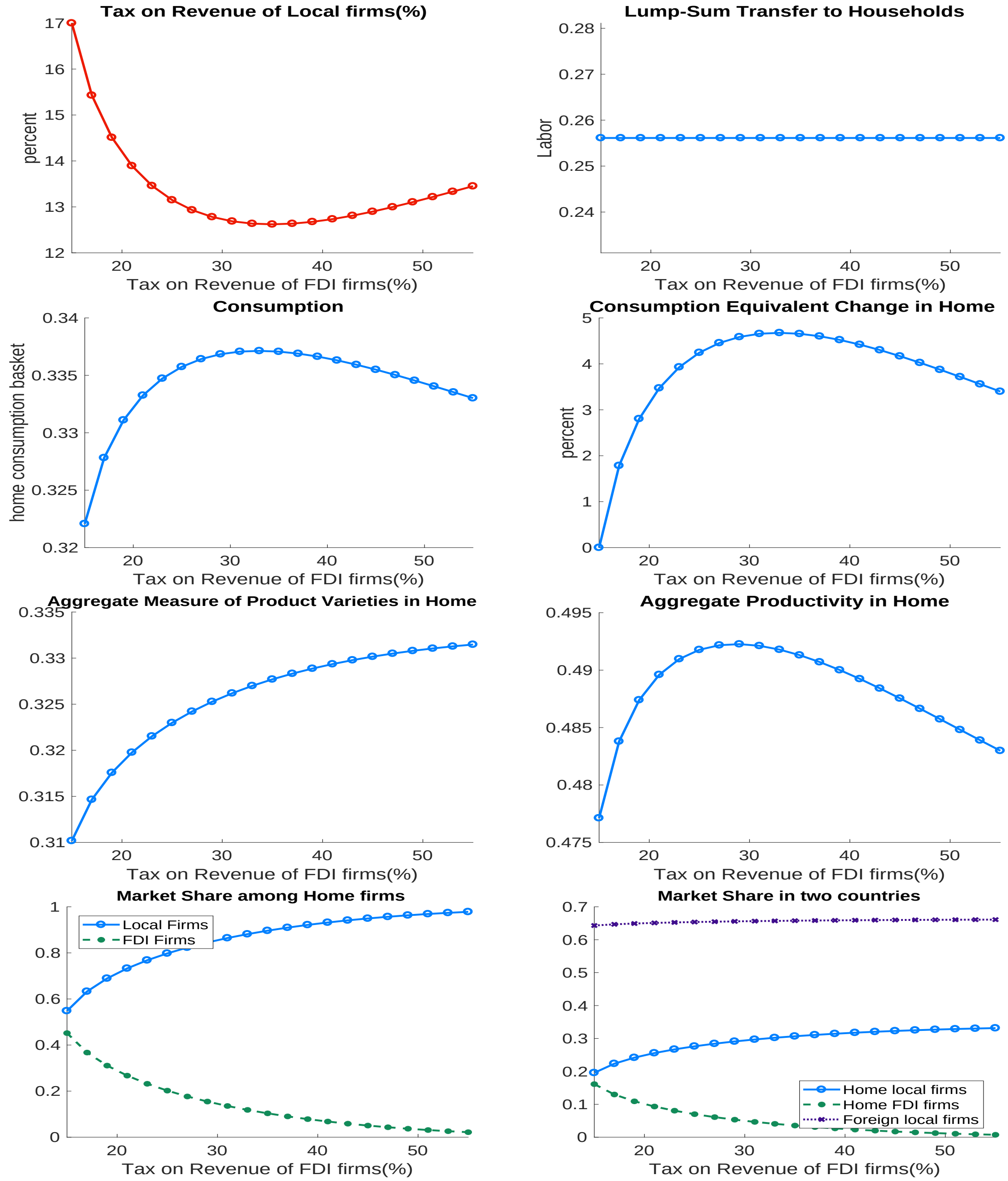
The market share of FDI firms is replaced with that of local firms and the market share of Foreign local firms increases. The tax reform drives out low-productivity FDI firms by raising its cutoff productivity, but the revenue tax changes do not affect the cutoff productivity of Home local firms and it attracts more low-productivity local firms in Foreign.

A humped shape in aggregate productivity: The appendix A.2.1 shows that the aggregate productivity in Home, $\widetilde{Z}^{HF}$, is defined as

$$\log\left(\widetilde{Z}^{HF}\right) \equiv \nu \log\left(\widetilde{Z}^{DI}\right) + (1-\nu)\log\left(\widetilde{Z}^{D*}\right),$$

[3] Note that aggregate consumption, C, is periodic utility itself since our framework is static: $C = V = \Phi {C_0}^{\theta_0} {C_1}^{\theta} = \Phi\left(C_0\right)^{\theta_0}\left(\left(M^{HF}\right)^{\frac{1}{\sigma-1}}\left(\rho\widetilde{Z}^{HF}\right)\theta\left(L+\frac{T}{W}\right)\right)^{\theta}$.

Figure A.1: Value-Added Tax Reform with varying τ_V^D under $\lambda = 0.70$, $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$



The Home aggregate productivity, $\widetilde{Z}^{HF}$, puts a larger weight on average productivity measures of Home local and FDI firms relative to that of Foreign local firms due to the presence of home bias, $\nu = 0.83$. Among Home local and FDI firms, the productivity aggregator, $\widetilde{Z}^{DI}$, gives more importance on those firms who have larger mass.

Figure A.2 shows the effect of the tax reform on productivity. The top two subfigures plot the Home aggregate productivity, $\widetilde{Z}^{HF}$, on the left and productivity aggregator over local and FDI firms, $\widetilde{Z}^{DI}$ on the right. Subfigures in the second row present effective productivity measures which are adjusted by after-tax wedges: $\tau_V^D \widetilde{Z}^D$ and $\tau_V^I \widetilde{Z}^I$. We plot such revenue wedges on local and FDI firms in the third row: τ_V^D on the left and τ_V^I on the right. The bottom two subfigures plot cutoff productivity levels of Home local and FDI firms, and Foreign local firms.

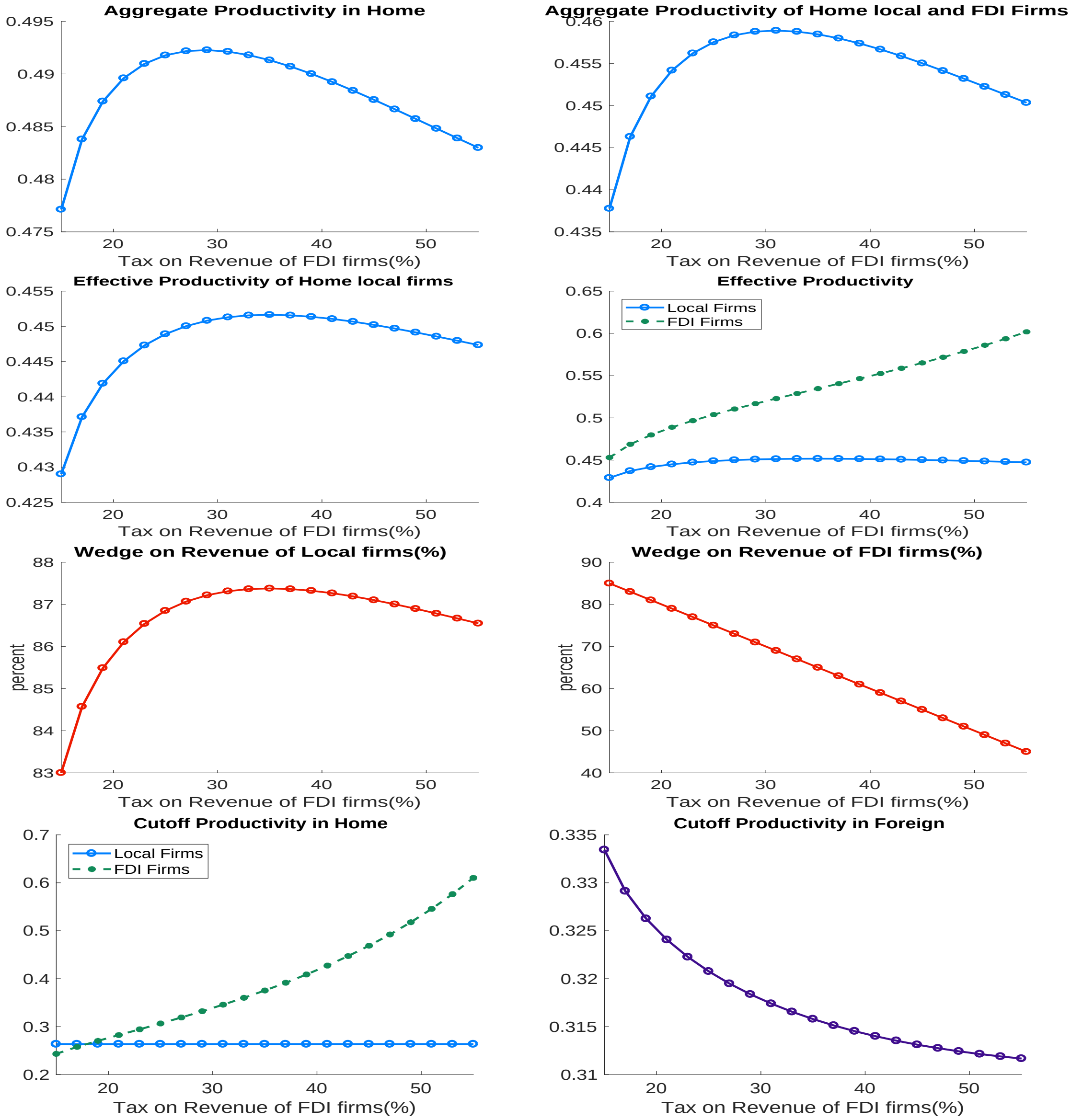
Be reminded that unlike the standard trade literature, we assume exporting is costless and our model abstracts from trade costs. We adopt this approach since our focus is to evaluate the effect of government policies on reallocations between local and FDI firms under financial market imperfection and tax distortions in the FDI host country. Tax on revenue, $1 - \tau_V$, reduces marginal revenue of firms, and hence it aggravates effective productivity by requiring more labor in producing one unit of product. Higher revenue tax, or lower wedge on revenue ($\tau_V \Downarrow$), negatively affects aggregate productivity $\widetilde{Z}^{HF}$.

Under the tax reform, effective productivity of Home local firms dominates those of FDI and Foreign local firms due to their larger mass and home bias. The variation in taxes on Home local and FDI firms does not change local firms' cutoff productivity. However, the change in the wedge on local firms' revenue mainly drives the change in Home effective productivity. Hence, the non-monotone feature of aggregate productivity is mainly driven by the wedge on revenue of Home local firms.

The appendix A.2 derives cutoff productivity levels of all firms. For Home firms, they are given by

$$
\begin{aligned}
Z^D &= \left\{ \left(\frac{\tau_C^D}{\delta} \right) \left(\frac{f^D}{F^D} \right) \left(\frac{(\sigma - 1)\,(z_{min})^\eta}{\eta - \sigma + 1} \right) \left[1 + \frac{\eta}{\sigma - 1} \left(\frac{1}{\lambda} - 1 \right) \left(\zeta - \chi \frac{F^D}{f^D} \right) \right] \right\}^{\frac{1}{\eta}}, \\
\alpha Z^I &= Z^D \left(\frac{f^I}{\left[f^D + \left(\frac{1}{\lambda} - 1 \right) \left(\zeta f^D - \chi F^D \right) \right]} \right)^{\frac{1}{\sigma - 1}} \left(\frac{\tau_V^D}{\tau_V^I} \right)^{\frac{\sigma}{\sigma - 1}}.
\end{aligned}
$$

Figure A.2: Value-Added Tax Reform with varying τ_V^D under $\lambda = 0.70$, $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$



It shows that the increase in revenue tax on FDI relative to that on local firms, $\frac{\tau_V^I}{\tau_V^D} \Downarrow$, raises FDI cutoff productivity, $Z^I \Uparrow$. Higher cutoff for FDI firms, $Z^I \Uparrow$, leads to lower cutoff for Foreign firms, $Z^{D*} \Downarrow$, as shown by the free entry condition in Foreign, given by:

$$F^{D*} = \left(\frac{1 - G^*(Z^{D*})}{\delta} \right) \frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} + \left(\frac{1 - G^*(Z^I)}{\delta} \right) \frac{\pi^I(\widetilde{Z}^I)}{W}.$$

This condition implies that if the market environment requires FDI firms to be highly productive for their survival ($Z^I \Uparrow$), then more low-productivity local firms in Foreign can enter ($Z^{D*} \Downarrow$).[4] Therefore, the rise of revenue tax on FDI firms has negative spillover effect on cutoff productivity of Foreign firms.

Decrease in composite varieties: Now we move on to the competition effect. We define the aggregate mass of Home firms as

$$\log\left(M^{HF}\right) \equiv \nu \log\left(M + M^I\right) + (1-\nu)\log\left(M^*\right).$$

The top-left subfigure in Figure A.3 shows the aggregate measure for Home variety, M^{HF}, strictly increases. This is due to the dominant effect of the increase in the mass of Home local firms, M. The combination of labor market clearing conditions and free entry conditions leads to the following equilibrium conditions as shown in the appendix A.2:

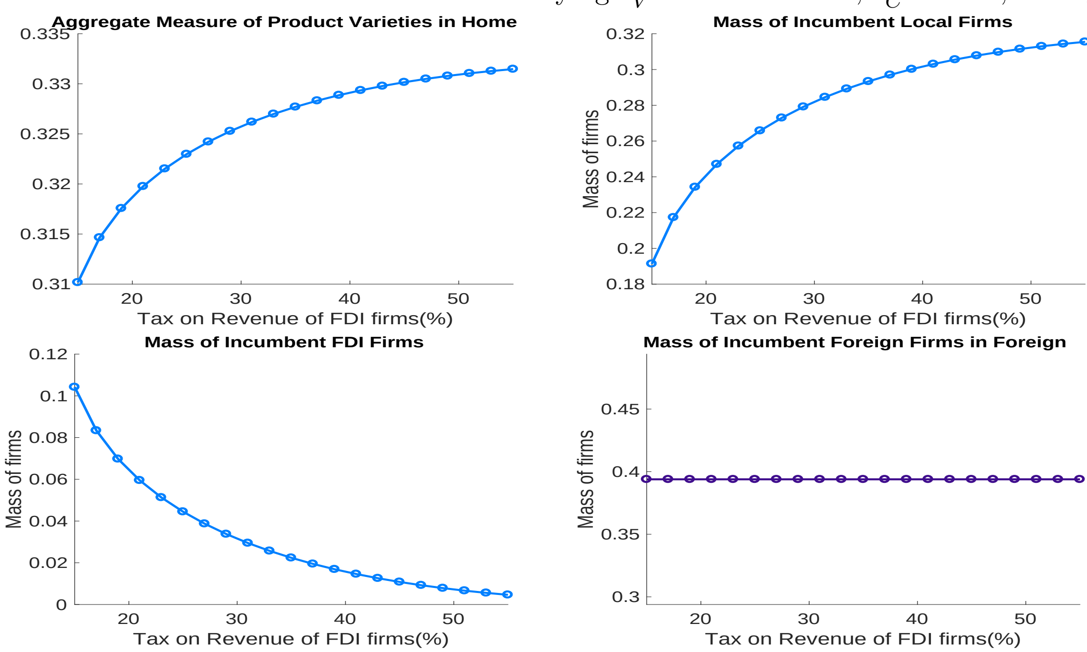
$$\begin{aligned} M &= \frac{(L-L_0) - \boldsymbol{M^I}\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]}{\left(\frac{\tau_C^D}{\sigma-1}+1\right)\boldsymbol{l^D}\left(\boldsymbol{\widetilde{Z}^D}\right) + (1-\tau_C^D)\, f^D}, \\ M^* &= \frac{L^* - \boldsymbol{L_0^*}}{\left\{\left(\frac{\tau_C^{D*}}{\sigma-1}+1\right) l^{D*}(\widetilde{Z}^{D*}) + (1-\tau_C^{D*})\, f^{D*} + \left(\frac{\boldsymbol{1-G^*(Z^I)}}{\boldsymbol{1-G^*(Z^{D*})}}\right)\frac{\tau_C^I}{Q_L}\left[\frac{1}{\sigma-1} l^I(\widetilde{Z}^I) - f^I\right]\right\}}, \end{aligned}$$

where $\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) = \left(\frac{Z^{D*}}{Z^I}\right)^{\eta^*}$ holds. The average labor demand can be derived as $l^D(\widetilde{Z}^D) = \left(f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right)(\sigma-1)\left(\frac{\eta}{\eta-\sigma+1}\right)$ for Home local firms, $l^I\left(\widetilde{Z}^I\right) = f^I(\sigma-1)\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)$ for Home FDI firms, and $l^{D*}\left(\widetilde{Z}^{D*}\right) = f^{D*}(\sigma-1)\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)$ for Foreign domestic firms (see the appendix A.2.3).

