

The Online Appendix for Benefits of FDI subsidies: The role of funding sources

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This technical appendix provides total equilibrium conditions and mathematical derivations. We only show the characterization of our model with the separable utility since the equilibrium under the CES utility can be obtained analogously. Note that all tax variables in the online appendix are expressed as wedge terms for algebraic simplicity. The following table displays the mapping of tax variables from the main text to the online appendix.

Table 1: Notations for tax variables in the online appendix

Main Text	Online Appendix
Labor-income tax rate τ_L	Labor-income tax wedge $\tau'_L = 1 - \tau_L$
Consumption tax rate τ_C	Consumption tax wedge $\tau'_C = 1 + \tau_C$
Firm-revenue tax rate τ_R	Firm-revenue tax wedge $\tau'_R = 1 - \tau_R$

In what follows, we use τ'_L , τ'_C , and τ'_R and suppress the superscript ‘ $'$ in all equilibrium conditions. The notation for a FDI subsidy indicates a rate as in the main text.

1 Total Equilibrium Conditions

We have 15 equations to solve for 15 variables: C^H , C^{H*} , C^{F*} , C^F , V , V^* , Z^D , Z^X , Z^I , Z^{D*} , Z^{X*} , Z^{I*} , M , M^* , and one tax variable from $\{\tau_L, \tau_C, \tau_R\}$. There is no government in Foreign: $s_V^* = 0$ and $\tau_L^* = \tau_C^* = \tau_R^* = 1$. We take the partial equilibrium analysis by excluding labor market clearing conditions: $\frac{W^*}{W} = 1$. $\frac{W}{P_0} = \frac{W^*}{P_0^*} = 1$ holds due to the CRTS technology of homogeneous goods. Home homogeneous good is the numeraire in Home and Foreign homogeneous good is the numeraire in Foreign: $P_0 = P_0^* = 1$.

Market Demand Shifters:

$$A^H \equiv (\tau_C)^{-\sigma} (C^H)^{\theta\sigma+1-\sigma}, \quad A^{H*} \equiv (\tau_C^*)^{-\sigma} (C^{H*})^{\theta\sigma+1-\sigma},$$

$$A^{F*} \equiv (\tau_C^*)^{-\sigma} (C^{F*})^{\theta\sigma+1-\sigma}, \quad A^F \equiv (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma}.$$

Average Productivity and Useful Definitions under the Pareto distribution:

$$\frac{J(Z^D, Z^X)}{G(Z^X) - G(Z^D)} = \left(\tilde{Z}^L \right)^{\sigma-1} = \left(\frac{\eta}{\eta-\sigma+1} \right) \left(\frac{(Z^D)^{-(\eta-\sigma+1)} - (Z^X)^{-(\eta-\sigma+1)}}{(Z^D)^{-\eta} - (Z^X)^{-\eta}} \right),$$

$$\frac{J(Z^X, Z^I)}{G(Z^I) - G(Z^X)} = \left(\tilde{Z}^X \right)^{\sigma-1} = \left(\frac{\eta}{\eta-\sigma+1} \right) \left(\frac{(Z^X)^{-(\eta-\sigma+1)} - (Z^I)^{-(\eta-\sigma+1)}}{(Z^X)^{-\eta} - (Z^I)^{-\eta}} \right),$$

$$\frac{J(Z^I)}{1-G(Z^I)} = \left(\tilde{Z}^I \right)^{\sigma-1} = \left(\frac{\eta}{\eta-\sigma+1} \right) (Z^I)^{\sigma-1},$$

$$\frac{J(Z^D)}{1-G(Z^D)} = \left(\tilde{Z}^D \right)^{\sigma-1} = \left(\frac{\eta}{\eta-\sigma+1} \right) (Z^D)^{\sigma-1}.$$

$$\frac{J^*(Z^{D*}, Z^{X*})}{G^*(Z^{X*}) - G^*(Z^{D*})} = \left(\tilde{Z}^{L*} \right)^{\sigma-1} = \left(\frac{\eta^*}{\eta^*-\sigma+1} \right) \left(\frac{(Z^{D*})^{-(\eta^*-\sigma+1)} - (Z^{X*})^{-(\eta^*-\sigma+1)}}{(Z^{D*})^{-\eta^*} - (Z^{X*})^{-\eta^*}} \right),$$

$$\frac{J^*(Z^{X*}, Z^{I*})}{G^*(Z^{I*}) - G^*(Z^{X*})} = \left(\tilde{Z}^{X*} \right)^{\sigma-1} = \left(\frac{\eta^*}{\eta^*-\sigma+1} \right) \left(\frac{(Z^{X*})^{-(\eta^*-\sigma+1)} - (Z^{I*})^{-(\eta^*-\sigma+1)}}{(Z^{X*})^{-\eta^*} - (Z^{I*})^{-\eta^*}} \right),$$