[4] In the appendix A.2.3, we show average profits can be derived as $\frac{\pi^{D*}(\widetilde{Z}^{D*})}{W^*} = \tau_C^{D*} f^{D*}\left(\frac{\sigma-1}{\eta^*-\sigma+1}\right)$ and $\frac{\pi^I(\widetilde{Z}^I)}{W} = \tau_C^I f^I\left(\frac{\sigma-1}{\eta^*-\sigma+1}\right)$. That is, average profits do not depend on cutoff productivity. To be concrete, the cutoff productivity for Foreign firms is determined by $Z^{D*} = \left\{\frac{\tau_C^{D*} f^{D*}\frac{(\sigma-1)(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1}}{\delta F^{D*} - \tau_C^I f^I \frac{(\sigma-1)(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1}\boldsymbol{\left(\frac{1}{Z^I}\right)^{\eta^*}}}\right\}^{\frac{1}{\eta^*}}$, which shows its negative association with Z^I clearly.

There are two determinants for the mass of Home local firms, M. Firstly, when average labor demand among Home local firms, $l^D(\widetilde{Z}^D)$, is higher, it makes competition among local firms in hiring labor harder, and thus the mass of firms decreases. Second, if there are more FDI firms in operation, $M^I \Uparrow$, then this leads to stronger competition for hiring Home labor and so the mass of Home local firms gets smaller. In the counterfactual experiment, the mass of FDI firms decreases due to the rise in its cutoff productivity[5] and average labor demand stays constant. Therefore, the mass of Home local firms, M, increases under the tax reform. The mass of Foreign firms, M^*, stays constant since the effect from the increase in the labor demand in the homogeneous sector, $L_0^* \Uparrow$, and the effect from the decrease in $\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})} \Downarrow$ are cancelled out.

Figure A.3: Value-Added Tax Reform with varying τ_V^D under $\lambda = 0.70$, $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$



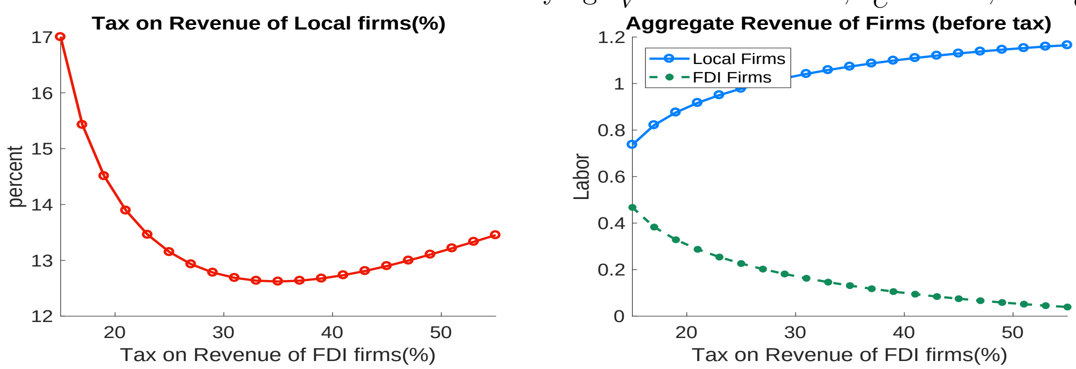
[5] The mass of FDI firms is defined as $M^I \equiv M^*\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right)$. The negative relation of the mass M^I to its cutoff Z^I can be shown as $\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) = \frac{(z^*_{min})^{\eta^*}\frac{\pi^{D*}(\tilde{Z}^{D*})}{W^*}}{(Z^I)^{\eta^*}\delta F^{D*} - \left(\frac{1}{Q_L}\right)(z^*_{min})^{\eta^*}\frac{\pi^I(\tilde{Z}^I)}{W}} = \frac{\tau_C^{D*} f^{D*}\frac{(\sigma-1)(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1}}{(Z^I)^{\eta^*}\delta F^{D*} - \left(\frac{1}{Q_L}\right)\tau_C^I f^I\frac{(\sigma-1)(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1}}$.

A humped shape in wedge on revenue of Home local firms, τ_V^D: We find that revenue tax on Home local firms, $1-\tau_V^D$, exhibits a humped shape as reproduced in Figure A.4. To see why this occurs, we also plot before-tax aggregate revenues of local and FDI firms. The appendix A.2.3 shows before-tax revenues can be written out as:

$$\begin{aligned}\left(\frac{R^D}{W}\right)^{B.T} &= \frac{M}{\tau_V^D}\sigma\left(f^D+\left(\frac{1}{\lambda}-1\right)(\zeta f^D-\chi F^D)\right)\left(\frac{\eta}{\eta-\sigma+1}\right),\\ \left(\frac{R^I}{W}\right)^{B.T} &= \frac{M^I}{\tau_V^I}\sigma f^I\left(\frac{\eta^*}{\eta^*-\sigma+1}\right).\end{aligned}$$

The mechanism is clear. Before-tax aggregate revenue of Home local firms increases, but that

Figure A.4: Value-Added Tax Reform with varying τ_V^D under $\lambda=0.70$, $\tau_C^D=0.67$, and $\tau_C^I=0.85$



of FDI firms declines. Keeping the same amount of government revenues, the government may reduce the tax on local firms initially as it imposes larger tax on FDI firms. However, as more and more FDI firms exit and hence tax revenues from FDI firms declines, the government eventually needs to finance revenues by increasing tax on local firms after some threshold at around 33% tax on FDI firms.

A.4.2 Financial Market Reform under Tax Distortions

Increase in composite varieties: We define the composite mass of Home firms as $\log\left(M^{HF}\right) \equiv \nu \log\left(M + M^I\right) + (1-\nu)\log\left(M^*\right)$. The left subfigure in the middle of Figure A.5 shows M^{HF} increases. This is due to the dominant effect of the increase in M. The combination of labor market clearing conditions and free entry conditions leads to the following equilibrium conditions:

$$\begin{aligned}
M &= \frac{(L - L_0) - \boldsymbol{M^I}\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]}{\left(\frac{\tau_C^D}{\sigma-1}+1\right)\boldsymbol{l^D}\left(\widetilde{\boldsymbol{Z}}^{\boldsymbol{D}}\right) + \left(1-\tau_C^D\right) f^D}, \\
M^* &= \frac{L^* - \boldsymbol{L_0^*}}{\left\{\left(\frac{\tau_C^{D*}}{\sigma-1}+1\right) l^{D*}(\widetilde{Z}^{D*}) + \left(1-\tau_C^{D*}\right) f^{D*} + \left(\frac{\boldsymbol{1-G^*(Z^I)}}{\boldsymbol{1-G^*(Z^{D*})}}\right)\frac{\tau_C^I}{Q_L}\left[\frac{1}{\sigma-1} l^I(\widetilde{Z}^I) - f^I\right]\right\}},
\end{aligned}$$

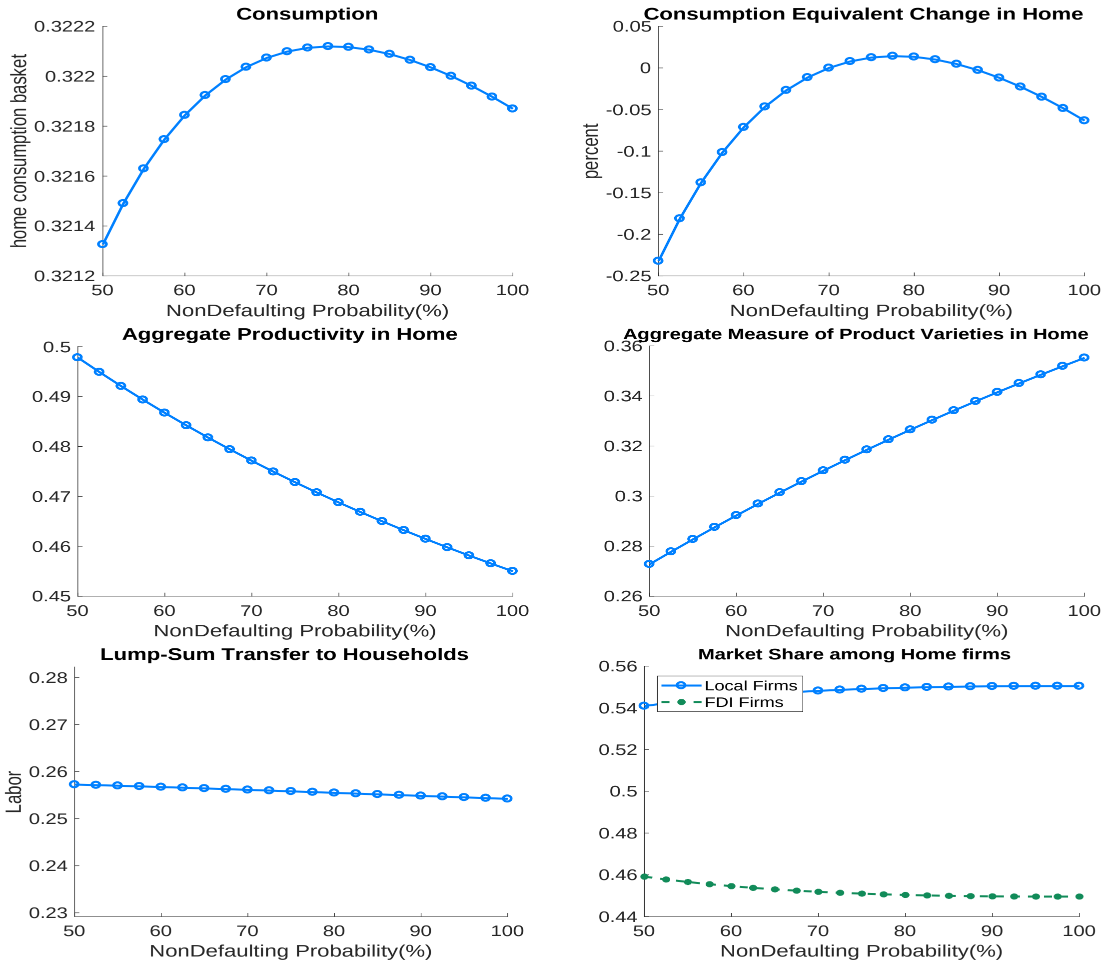
where $\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})}\right) = \left(\frac{Z^{D*}}{Z^I}\right)^{\eta^*}$ holds. The average labor demand can be derived as $l^D(\widetilde{Z}^D) = \left(f^D + \left(\frac{1}{\lambda}-1\right)\left(\zeta f^D - \chi F^D\right)\right)(\sigma-1)\left(\frac{\eta}{\eta-\sigma+1}\right)$ for Home local firms, $l^I\left(\widetilde{Z}^I\right) = f^I(\sigma-1)\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)$ for Home FDI firms, and $l^{D*}\left(\widetilde{Z}^{D*}\right) = f^{D*}(\sigma-1)\left(\frac{\eta^*}{\eta^*-\sigma+1}\right)$ for Foreign domestic firms (see the appendix A.2.3).

There are two determinants for the mass of Home local firms, M_k. Firstly, when average labor demand among Home local firms, $l^D(\widetilde{Z}^D)$, is higher, it makes competition among local firms in hiring labor harder, and thus the mass of firms decreases. Second, if there are more FDI firms in operation, $M^I \Uparrow$, then this leads to harder competition for hiring Home labor and the mass of Home local firms gets smaller. In the counterfactual experiment, both competition effects get smaller due to the decrease in average labor demand, $l^D(\widetilde{Z}^D)$, and the decrease in the mass of FDI firms. Therefore, M increases when λ rises as shown in Figure A.6.

The mass of Foreign firms, M^*, gets smaller mainly due to the increase in labor demand in the homogeneous sector, $L_0^* \Uparrow$, but its movement is negligible. Since the FDI cutoff Z^I

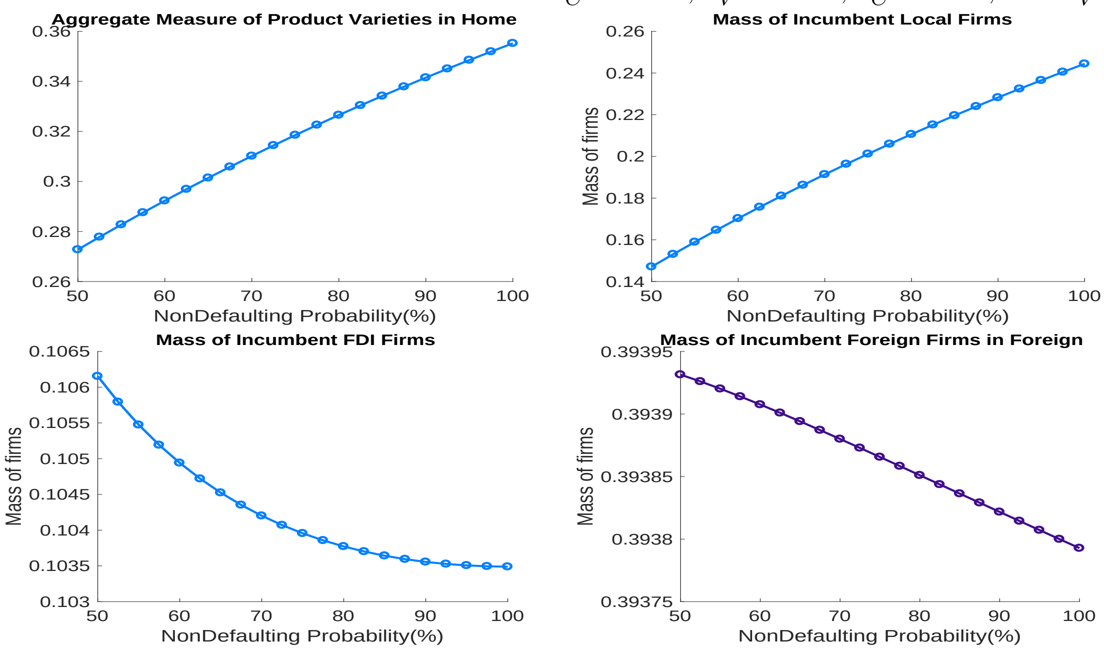
slightly increases, the mass of FDI firms decreases, $M^I \Downarrow$.[6]

Figure A.5: Financial Market Reform under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



[6] The mass of FDI firms is defined as $M^I \equiv M^* \left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})} \right)$. The negative relation of the mass M^I to its cutoff Z^I can be shown as $\left(\frac{1-G^*(Z^I)}{1-G^*(Z^{D*})} \right) = \frac{(z^*_{min})^{\eta^*} \frac{\pi^{D*}(\tilde{Z}^{D*})}{W^*}}{(Z^I)^{\eta^*} \delta F^{D*} - \left(\frac{1}{Q_L}\right) (z^*_{min})^{\eta^*} \frac{\pi^I(\tilde{Z}^I)}{W}} = \frac{\tau_C^{D*} f^{D*} \frac{(\sigma-1)(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1}}{(Z^I)^{\eta^*} \delta F^{D*} - \left(\frac{1}{Q_L}\right) \tau_C^I f^I \frac{(\sigma-1)(z^*_{min})^{\eta^*}}{\eta^*-\sigma+1}}$.

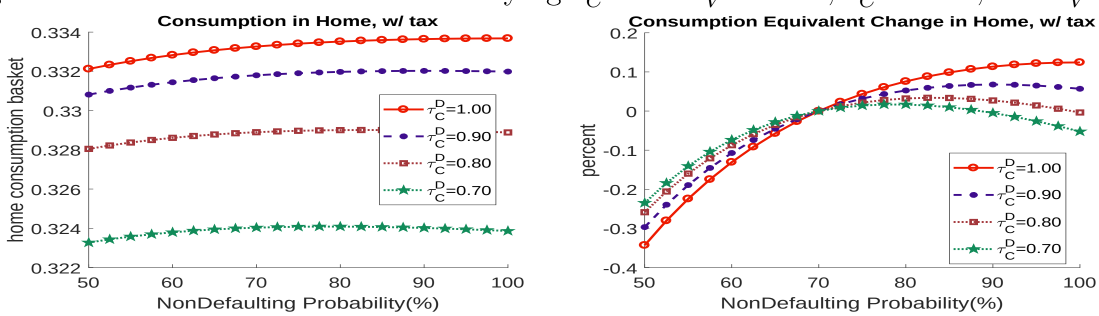
Figure A.6: Financial Market Reform under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



A.4.3 Financial Reform with varying taxes on local firms' profits

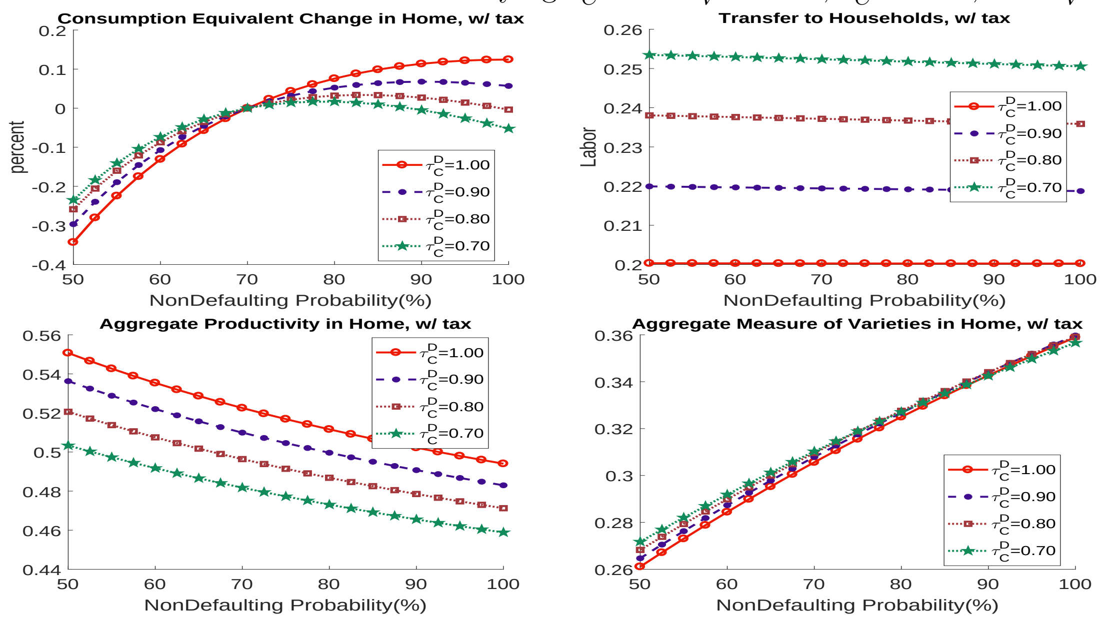
Figure A.7 shows changes in consumption according to financial reform with respect to different levels of a profit tax on Home local firms. When we normalize each line by dividing it by the value on $\lambda = 70\%$, then the consumption equivalent change clearly exhibits the pattern which becomes "humped" less and less as τ_C^D increases (decreasing profit tax on local firms).

Figure A.7: Financial Reform under varying τ_C^D with $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



Observation of Figure A.8 leads us to the main culprit for this pattern: variety effect. Variety effect increases with a steeper slope as τ_C^D increases(decreasing profit tax). Also, lump-sum transfer and aggregate productivity are all less than one, and it dampens the increase in the variety effects and consumption plot becomes smoother than the variety effect plot: $C_1 = \left[M^{HF}\right]^{\frac{1}{\sigma-1}} \rho \widetilde{Z}^{HF} \theta \left(L + \frac{T}{W}\right)$.

Figure A.8: Financial Reform under varying τ_C^D with $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



So why does the variety effect increase with a steeper slope as τ_C^D increases(decreasing profit tax)? This is mainly because the mass of Home local firms increases with a steeper slope as τ_C^D increases.

$$\begin{aligned} M &= \frac{(L-L_0) - \boldsymbol{M^I}\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]}{\left(\frac{\tau_C^D}{\sigma-1}+1\right)\boldsymbol{l^D}\left(\boldsymbol{\widetilde{Z}^D}\right) + (1-\tau_C^D)\, f^D}, \\ &= \underbrace{\left\{(L-L_0) - \boldsymbol{M^I}\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]\right\}}_{M_l} \underbrace{\left\{\left(\frac{\tau_C^D}{\sigma-1}+1\right)\boldsymbol{l^D}\left(\boldsymbol{\widetilde{Z}^D}\right) + \left(1-\tau_C^D\right) f^D\right\}^{-1}}_{M_r} \end{aligned}$$

where $l^D(\widetilde{Z}^D) = \left(f^D + \left(\frac{1}{\boldsymbol{\lambda}}-1\right)\left(\zeta f^D - \chi F^D\right)\right)(\sigma-1)\left(\frac{\eta}{\eta-\sigma+1}\right)$. The left-bottom chart in

Figure A.9: Financial Reform under varying τ_C^D with $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$

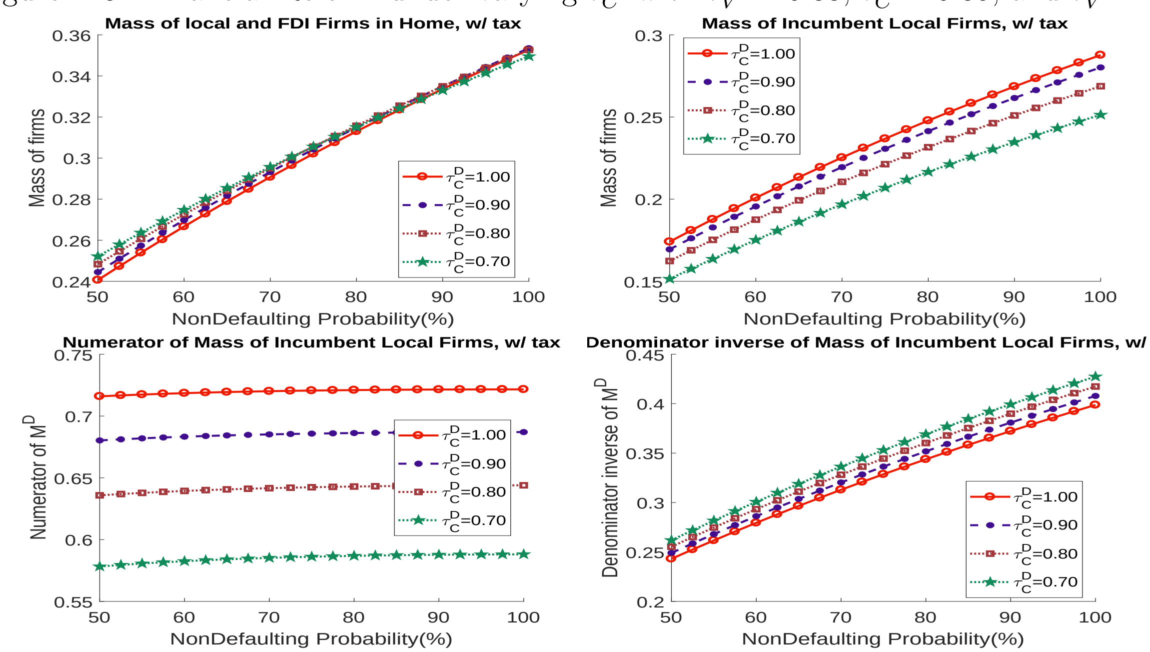


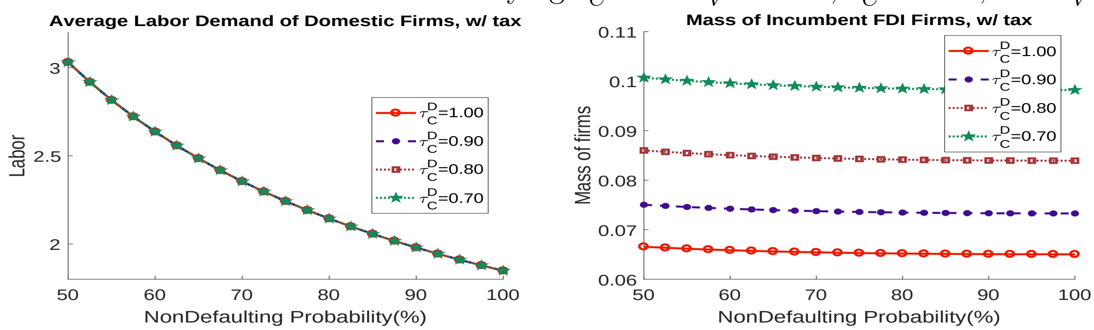
Fig A.9 plots $M_l = \left\{(L-L_0) - \boldsymbol{M^I}\left[f^I + l^I\left(\widetilde{Z}^I\right)\right]\right\}$ and the right-bottom chart plots $M_r = \left\{\left(\frac{\tau_C^D}{\sigma-1}+1\right)\boldsymbol{l^D}\left(\boldsymbol{\widetilde{Z}^D}\right) + \left(1-\tau_C^D\right) f^D\right\}^{-1}$. As λ increases, $l^D(\widetilde{Z}^D)$ decreases and thus M_r increases. It barely shifts according to the change in τ_C^D.

As τ_C^D increases (decreasing profit tax on local firms), M^I decreases due to the exit of

FDI firms, and M_l shifts up. Therefore, as τ_C^D increases, M_l acts as a multiplier which makes the slope of M_r steeper.

All in all, the main reason for the steeper slope of the variety effect is the decrease of FDI firm mass. According to the financial reform, λ increases and the average labor requirement $l^D(\widetilde{Z}^D)$ decreases since local firms pay less and less financial costs in labor term. The increase in M_r is multiplied by the upward shift in M_l as τ_C^D rises and this is mainly due to the exit of FDI firms: $M^I \downarrow$.

Figure A.10: Financial Reform under varying τ_C^D with $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



A.4.4 Financial Reform with varying taxes on FDI firms' profits

Figure A.11 shows changes in consumption according to financial reform with respect to different levels of a profit tax on FDI firms. The Home consumption gets larger in level when profit taxes on FDI firms increases (τ_C^I decreases). This is mainly due to the income effect: as the Home government gathers large taxes from FDI firms, Home households can earn more lump-sum transfers.

Figure A.11: Financial Reform under varying τ_C^I with $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_V^I = 0.85$

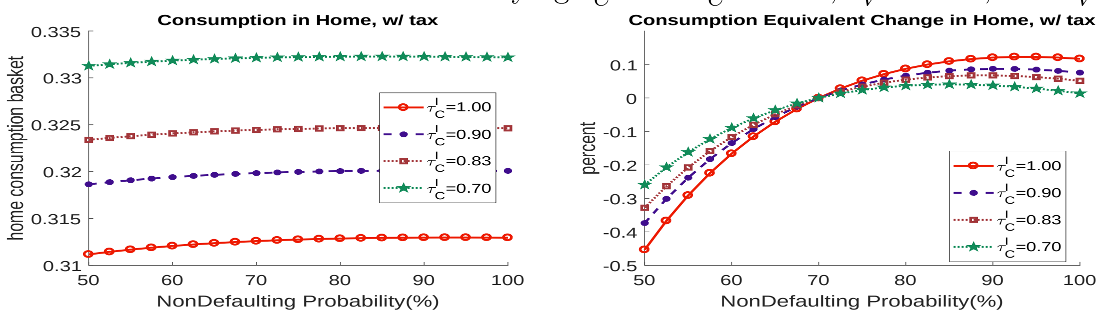


Figure A.12: Financial Reform under varying τ_C^I with $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_V^I = 0.85$

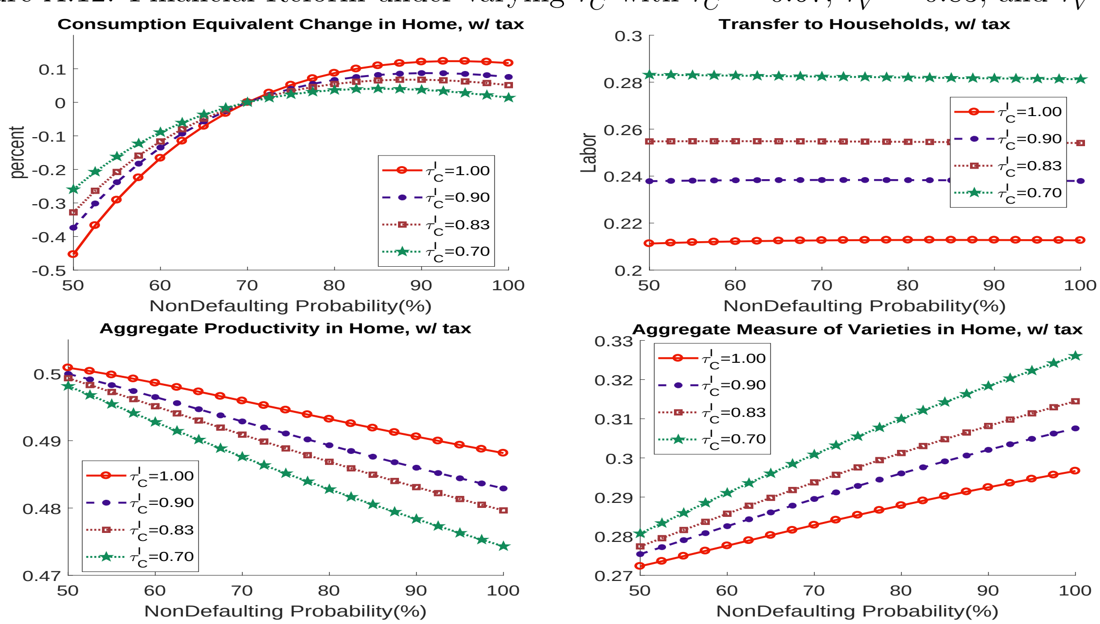


Figure A.13: Financial Reform under varying τ_C^I with $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_V^I = 0.85$