$$\frac{J^*(Z^{I*})}{1-G^*(Z^{I*})} = \left(\tilde{Z}^{I*} \right)^{\sigma-1} = \left(\frac{\eta^*}{\eta^*-\sigma+1} \right) (Z^{I*})^{\sigma-1},$$

$$\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})} = \left(\tilde{Z}^{D*} \right)^{\sigma-1} = \left(\frac{\eta^*}{\eta^*-\sigma+1} \right) (Z^{D*})^{\sigma-1}.$$

These are equivalent to

$$\frac{J(Z^D, Z^X)}{1-G(Z^D)} = \int_{Z^D}^{Z^X} z^{\sigma-1} \frac{dG(z)}{1-G(Z^D)} = \left(\frac{\eta}{\eta-\sigma+1} \right) \left(\frac{(Z^D)^{-\eta+\sigma-1} - (Z^X)^{-\eta+\sigma-1}}{(Z^D)^{-\eta}} \right),$$

$$\frac{J(Z^X, Z^I)}{1-G(Z^D)} = \int_{Z^X}^{Z^I} z^{\sigma-1} \frac{dG(z)}{1-G(Z^D)} = \left(\frac{\eta}{\eta-\sigma+1} \right) \left(\frac{(Z^X)^{-\eta+\sigma-1} - (Z^I)^{-\eta+\sigma-1}}{(Z^D)^{-\eta}} \right),$$

$$\frac{J(Z^I)}{1-G(Z^D)} = \int_{Z^I}^{\infty} z^{\sigma-1} \frac{dG(z)}{1-G(Z^D)} = \left(\frac{\eta}{\eta-\sigma+1} \right) \left(\frac{(Z^I)^{-\eta+\sigma-1}}{(Z^D)^{-\eta}} \right),$$

$$\frac{J(Z^D)}{1-G(Z^D)} = \int_{Z^D}^{\infty} z^{\sigma-1} \frac{dG(z)}{1-G(Z^D)} = \left(\frac{\eta}{\eta-\sigma+1} \right) (Z^D)^{\sigma-1}.$$

$$\frac{J^*(Z^{D*}, Z^{X*})}{1-G^*(Z^{D*})} = \int_{Z^{D*}}^{Z^{X*}} z^{\sigma-1} \frac{dG^*(z)}{1-G^*(Z^{D*})} = \left(\frac{\eta^*}{\eta^*-\sigma+1} \right) \left(\frac{(Z^{D*})^{-\eta^*+\sigma-1} - (Z^{X*})^{-\eta^*+\sigma-1}}{(Z^{D*})^{-\eta^*}} \right),$$

$$\frac{J^*(Z^{X*}, Z^{I*})}{1-G^*(Z^{D*})} = \int_{Z^{X*}}^{Z^{I*}} z^{\sigma-1} \frac{dG^*(z)}{1-G^*(Z^{D*})} = \left(\frac{\eta^*}{\eta^*-\sigma+1} \right) \left(\frac{(Z^{X*})^{-\eta^*+\sigma-1} - (Z^{I*})^{-\eta^*+\sigma-1}}{(Z^{D*})^{-\eta^*}} \right),$$

$$\frac{J^*(Z^{I*})}{1-G^*(Z^{D*})} = \int_{Z^{I*}}^{\infty} z^{\sigma-1} \frac{dG^*(z)}{1-G^*(Z^{D*})} = \left(\frac{\eta^*}{\eta^*-\sigma+1} \right) \left(\frac{(Z^{I*})^{-\eta^*+\sigma-1}}{(Z^{D*})^{-\eta^*}} \right),$$

$$\frac{J^*(Z^{D*})}{1-G^*(Z^{D*})} = \int_{Z^{D*}}^{\infty} z^{\sigma-1} \frac{dG^*(z)}{1-G^*(Z^{D*})} = \left(\frac{\eta^*}{\eta^*-\sigma+1} \right) (Z^{D*})^{\sigma-1}.$$