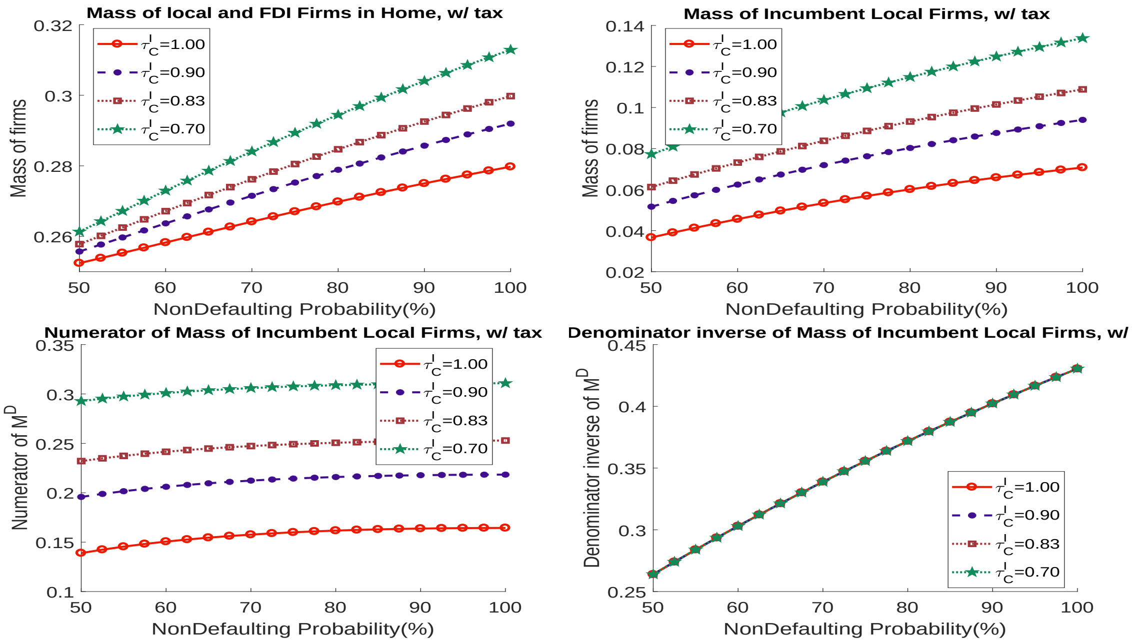
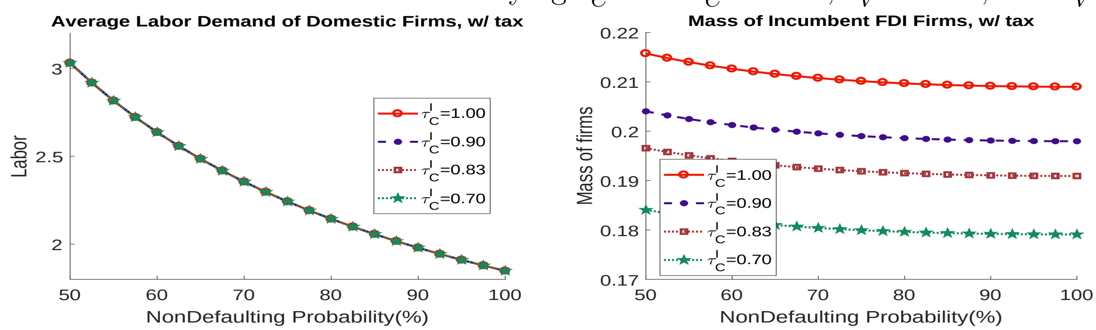


Figure A.14: Financial Reform under varying τ_C^I with $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_V^I = 0.85$



E Figure with $1 - \tau_V^I = 15\%$

Figure 15: Financial Market Reform under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$

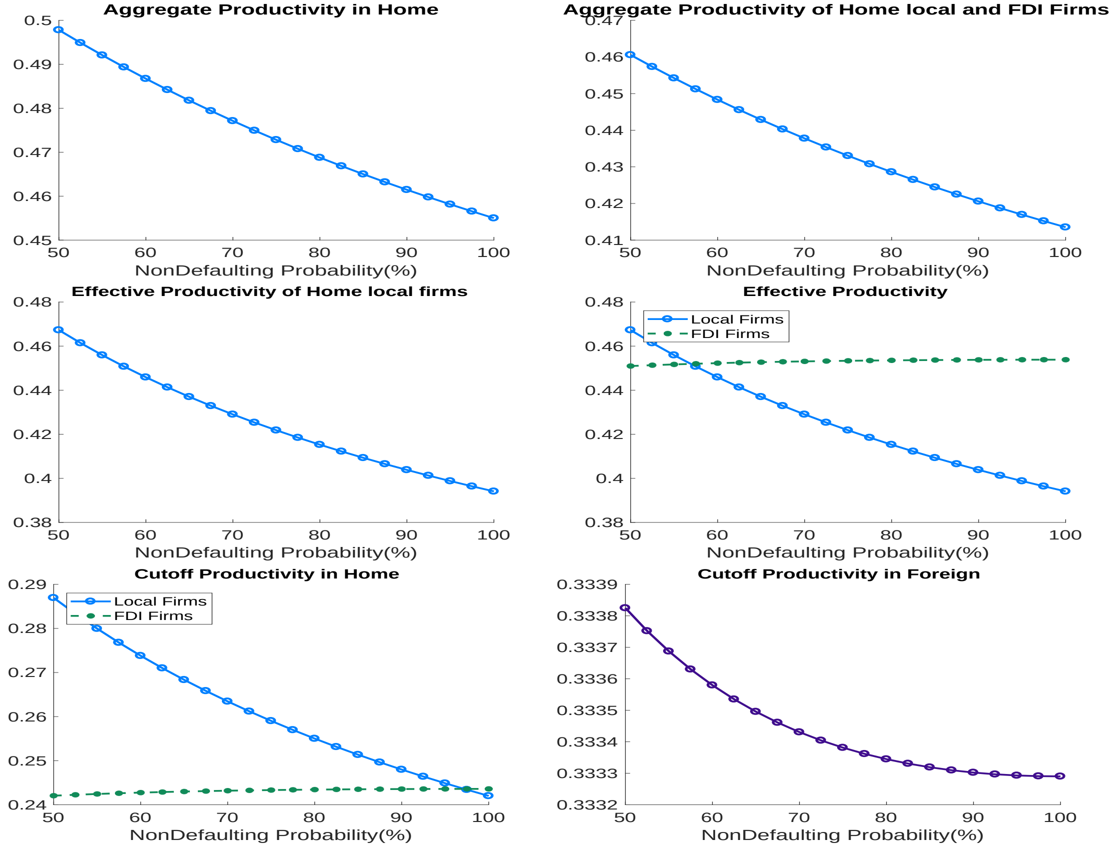


Figure 16: Financial Market Reform under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$

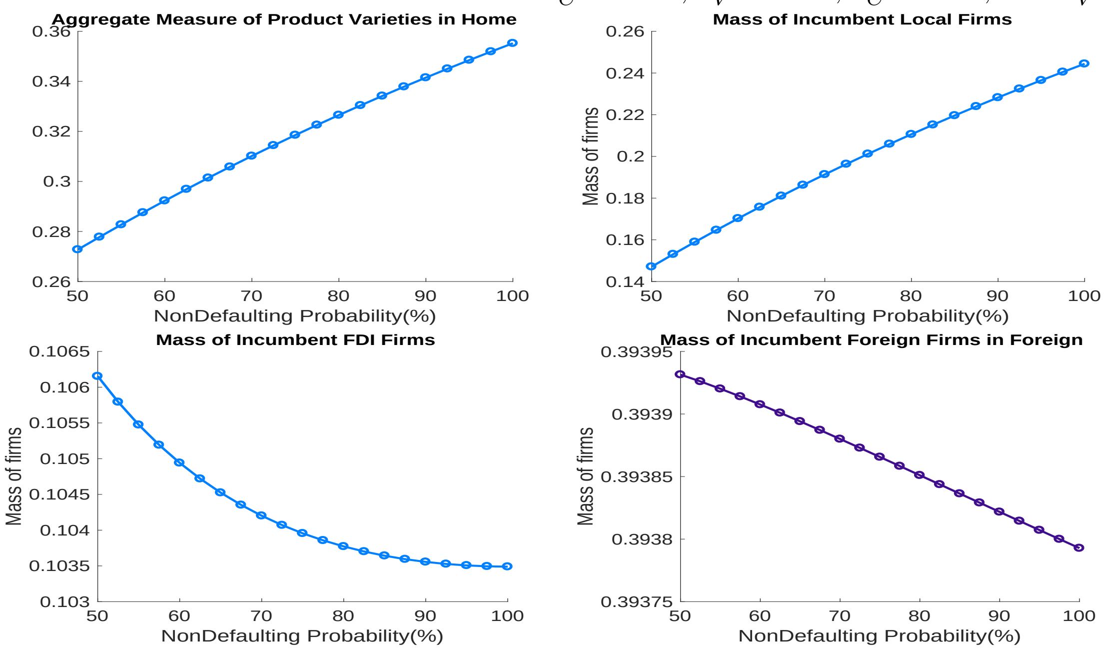
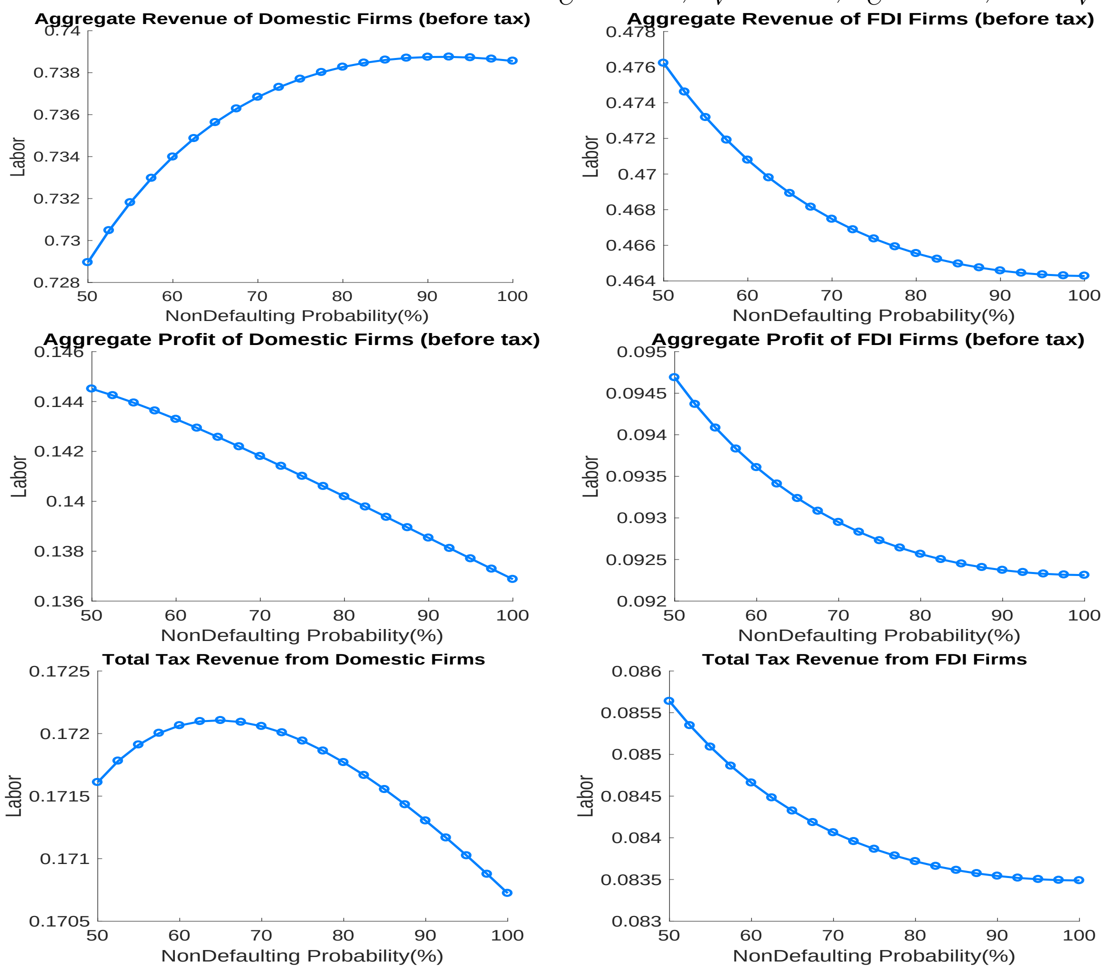


Figure 17: Financial Market Reform under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, $\tau_C^I = 0.85$, and $\tau_V^I = 0.85$



F Allocations with and without distortions

Figure 18: Value-Added Tax Reform under distortions and under no distortions.

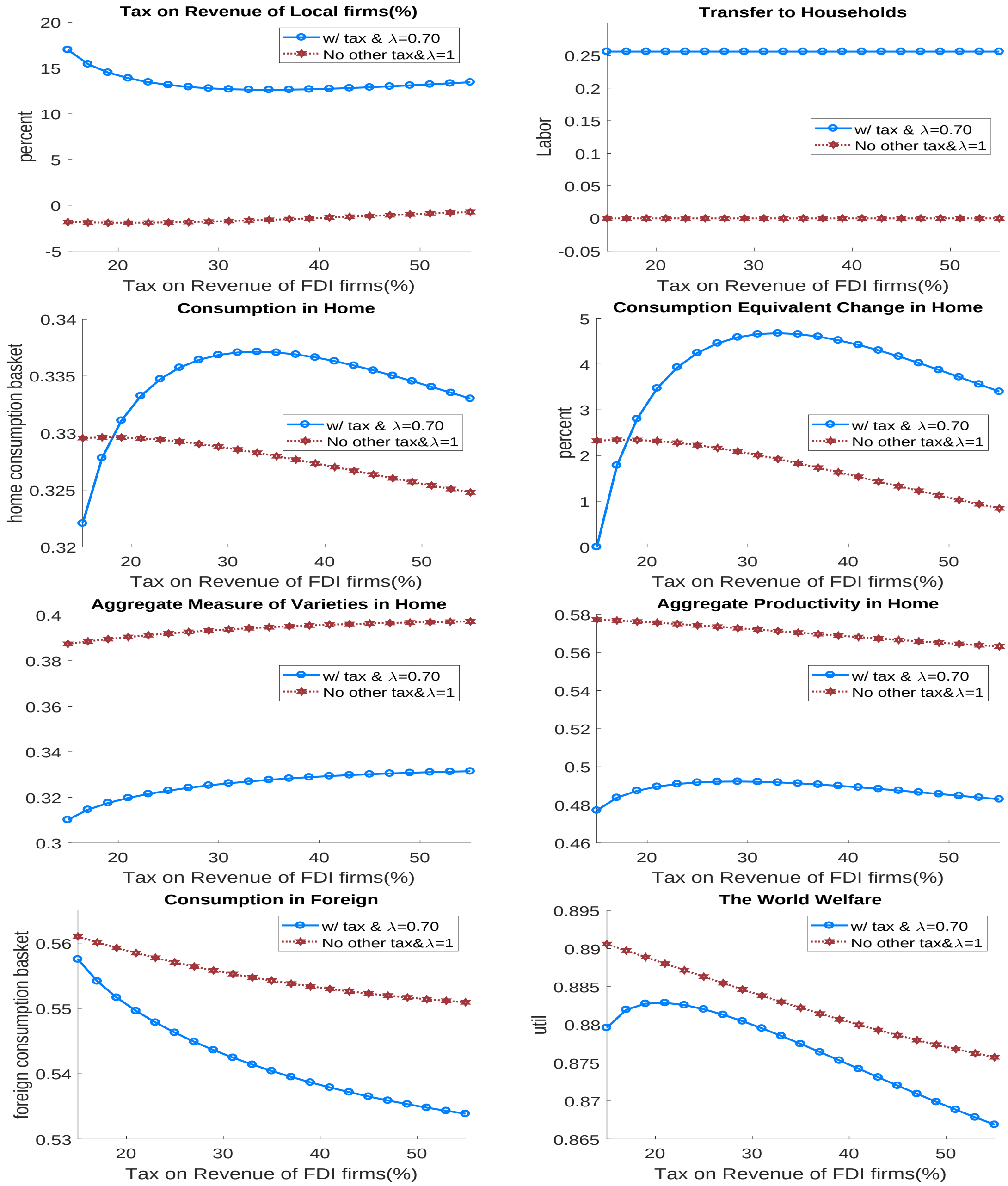


Figure 19: Value-Added Tax Reform under distortions and under no distortions.