Home Households (Equation 1, 2):

$$\begin{aligned}
& (C^H)^{(\sigma-1)(1-\theta)} \\
&= M^L \int_{Z^D}^{Z^X} \left[\frac{\tau_C p^D(z)}{P_0} \right]^{1-\sigma} \frac{dG(z)}{G(Z^X) - G(Z^D)} + M^X \int_{Z^X}^{Z^I} \left[\frac{\tau_C p^{D,X}(z)}{P_0} \right]^{1-\sigma} \frac{dG(z)}{G(Z^I) - G(Z^X)} + M^I \int_{Z^I}^{\infty} \left[\frac{\tau_C p^{D,I}(z)}{P_0} \right]^{1-\sigma} \frac{dG(z)}{1-G(Z^I)} \\
&= \frac{M}{1-G(Z^D)} \int_{Z^D}^{\infty} \left[(\tau_C) \left(\frac{1}{\rho} \right) \left(\frac{1}{\tau_R} \right) \left(\frac{W}{P_0} \right) \left(\frac{1}{z} \right) \right]^{1-\sigma} dG(z) \\
&= \frac{M}{1-G(Z^D)} (\tau_C)^{1-\sigma} \left(\frac{1}{\rho} \right)^{1-\sigma} \left(\frac{1}{\tau_R} \right)^{1-\sigma} \left(\frac{W}{P_0} \right)^{1-\sigma} J(Z^D) \\
&= \underbrace{\frac{M}{\text{Variety}} \underbrace{\left(\frac{1}{\rho} \frac{W}{P_0} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)}}_{\text{Effective Price}} \underbrace{\left(\tilde{Z}^D \right)^{\sigma-1}}_{\text{Average Productivity}}
\end{aligned}$$

where $\sigma > 1$, $M^L = \left(\frac{G(Z^X) - G(Z^D)}{1-G(Z^D)} \right) M$, $M^X = \left(\frac{G(Z^I) - G(Z^X)}{1-G(Z^D)} \right) M$, and $M^I = \left(\frac{1-G(Z^I)}{1-G(Z^D)} \right) M$.

$$\begin{aligned}
& (C^F)^{(\sigma-1)(1-\theta)} \\
&= M^{X*} \int_{Z^{X*}}^{Z^{I*}} \left[\frac{\tau_C p^X(z)}{P_0} \right]^{1-\sigma} \frac{dG^*(z)}{G^*(Z^{I*}) - G^*(Z^{X*})} + M^{I*} \int_{Z^{I*}}^{\infty} \left[\frac{\tau_C p^I(z)}{P_0} \right]^{1-\sigma} \frac{dG^*(z)}{1-G^*(Z^{I*})} \\
&= \left(\begin{array}{l} + M^{X*} \left[(\tau_C) \left(\frac{\tau^*}{\rho} \right) \left(\frac{1}{\tau_R^*} \right) \left(\frac{W^*}{P_0} \right) \right]^{1-\sigma} \frac{J^*(Z^{X*}, Z^{I*})}{G^*(Z^{I*}) - G^*(Z^{X*})} \\ + M^{I*} \left[(\tau_C) \left(\frac{1-s_V}{\rho} \right) \left(\frac{1}{\tau_R} \right) \left(\frac{W}{P_0} \right) \right]^{1-\sigma} \frac{J^*(Z^{I*})}{1-G^*(Z^{I*})} \end{array} \right) \\
&= \left(\begin{array}{l} + \frac{M^*}{1-G^*(Z^{D*})} (\tau_C)^{1-\sigma} \left(\frac{\tau^*}{\rho} \right)^{1-\sigma} \left(\frac{1}{\tau_R^*} \right)^{1-\sigma} \left(\frac{W^*}{P_0} \right)^{1-\sigma} J^*(Z^{X*}, Z^{I*}) \\ + \frac{M^*}{1-G^*(Z^{D*})} (\tau_C)^{1-\sigma} \left(\frac{1-s_V}{\rho} \right)^{1-\sigma} \left(\frac{1}{\tau_R} \right)^{1-\sigma} \left(\frac{W}{P_0} \right)^{1-\sigma} J^*(Z^{I*}) \end{array} \right) \\
&= \left(\begin{array}{l} + M^{X*} \left(\frac{\tau^*}{\rho} \frac{W^*}{P_0} \frac{\tau_C}{\tau_R^*} \right)^{-(\sigma-1)} \left(\tilde{Z}^{X*} \right)^{\sigma-1} \\ + \underbrace{M^{I*} \left(\frac{1-s_V}{\rho} \frac{W}{P_0} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)}}_{\text{Variety}} \underbrace{\left(\tilde{Z}^{I*} \right)^{\sigma-1}}_{\text{Effective Price}} \end{array} \right)
\end{aligned}$$

where $\sigma > 1$, $M^{L*} = \left(\frac{G^*(Z^{X*}) - G^*(Z^{D*})}{1-G^*(Z^{D*})} \right) M^*$, $M^{X*} = \left(\frac{G^*(Z^{I*}) - G^*(Z^{X*})}{1-G^*(Z^{D*})} \right) M^*$, and $M^{I*} = \left(\frac{1-G^*(Z^{I*})}{1-G^*(Z^{D*})} \right) M^*$.