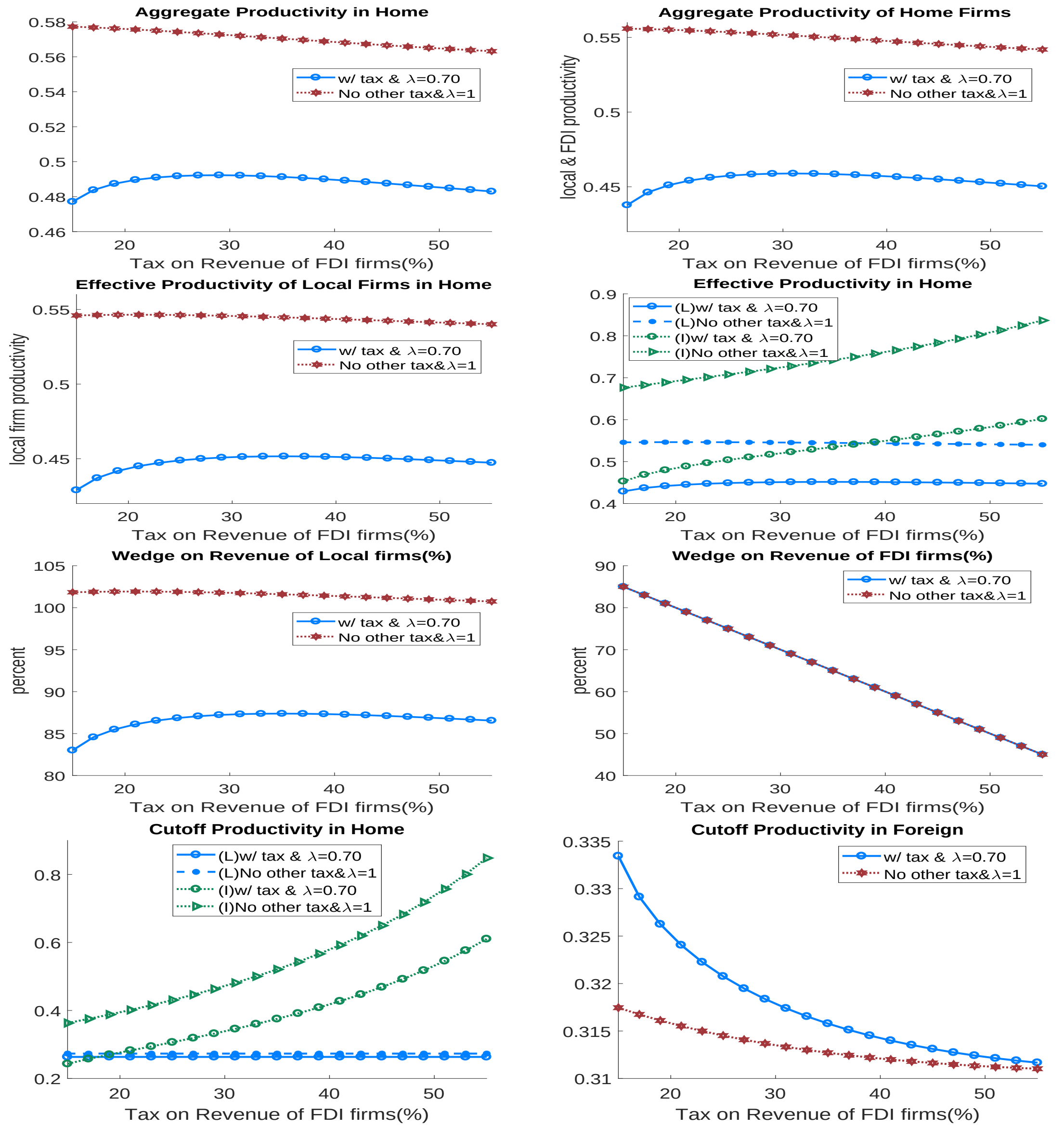


Figure 20: Value-Added Tax Reform under distortions and under no distortions.

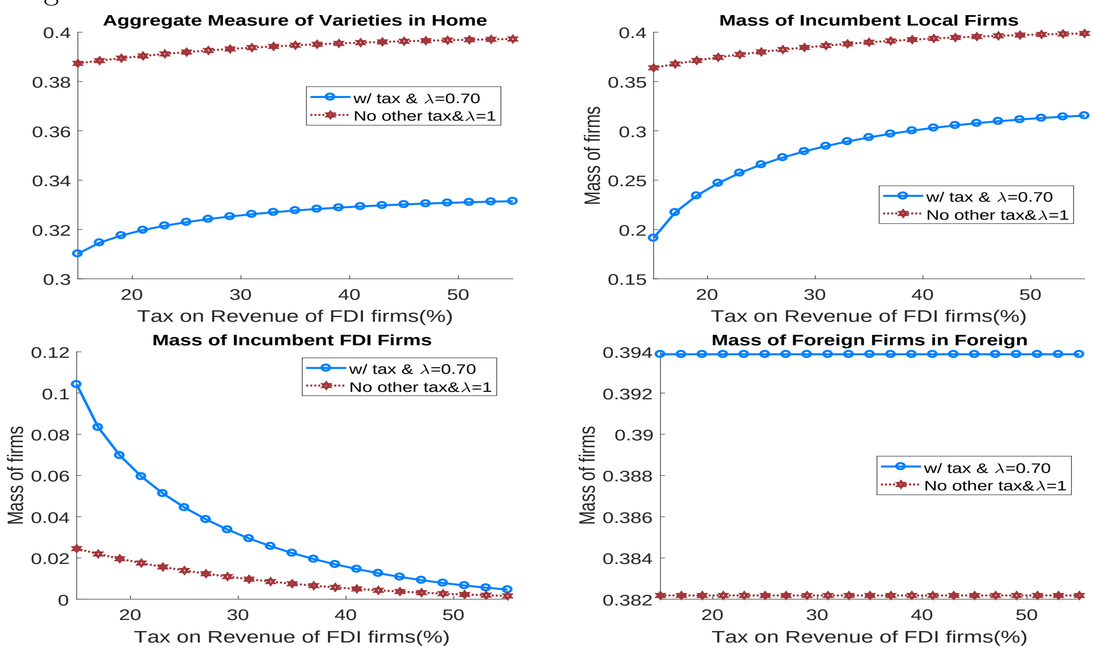


Figure 21: Financial Market Reform under distortions and under no distortions.

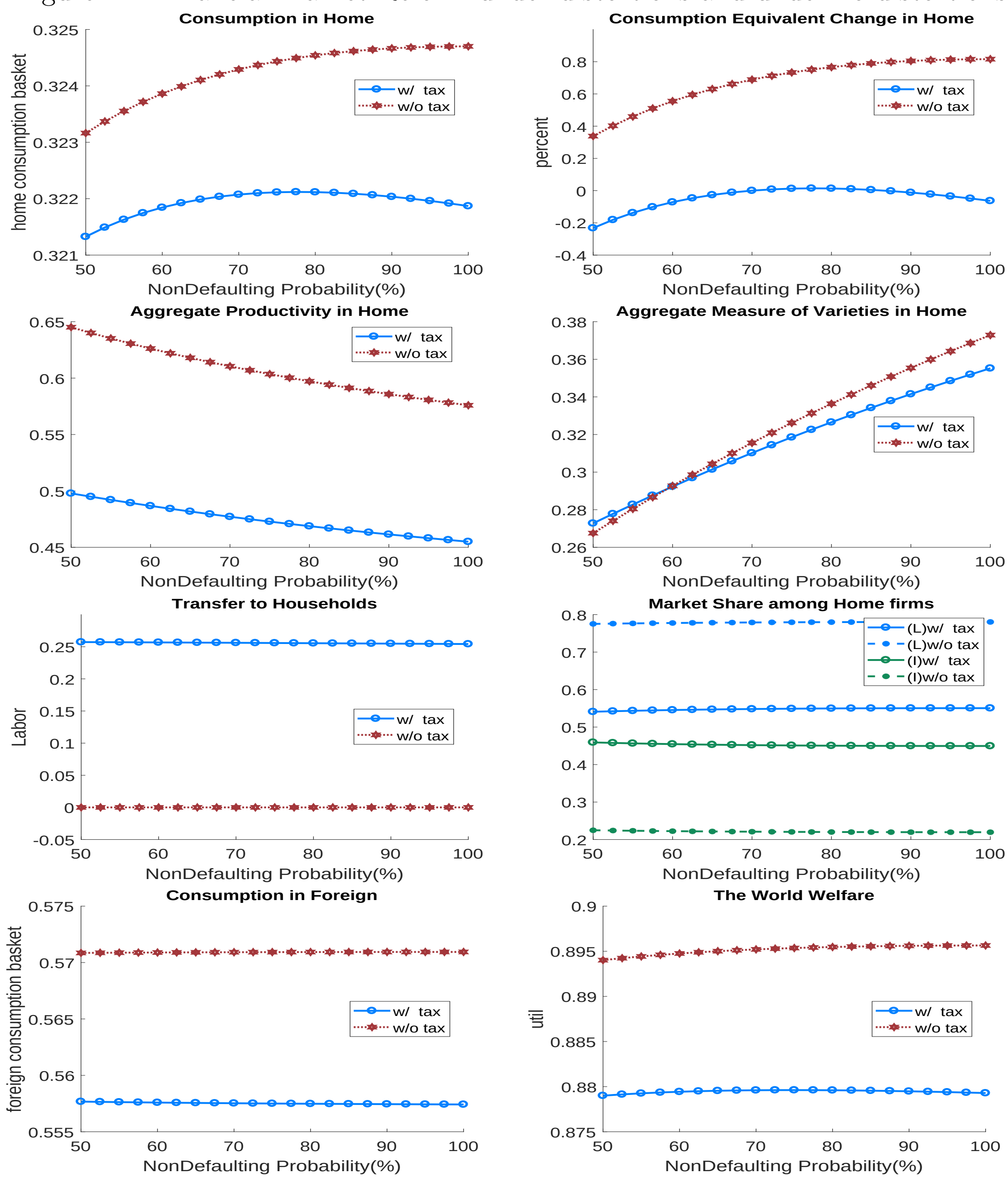


Figure 22: Financial Market Reform under distortions and under no distortions.

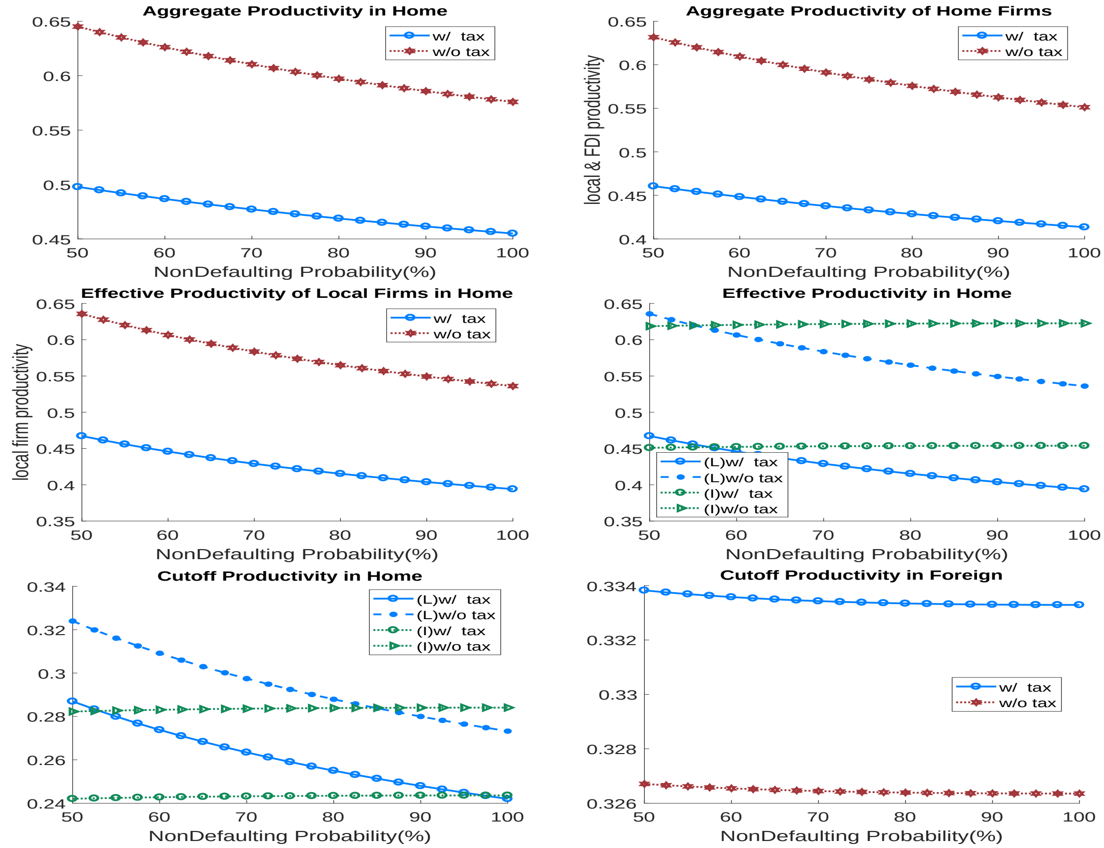


Figure 23: Financial Market Reform under distortions and under no distortions.