Foreign Households (Equation 3, 4):

$$\begin{aligned}
& (C^{F*})^{(\sigma-1)(1-\theta)} \\
&= \left(M^{L*} \int_{Z^{D*}}^{Z^{X*}} \left[\frac{\tau_C^* p^{D*}(z)}{P_0^*} \right]^{1-\sigma} \frac{dG^*(z)}{G^*(Z^{X*}) - G^*(Z^{D*})} \right. \\
&\quad \left. + M^{X*} \int_{Z^{X*}}^{Z^{I*}} \left[\frac{\tau_C^* p^{D,X*}(z)}{P_0^*} \right]^{1-\sigma} \frac{dG^*(z)}{G^*(Z^{I*}) - G^*(Z^{X*})} + M^{I*} \int_{Z^{I*}}^{\infty} \left[\frac{\tau_C^* p^{D,I*}(z)}{P_0^*} \right]^{1-\sigma} \frac{dG^*(z)}{1-G^*(Z^{I*})} \right) \\
&= \frac{M^*}{1-G^*(Z^{D*})} \int_{Z^{D*}}^{\infty} \left[(\tau_C^*) \left(\frac{1}{\rho} \right) \left(\frac{1}{\tau_R^*} \right) \left(\frac{W^*}{P_0^*} \right) \left(\frac{1}{z} \right) \right]^{1-\sigma} dG^*(z) \\
&= \frac{M^*}{1-G^*(Z^{D*})} (\tau_C^*)^{1-\sigma} \left(\frac{1}{\rho} \right)^{1-\sigma} \left(\frac{1}{\tau_R^*} \right)^{1-\sigma} \left(\frac{W^*}{P_0^*} \right)^{1-\sigma} J^*(Z^{D*}) \\
&= \underbrace{\frac{M^*}{\text{Variety}} \left(\frac{1}{\rho} \frac{W^*}{P_0^*} \frac{\tau_C^*}{\tau_R^*} \right)^{-(\sigma-1)}}_{\text{Effective Price}} \underbrace{\left(\tilde{Z}^{D*} \right)^{\sigma-1}}_{\text{Average Productivity}}
\end{aligned}$$

where $\sigma > 1$, $M^{L*} = \left(\frac{G^*(Z^{X*}) - G^*(Z^{D*})}{1-G^*(Z^{D*})} \right) M^*$, $M^{X*} = \left(\frac{G^*(Z^{I*}) - G^*(Z^{X*})}{1-G^*(Z^{X*})} \right) M^*$, and $M^{I*} = \left(\frac{1-G^*(Z^{I*})}{1-G^*(Z^{D*})} \right) M^*$.

$$\begin{aligned}
& (C^{H*})^{-(1-\sigma)(1-\theta)} \\
&= M^X \int_{Z^X}^{Z^I} \left[\frac{\tau_C^* p^{X*}(z)}{P_0^*} \right]^{1-\sigma} \frac{dG(z)}{G(Z^I) - G(Z^X)} + M^I \int_{Z^I}^{\infty} \left[\frac{\tau_C^* p^{I*}(z)}{P_0^*} \right]^{1-\sigma} \frac{dG(z)}{1-G(Z^I)} \\
&= \left(+ M^X \left[(\tau_C^*) \left(\frac{1}{\rho} \right) \left(\frac{W}{P_0^*} \right) \right]^{1-\sigma} \frac{J(Z^X, Z^I)}{G(Z^I) - G(Z^X)} \right) \\
&\quad + M^I \left[(\tau_C^*) \left(\frac{(1-s_V^*)}{\rho} \right) \left(\frac{1}{\tau_R^*} \right) \left(\frac{W^*}{P_0^*} \right) \right]^{1-\sigma} \frac{J(Z^I)}{1-G(Z^I)} \\
&= \left(+ \frac{M}{1-G(Z^D)} (\tau_C^*)^{1-\sigma} \left(\frac{1}{\rho} \right)^{1-\sigma} \left(\frac{1}{\tau_R^*} \right)^{1-\sigma} \left(\frac{W}{P_0^*} \right)^{1-\sigma} J(Z^X, Z^I) \right) \\
&\quad + \frac{M}{1-G(Z^D)} (\tau_C^*)^{1-\sigma} \left(\frac{1-s_V^*}{\rho} \right)^{1-\sigma} \left(\frac{1}{\tau_R^*} \right)^{1-\sigma} \left(\frac{W^*}{P_0^*} \right)^{1-\sigma} J(Z^I) \\
&= \left(+ M^X \left(\frac{1}{\rho} \frac{W}{P_0^*} \frac{\tau_C^*}{\tau_R^*} \right)^{-(\sigma-1)} \left(\tilde{Z}^X \right)^{\sigma-1} \right. \\
&\quad \left. + \underbrace{\frac{M^I}{\text{Variety}} \left(\frac{1-s_V^*}{\rho} \frac{W^*}{P_0^*} \frac{\tau_C^*}{\tau_R^*} \right)^{-(\sigma-1)}}_{\text{Effective Price}} \underbrace{\left(\tilde{Z}^I \right)^{\sigma-1}}_{\text{Average Productivity}} \right)
\end{aligned}$$

where $\sigma > 1$, $M^L = \left(\frac{G(Z^X) - G(Z^D)}{1-G(Z^D)} \right) M$, $M^X = \left(\frac{G(Z^I) - G(Z^X)}{1-G(Z^D)} \right) M$, and $M^I = \left(\frac{1-G(Z^I)}{1-G(Z^D)} \right) M$.

The Indirect Utility (Equation 5, 6):

$$V = \frac{W}{P_0} L \tau_L + \left(\frac{1}{\theta} - 1 \right) (C^H)^\theta + \left(\frac{1}{\theta} - 1 \right) (C^F)^\theta$$

$$V^* = \frac{W^*}{P_0^*} L^* \tau_L^* + \left(\frac{1}{\theta} - 1 \right) (C^{F*})^\theta + \left(\frac{1}{\theta} - 1 \right) (C^{H*})^\theta$$

Cutoff Productivity (Equation 7, 8, 9, 10, 11, 12):

$$\begin{aligned}
Z^D &= \left(\frac{\frac{W}{P_0} f^D}{(\tau_R)^\sigma (\frac{1}{\sigma}) (\frac{1}{\rho})^{1-\sigma} (\frac{W}{P_0})^{1-\sigma} A^H} \right)^{\frac{1}{\sigma-1}} \\
Z^X &= \left(\frac{\frac{W}{P_0^*} f^X}{(\tau_R)^\sigma (\frac{1}{\tau})^{\sigma-1} (\frac{1}{\sigma})^{1-\sigma} (\frac{W}{W^*})^{1-\sigma} (\frac{W^*}{P_0^*})^{1-\sigma} A^{H*}} \right)^{\frac{1}{\sigma-1}} \\
Z^I &= \left(\frac{\frac{W^*}{P_0^*} f^{I*} - \frac{W}{P_0^*} f^X}{(\frac{1}{\sigma}) (\frac{1}{\rho})^{1-\sigma} \left[(\tau_R^*)^\sigma (1-s_V^*)^{1-\sigma} (\frac{W^*}{P_0^*})^{1-\sigma} A^{H*} - (\tau_R)^\sigma (\frac{1}{\tau})^{\sigma-1} (\frac{W}{W^*})^{1-\sigma} (\frac{W^*}{P_0^*})^{1-\sigma} A^{H*} \right]} \right)^{\frac{1}{\sigma-1}} \\
Z^{D*} &= \left(\frac{\left(\frac{W^*}{P_0^*} \right) f^{D*}}{(\tau_R^*)^\sigma (\frac{1}{\sigma}) (\frac{1}{\rho})^{1-\sigma} (\frac{W^*}{P_0^*})^{1-\sigma} A^{F*}} \right)^{\frac{1}{\sigma-1}} \\
Z^{X*} &= \left(\frac{\frac{W}{P_0^*} f^{X*}}{(\tau_R^*)^\sigma (\frac{1}{\tau^*})^{\sigma-1} (\frac{1}{\sigma}) (\frac{1}{\rho})^{1-\sigma} (\frac{W^*}{W})^{1-\sigma} (\frac{W}{P_0^*})^{1-\sigma} A^F} \right)^{\frac{1}{\sigma-1}} \\
Z^{I*} &= \left(\frac{\frac{W}{P_0} f^I - \frac{W^*}{P_0^*} f^{X*}}{(\frac{1}{\sigma}) (\frac{1}{\rho})^{1-\sigma} \left[(\tau_R)^\sigma (1-s_V)^{1-\sigma} (\frac{W}{P_0})^{1-\sigma} A^F - (\tau_R^*)^\sigma (\frac{1}{\tau^*})^{\sigma-1} (\frac{W^*}{W})^{1-\sigma} (\frac{W}{P_0})^{1-\sigma} A^F \right]} \right)^{\frac{1}{\sigma-1}}
\end{aligned}$$