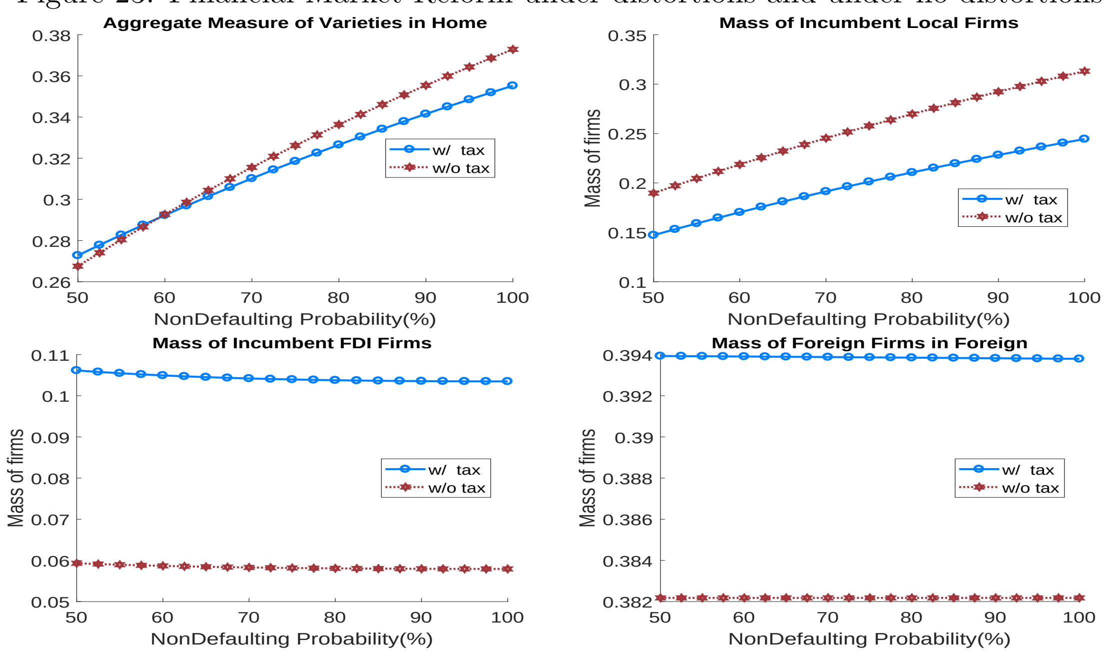
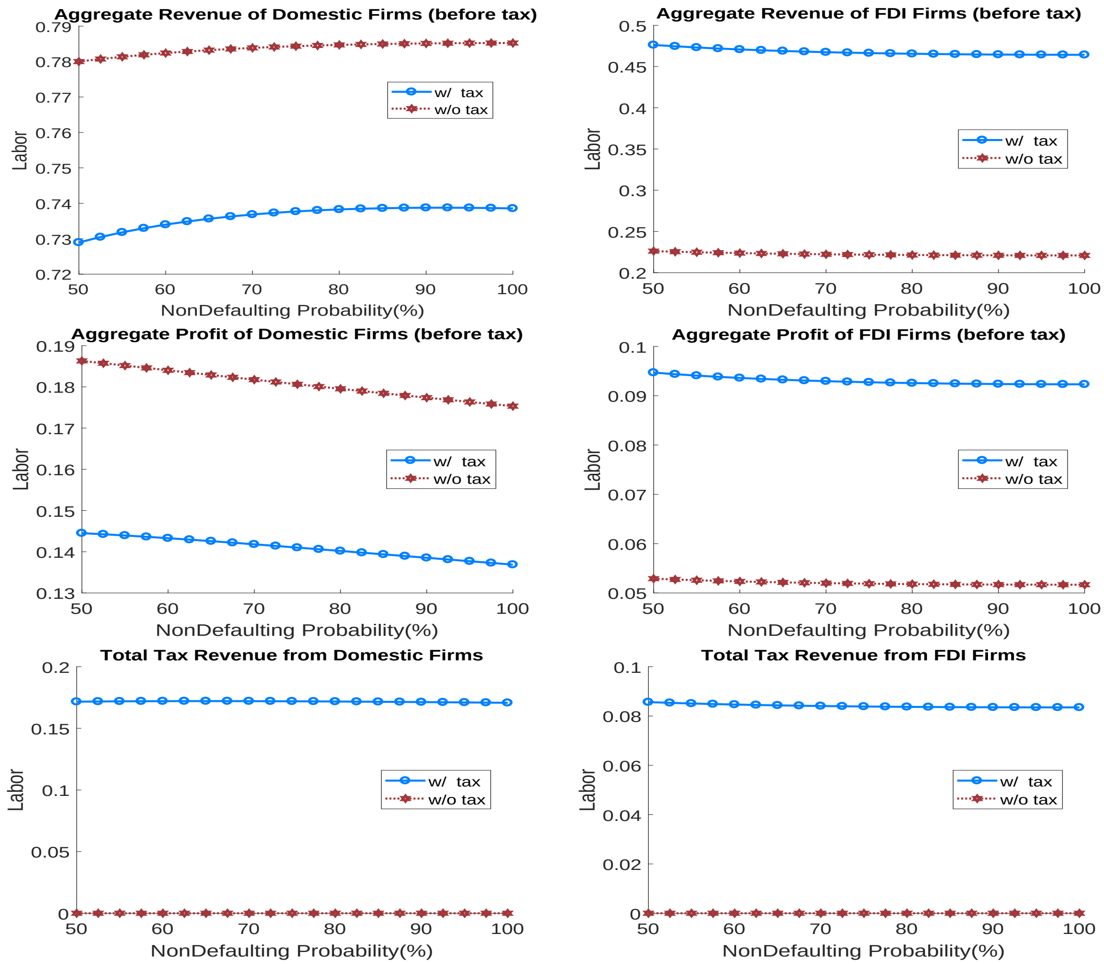


Figure 24: Financial Market Reform under distortions and under no distortions.



G Figure under varying degrees of frictions

Figure 25: Value-Added Tax Reform with varying λ and τ_V^D under $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$.

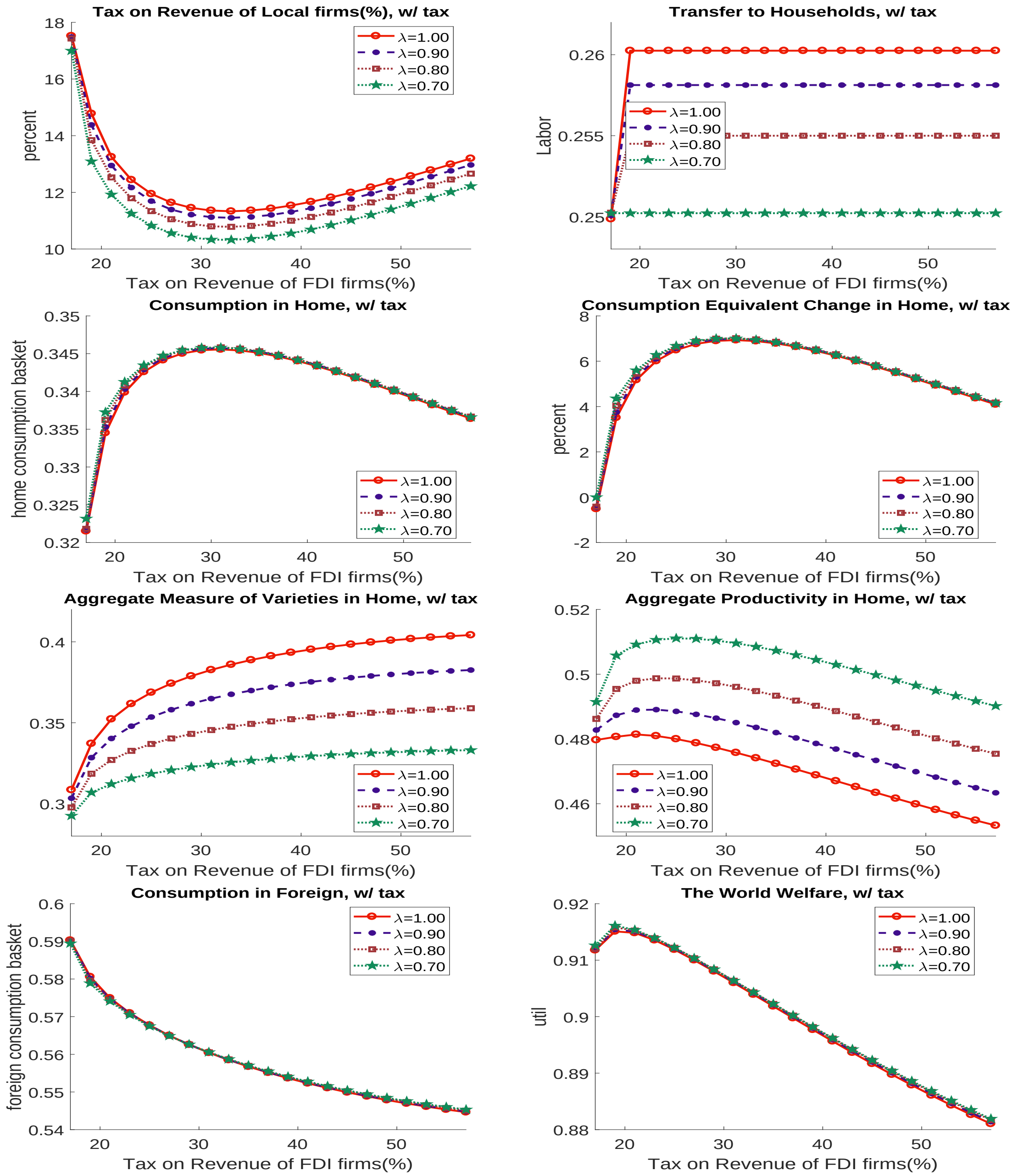


Figure 26: Value-Added Tax Reform with varying λ and τ_V^D under $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$.

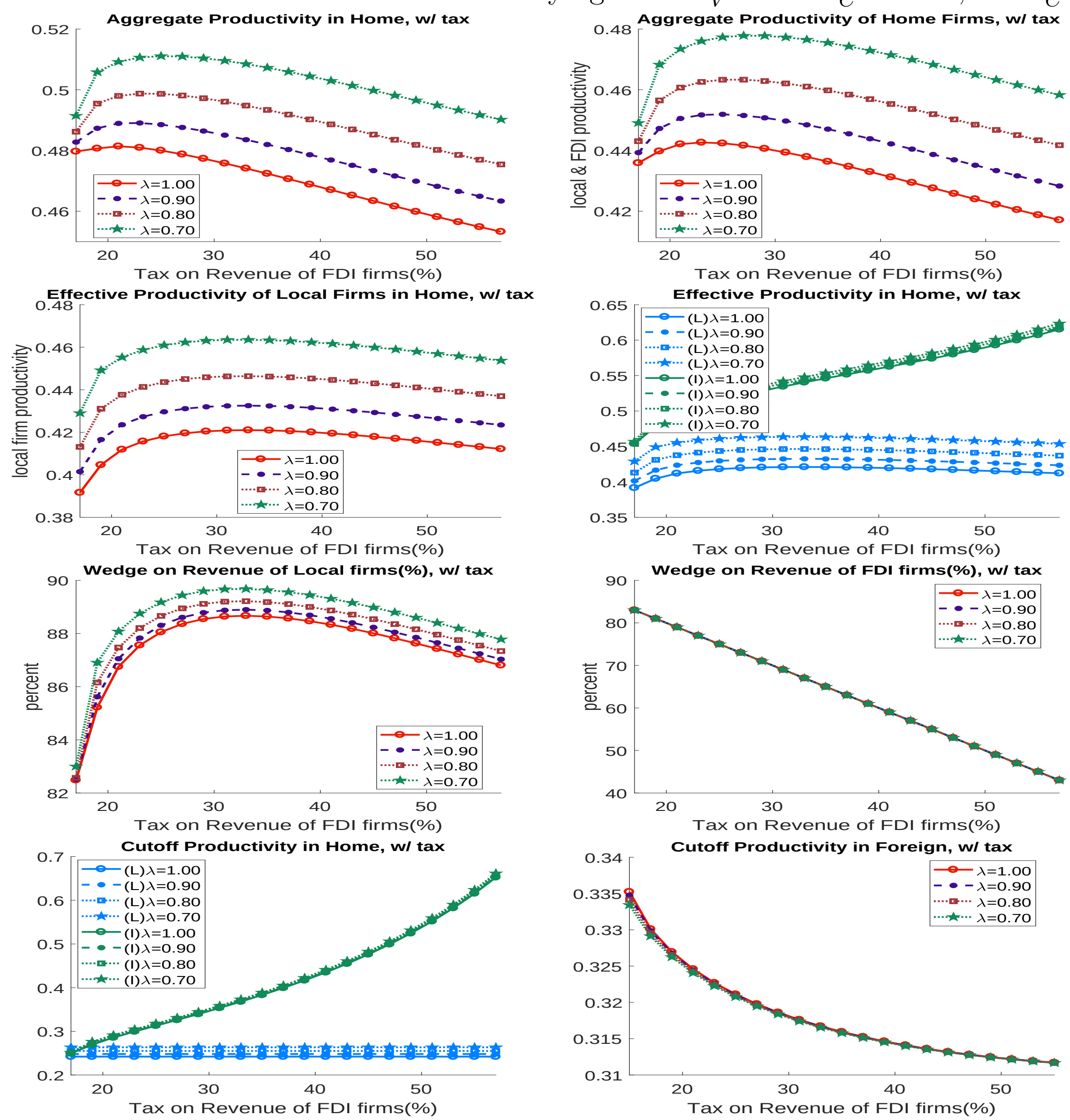


Figure 27: Value-Added Tax Reform with varying λ and τ_V^D under $\tau_C^D = 0.67$, and $\tau_C^I = 0.85$.

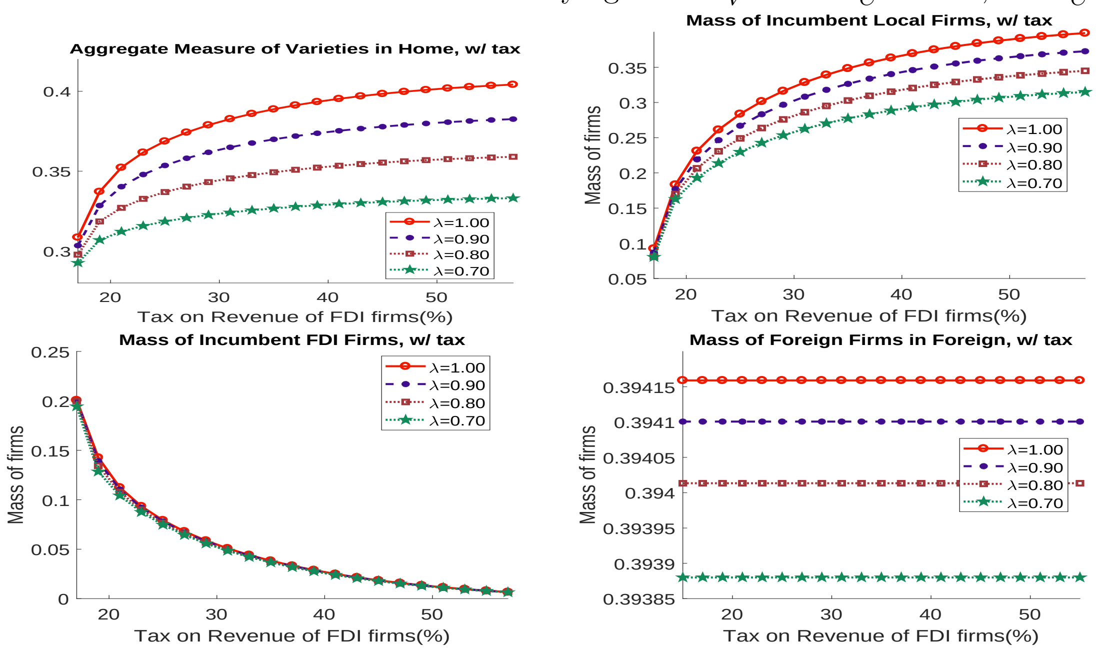


Figure 28: Financial Market Reform with varying τ_V^I under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_C^I = 0.85$.