Free Entries (Equation 13, 14):

$$\delta_{\frac{W}{P_0}} F^D = \left(\begin{array}{l} + J(Z^D) (\tau_R)^\sigma (\frac{1}{\sigma}) \left(\frac{1}{\rho} \right)^{1-\sigma} \left[\left(\frac{W}{P_0} \right)^{1-\sigma} (\tau_C)^{-\sigma} (C^H)^{\theta\sigma+1-\sigma} \right] \\ + \left(\frac{P_0^*}{P_0} \right) J(Z^X, Z^I) (\tau_R)^\sigma (\frac{1}{\sigma}) \left(\frac{1}{\rho} \right)^{1-\sigma} \left(\frac{1}{\tau} \right)^{\sigma-1} \left[\left(\frac{W}{P_0^*} \right)^{1-\sigma} (\tau_C^*)^{-\sigma} (C^{H*})^{\theta\sigma+1-\sigma} \right] \\ + \left(\frac{P_0^*}{P_0} \right) J(Z^I) (\tau_R^*)^\sigma (\frac{1}{\sigma}) \left(\frac{1}{\rho} \right)^{1-\sigma} (1-s_V^*)^{1-\sigma} \left[\left(\frac{W^*}{P_0^*} \right)^{1-\sigma} (\tau_C^*)^{-\sigma} (C^{H*})^{\theta\sigma+1-\sigma} \right] \\ - \frac{W}{P_0} f^D (1 - G(Z^D)) \\ - \frac{W}{P_0} f^X (G(Z^I) - G(Z^X)) \\ - \left(\frac{P_0^*}{P_0} \right) \frac{W^*}{P_0^*} f^{I*} (1 - G(Z^I)) \end{array} \right)$$

$$\delta \frac{W^*}{P_0^*} F^{D*} = \left(\begin{array}{l} + J^* (Z^{D*}) (\tau_R^*)^\sigma \left(\frac{1}{\sigma}\right) \left(\frac{1}{\rho}\right)^{1-\sigma} \left[\left(\frac{W^*}{P_0^*}\right)^{1-\sigma} (\tau_C^*)^{-\sigma} (C^{F*})^{\theta\sigma+1-\sigma} \right] \\ + \left(\frac{P_0}{P_0^*}\right) J^* (Z^{X*}, Z^{I*}) (\tau_R^*)^\sigma \left(\frac{1}{\sigma}\right) \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau^*}\right)^{\sigma-1} \left(\frac{W^*}{W}\right)^{1-\sigma} \left[\left(\frac{W}{P_0}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right] \\ + \left(\frac{P_0}{P_0^*}\right) J^* (Z^{I*}) (\tau_R)^\sigma \left(\frac{1}{\sigma}\right) \left(\frac{1}{\rho}\right)^{1-\sigma} (1-s_V)^{1-\sigma} \left[\left(\frac{W}{P_0}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right] \\ - \frac{W^*}{P_0^*} f^{D*} (1-G^*(Z^{D*})) \\ - \frac{W^*}{P_0^*} f^{X*} (G^*(Z^{I*}) - G^*(Z^{X*})) \\ - \left(\frac{P_0}{P_0^*}\right) \frac{W}{P_0} f^I (1-G^*(Z^{I*})) \end{array} \right)$$

Mass of Firms:

$$\begin{aligned} M^E &= \frac{\delta M}{1-G(Z^D)} & M^{E*} &= \frac{\delta M^*}{1-G^*(Z^{D*})} \\ M^L &= \left(\frac{G(Z^X)-G(Z^D)}{1-G(Z^D)}\right) M = \left(\frac{(Z^D)^{-\eta}-(Z^X)^{-\eta}}{(Z^D)^{-\eta}}\right) M & M^{L*} &= \left(\frac{G^*(Z^{X*})-G^*(Z^{D*})}{1-G^*(Z^{D*})}\right) M^* = \left(\frac{(Z^{D*})^{-\eta^*}-(Z^{X*})^{-\eta^*}}{(Z^{D*})^{-\eta^*}}\right) M^* \\ M^X &= \left(\frac{G(Z^I)-G(Z^X)}{1-G(Z^D)}\right) M = \left(\frac{(Z^X)^{-\eta}-(Z^I)^{-\eta}}{(Z^D)^{-\eta}}\right) M & M^{X*} &= \left(\frac{G^*(Z^{I*})-G^*(Z^{X*})}{1-G^*(Z^{D*})}\right) M^* = \left(\frac{(Z^{X*})^{-\eta^*}-(Z^{I*})^{-\eta^*}}{(Z^{D*})^{-\eta^*}}\right) M^* \\ M^I &= \left(\frac{1-G(Z^I)}{1-G(Z^D)}\right) M = \left(\frac{Z^I}{Z^D}\right)^{-\eta} M & M^{I*} &= \left(\frac{1-G^*(Z^{I*})}{1-G^*(Z^{D*})}\right) M^* = \left(\frac{Z^{I*}}{Z^{D*}}\right)^{-\eta^*} M^* \end{aligned}$$

Home Government Budget Balance (Equation 15):

$$\begin{aligned} & \left((1-\tau_L) \frac{W}{P_0} L + M(\tau_C-1) \frac{J(Z^D)}{1-G(Z^D)} \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} \left(\frac{W}{P_0}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^H)^{\theta\sigma+1-\sigma} \right. \\ & + M^{X*}(\tau_C-1) \frac{J^*(Z^{X*}, Z^{I*})}{G^*(Z^{I*})-G^*(Z^{X*})} \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R^*}\right)^{1-\sigma} \left(\frac{1}{\tau^*}\right)^{\sigma-1} \left(\frac{W^*}{P_0}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \\ & + M^{I*}(\tau_C-1) \frac{J^*(Z^{I*})}{1-G^*(Z^{I*})} \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} (1-s_V)^{1-\sigma} \left(\frac{W}{P_0}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \\ & + M(1-\tau_R) \frac{J(Z^D)}{1-G(Z^D)} \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} \left(\frac{W}{P_0}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^H)^{\theta\sigma+1-\sigma} \\ & + M(1-\tau_R) \frac{J(Z^X, Z^I)}{1-G(Z^D)} \left(\frac{1}{\tau}\right)^{\sigma-1} \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} \left(\frac{W}{P_0^*}\right)^{1-\sigma} (\tau_C^*)^{-\sigma} (C^{H*})^{\theta\sigma+1-\sigma} \\ & \left. + M^*(1-\tau_R) \left(\frac{J^*(Z^{I*})}{1-G^*(Z^{D*})}\right) \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} (1-s_V)^{1-\sigma} \left(\frac{W}{P_0}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right) \\ & = \left(+ M^{I*} \frac{J^*(Z^{I*})}{1-G^*(Z^{I*})} s_V \left(\frac{1}{\rho}\right)^{-\sigma} \left(\frac{1}{\tau_R}\right)^{-\sigma} (1-s_V)^{-\sigma} \left(\frac{W}{P_0}\right)^{-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right) \end{aligned}$$

Nontradable Homogeneous Goods Market Clearing and Household Budget Constraints:

The following two conditions pin down labor used in the homogeneous sector, L_0 and L_0^* .

$$\begin{aligned} L_0 = C_0 &= \tau_L \frac{W}{P_0} L - (C^H)^\theta - (C^F)^\theta \\ L_0^* = C_0^* &= \tau_L^* \frac{W^*}{P_0^*} L^* - (C^{F*})^\theta - (C^{H*})^\theta \end{aligned}$$

Labor Market Clearing Conditions: By Walras' law, we need only one market clearing condition from the two labor markets in the equilibrium system, and it pins down the world relative wage, $\frac{W^*}{W} = \frac{P_0^*}{P_0}$, which can be interpreted as the real exchange rate in our model economy.

$$\begin{aligned} L - L_0 &= \left(\begin{array}{l} M^E F^D \\ + M^L \int_{Z^D}^{Z^X} (l^D(z) + f^D) \frac{dG(z)}{G(Z^X) - G(Z^D)} \\ + M^X \int_{Z^X}^{Z^I} (l^{D,X}(z) + f^D) \frac{dG(z)}{G(Z^I) - G(Z^X)} \\ + M^I \int_{Z^I}^{\infty} (l^{D,I}(z) + f^D) \frac{dG(z)}{1 - G(Z^I)} \\ + M^X \int_{Z^X}^{Z^I} (l^X(z) + f^X) \frac{dG(z)}{G(Z^I) - G(Z^X)} \\ + M^{I*} \int_{Z^{I*}}^{\infty} (l^I(z) + f^I) \frac{dG^*(z)}{1 - G^*(Z^{I*})} \end{array} \right) \\ L^* - L_0^* &= \left(\begin{array}{l} M^{E*} F^{D*} \\ + M^{L*} \int_{Z^{D*}}^{Z^{X*}} (l^{D*}(z) + f^{D*}) \frac{dG^*(z)}{G^*(Z^{X*}) - G^*(Z^{D*})} \\ + M^{X*} \int_{Z^{X*}}^{Z^{I*}} (l^{D,X*}(z) + f^{D*}) \frac{dG^*(z)}{G^*(Z^{I*}) - G^*(Z^{X*})} \\ + M^{I*} \int_{Z^{I*}}^{\infty} (l^{D,I*}(z) + f^{D*}) \frac{dG^*(z)}{1 - G^*(Z^{I*})} \\ + M^{X*} \int_{Z^{X*}}^{Z^{I*}} (l^{X*}(z) + f^{X*}) \frac{dG^*(z)}{G^*(Z^{I*}) - G^*(Z^{X*})} \\ + M^I \int_{Z^I}^{\infty} (l^{I*}(z) + f^{I*}) \frac{dG(z)}{1 - G(Z^I)} \end{array} \right) \end{aligned}$$

Since we conduct the partial-equilibrium analysis, we exclude the labor market clearing condition. The relative wage is assumed to be unity: $\frac{W^*}{W} = \frac{P_0^*}{P_0} = 1$.