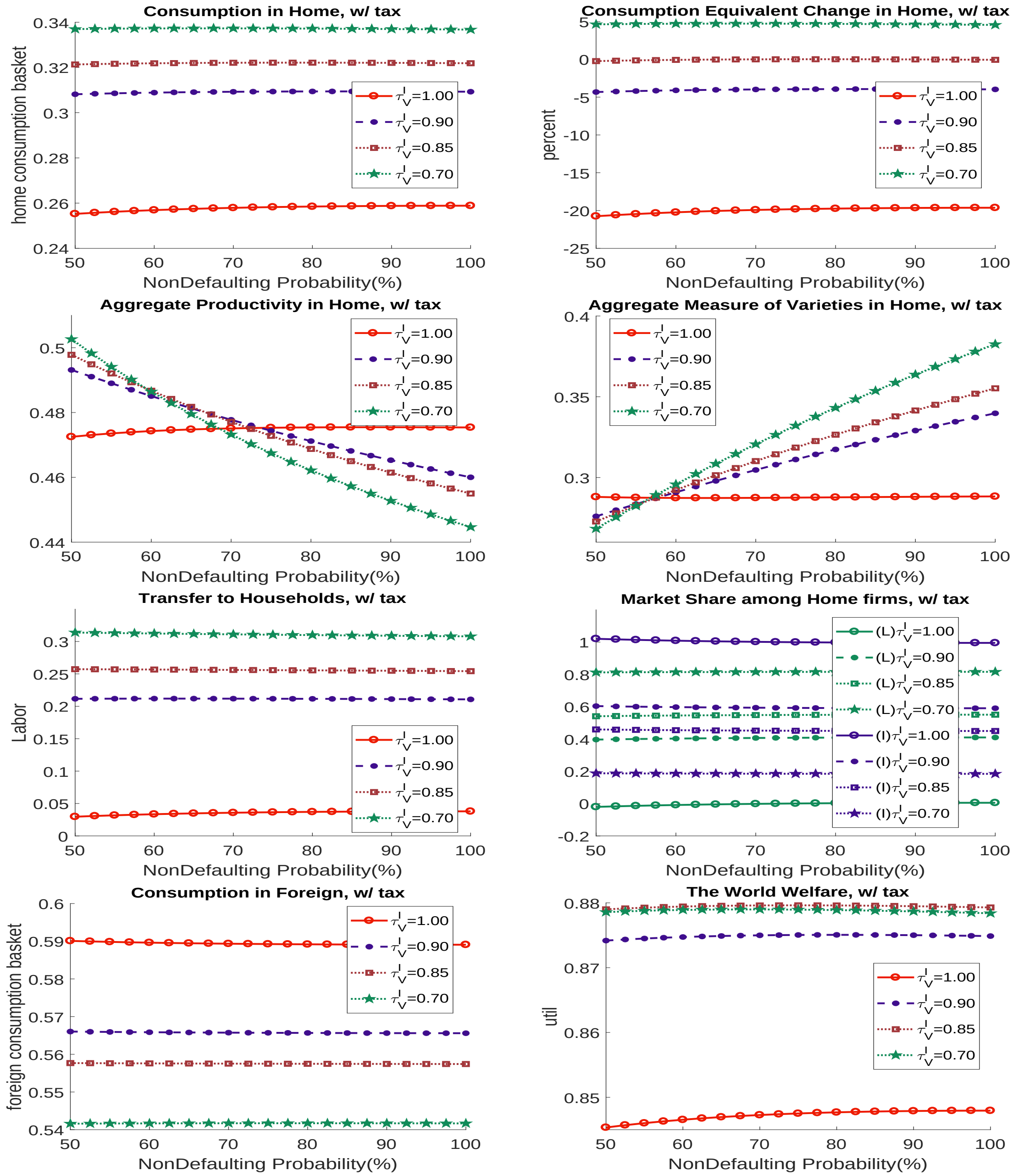


Figure 29: Financial Market Reform with varying τ_V^I under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_C^I = 0.85$.

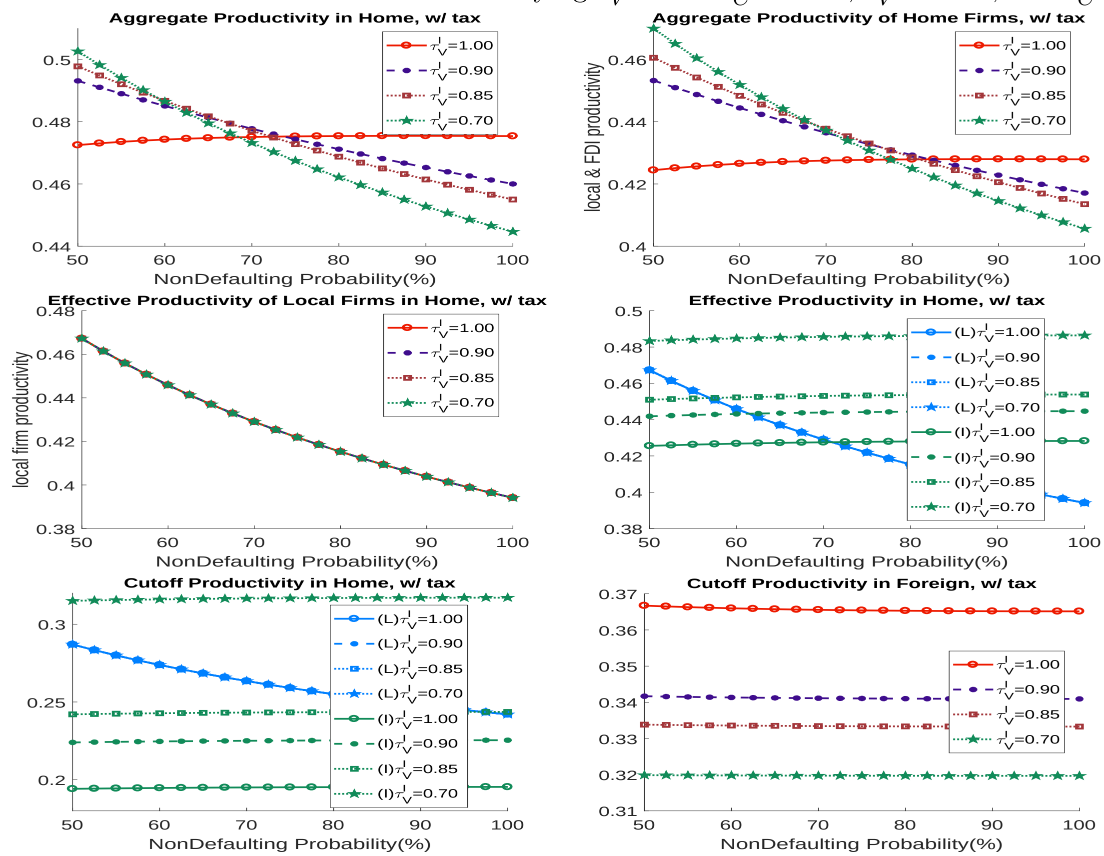


Figure 30: Financial Market Reform with varying τ_V^I under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_C^I = 0.85$.

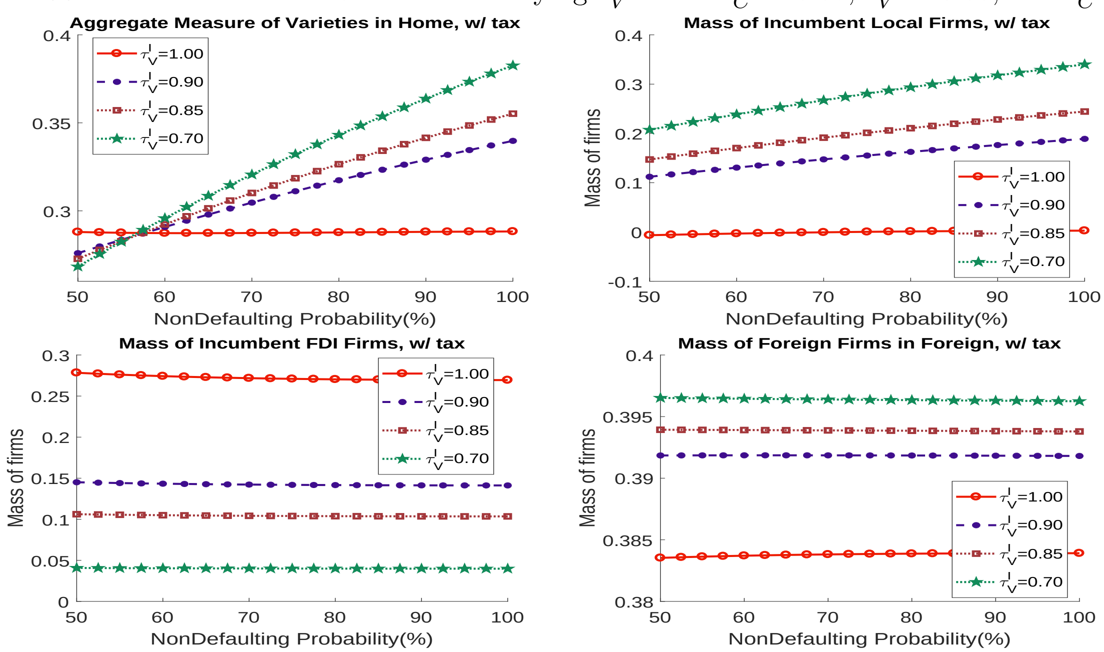
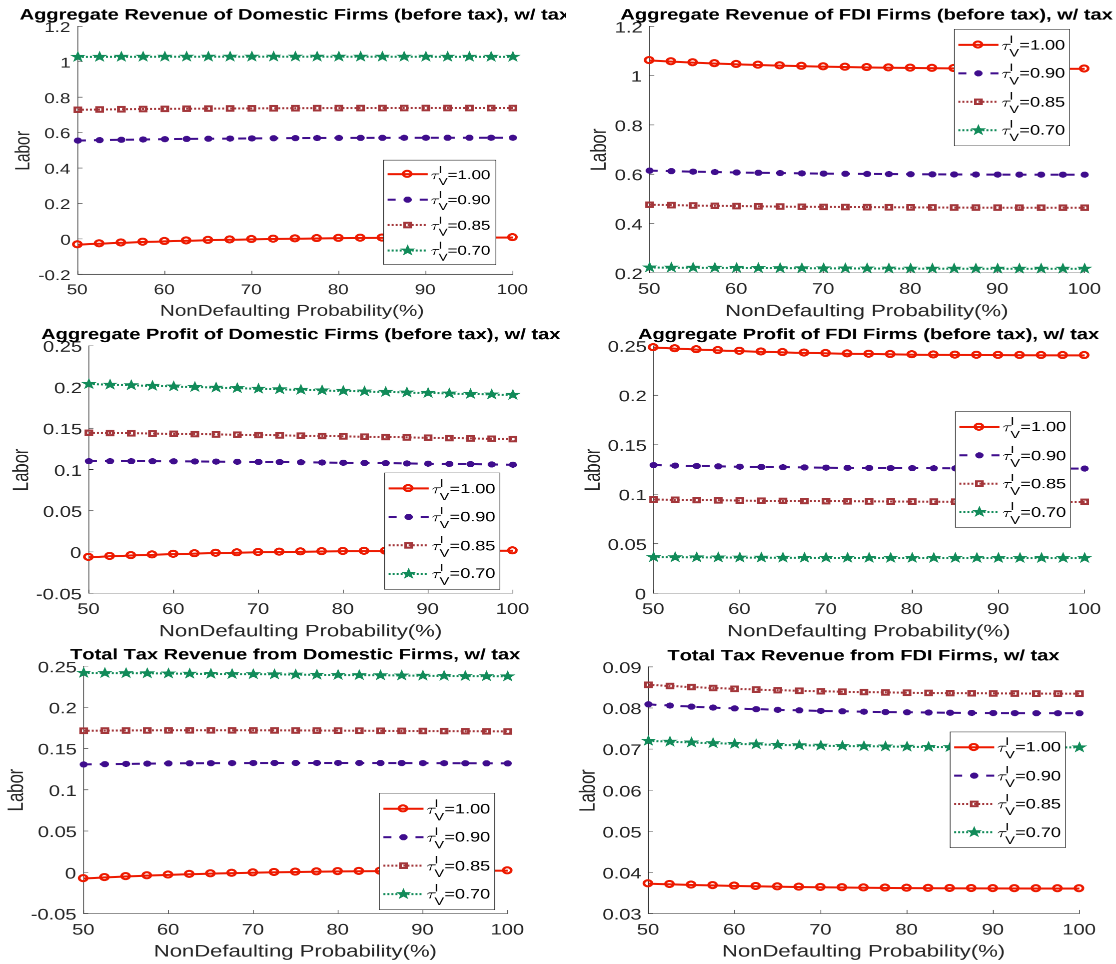


Figure 31: Financial Market Reform with varying τ_V^I under $\tau_C^D = 0.67$, $\tau_V^D = 0.83$, and $\tau_C^I = 0.85$.



H Figure under varying degrees of frictions with no other taxes

Figure 32: Value-Added Tax Reform with varying λ and τ_V^D under $\tau_C^D = 1.00$, and $\tau_C^I = 1.00$.

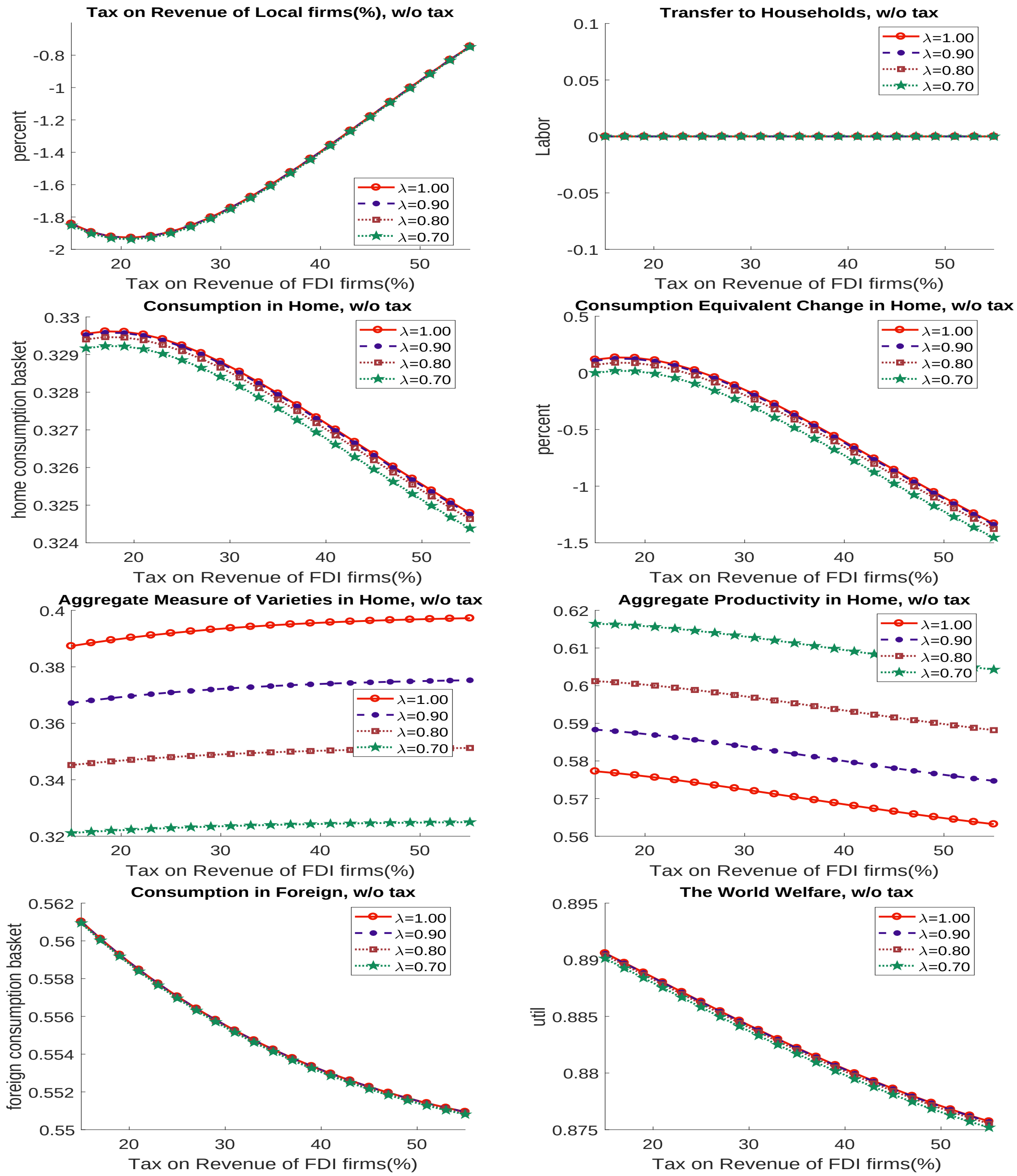


Figure 33: Value-Added Tax Reform with varying λ and τ_V^D under $\tau_C^D = 1.00$, and $\tau_C^I = 1.00$.

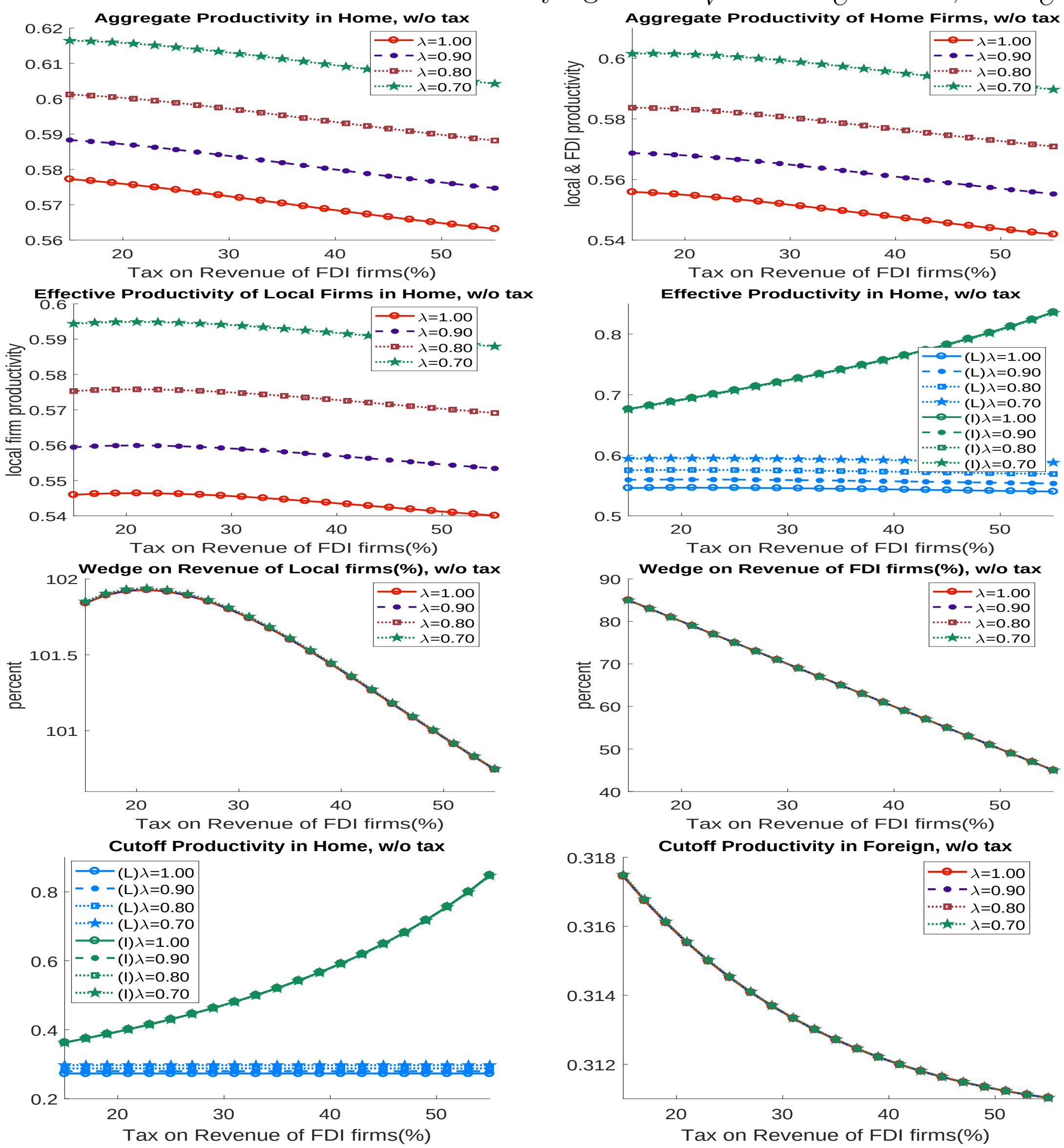


Figure 34: Value-Added Tax Reform with varying λ and τ_V^D under $\tau_C^D = 1.00$, and $\tau_C^I = 1.00$.

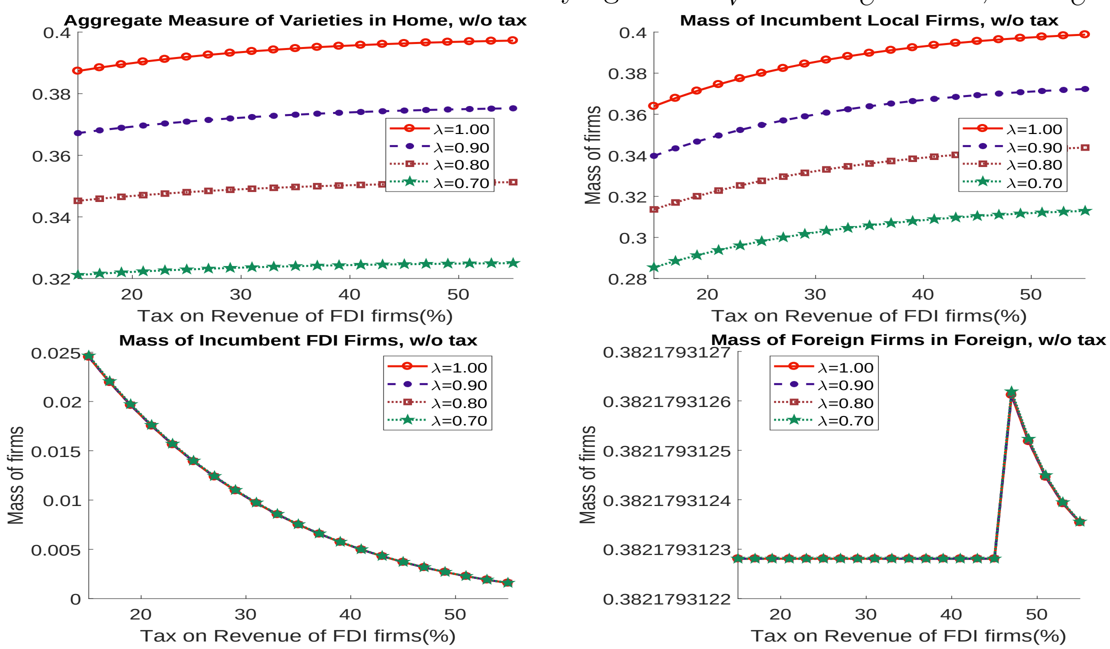


Figure 35: Financial Market Reform with varying τ_V^I under $\tau_C^D = 1.00$, $\tau_V^D = 1.00$, and $\tau_C^I = 1.00$.

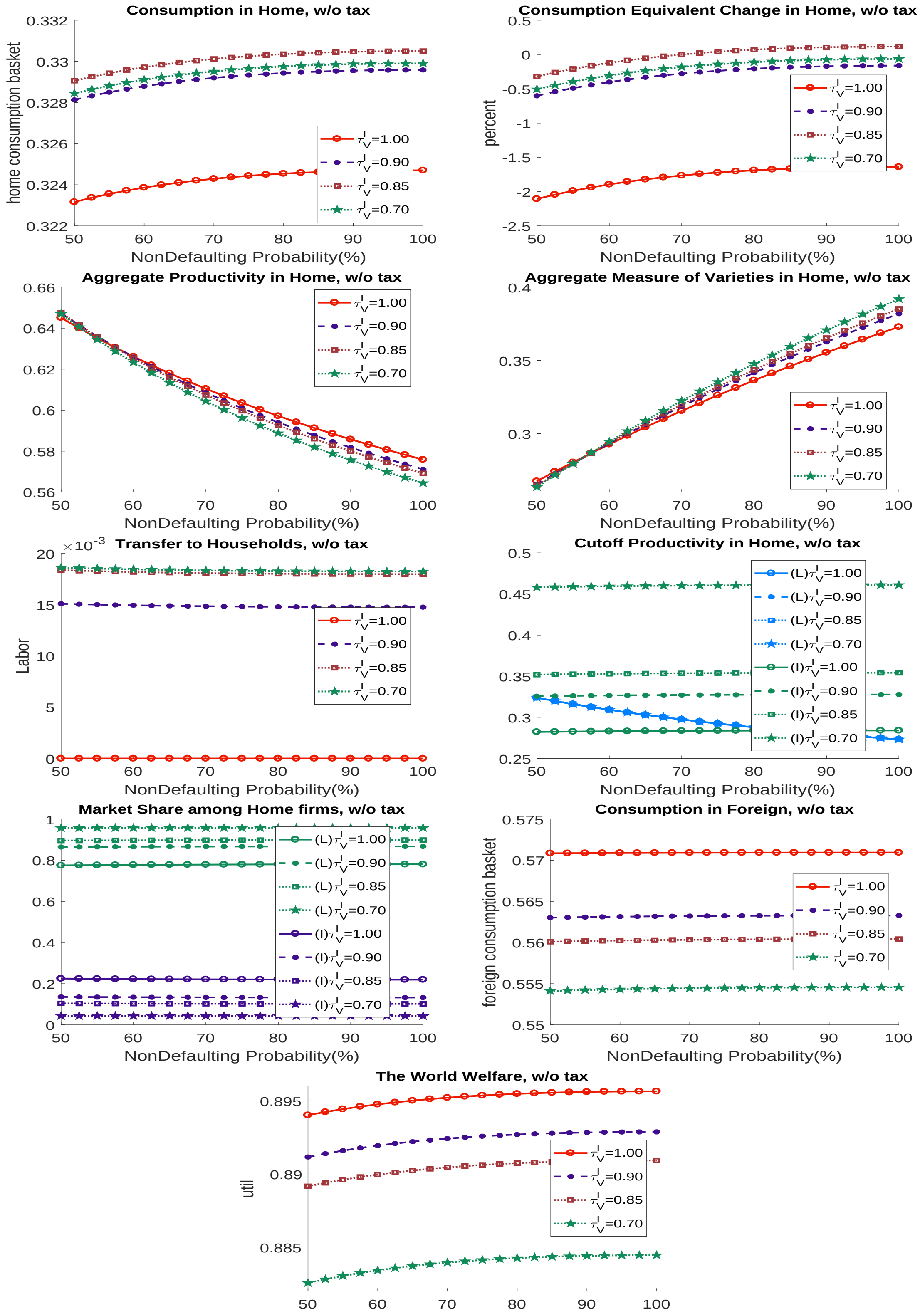


Figure 36: Financial Market Reform with varying τ_V^I under $\tau_C^D = 1.00$, $\tau_V^D = 1.00$, and $\tau_C^I = 1.00$.

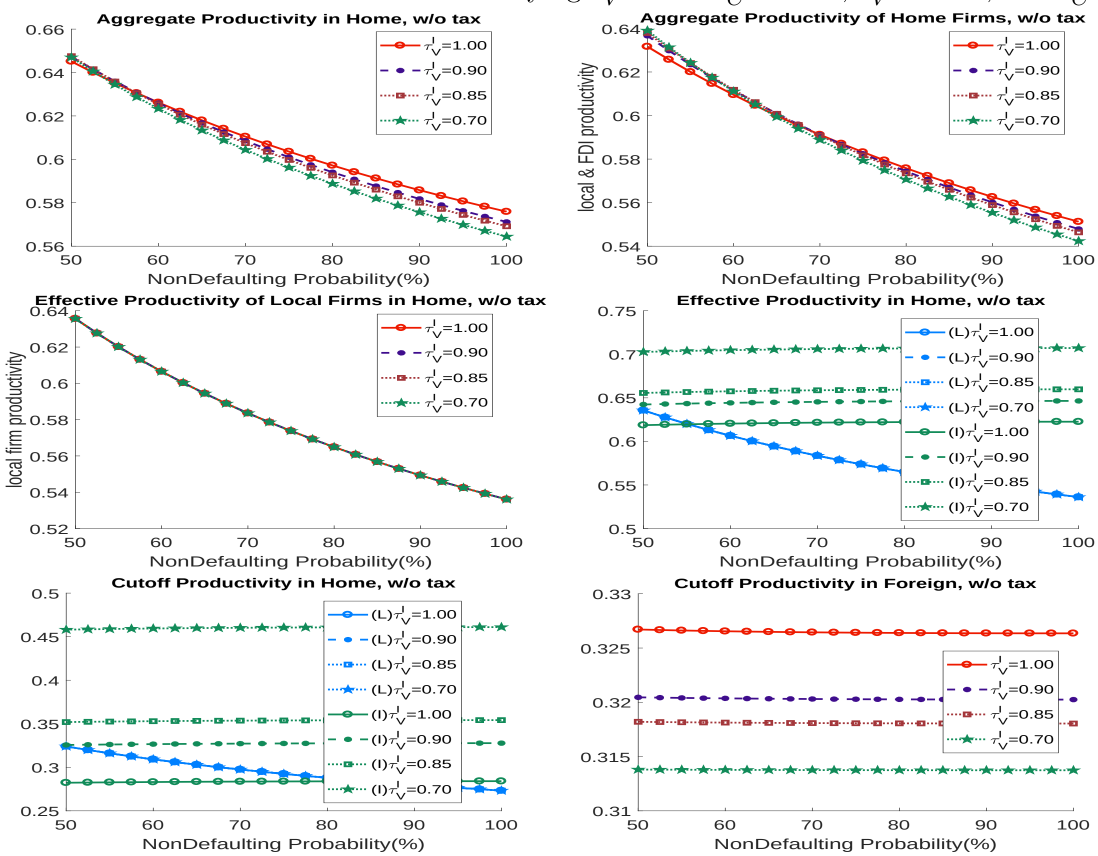


Figure 37: Financial Market Reform with varying τ_V^I under $\tau_C^D = 1.00$, $\tau_V^D = 1.00$, and $\tau_C^I = 1.00$.

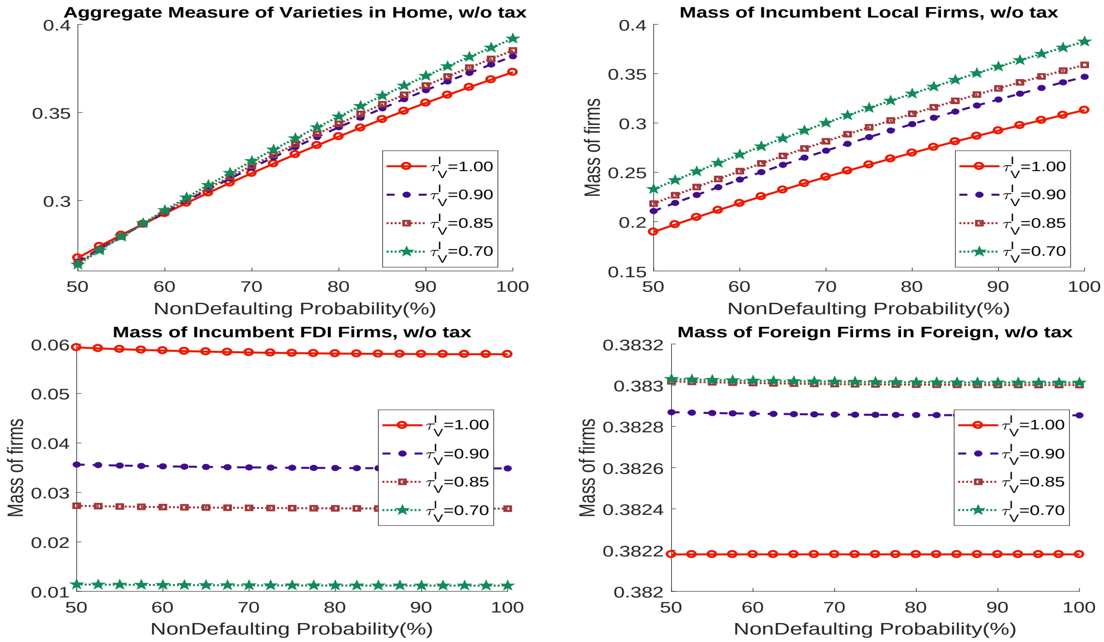


Figure 38: Financial Market Reform with varying τ_V^I under $\tau_C^D = 1.00$, $\tau_V^D = 1.00$, and $\tau_C^I = 1.00$.