2 Characterization of the Equilibrium

This section presents derivations for the analytical solutions for 15 variables: C^H , C^{H*} , C^{F*} , C^F , V , V^* , Z^D , Z^X , Z^I , Z^{D*} , Z^{X*} , Z^{I*} , M , M^* and one tax variable from $\{\tau_L, \tau_C, \tau_R\}$. We take the partial equilibrium analysis: $\frac{W^*}{W} = \frac{P_0^*}{P_0} = 1$ with $\frac{W}{P_0} = \frac{W^*}{P_0^*} = 1$. There is no government in Foreign: $s_V^* = 0$ and $\tau_L^* = \tau_C^* = \tau_R^* = 1$. Given a subsidy rate, the government tax is found numerically from the balanced government budget. Parameters are restricted by

$$\sigma > 1, \quad \tau > 1, \quad \tau^* > 1, \quad \eta > \sigma - 1, \quad \eta^* > \sigma - 1.$$

2.1 Find the equilibrium under exogenous firm mass

We first take M and M^* as given and solve for 13 variables in terms of firm masses.

Find C^H , A^H , Z^D : C^H can be solved out by

$$\begin{aligned} & (C^H)^{(\sigma-1)(1-\theta)} \\ &= M \left(\frac{1}{\rho} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)} (\tilde{Z}^D)^{\sigma-1} \\ &= M \left(\frac{1}{\rho} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)} \frac{\eta}{\eta-\sigma+1} (Z^D)^{\sigma-1} \\ &= M \left(\frac{1}{\rho} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)} \frac{\eta}{\eta-\sigma+1} \left(\frac{f^D}{(\tau_R)^\sigma \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} A^H} \right) \\ &= M \left(\frac{1}{\rho} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)} \frac{\eta}{\eta-\sigma+1} \left(\frac{f^D}{(\tau_R)^\sigma \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} (\tau_C)^{-\sigma} (C^H)^{\theta\sigma+1-\sigma}} \right) \\ &= M \left(\sigma \frac{\tau_C}{\tau_R} \right) \left(\frac{\eta}{\eta-\sigma+1} \right) \left(\frac{f^D}{(C^H)^{\theta\sigma+1-\sigma}} \right) \\ \\ &\therefore (C^H)^{\theta\sigma+1-\sigma+(\sigma-1)(1-\theta)} = (C^H)^\theta = M \left(\sigma \frac{\tau_C}{\tau_R} \right) \left(\frac{\eta}{\eta-\sigma+1} \right) f^D \\ &\text{i.e. } C^H = \left(M \left(\sigma \frac{\tau_C}{\tau_R} \right) \left(\frac{\eta}{\eta-\sigma+1} \right) f^D \right)^{\frac{1}{\theta}} \end{aligned}$$

where $\sigma \equiv \frac{1}{1-\rho}$, $1-\sigma = \frac{-\rho}{1-\rho}$, and $\rho = \frac{\sigma-1}{\sigma}$ with $\sigma > 1$ and $0 < \rho < 1$.

Hence, we obtain:

$$\begin{aligned} A^H &= (\tau_C)^{-\sigma} (C^H)^{\theta\sigma+1-\sigma} \\ Z^D &= \left(\frac{f^D}{(\tau_R)^\sigma \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} A^H} \right)^{\frac{1}{\sigma-1}} \end{aligned}$$

Find C^{F*} , A^{F*} , Z^{D*} : C^{F*} can be solved out by

$$\begin{aligned}
& (C^{F*})^{(\sigma-1)(1-\theta)} \\
&= M^* \left(\frac{1}{\rho} \right)^{-(\sigma-1)} \left(\frac{\tau_C^*}{\tau_R^*} \right)^{-(\sigma-1)} \left(\tilde{Z}^{D*} \right)^{\sigma-1} \\
&= M^* \left(\frac{1}{\rho} \right)^{-(\sigma-1)} \left(\frac{\tau_C^*}{\tau_R^*} \right)^{-(\sigma-1)} \frac{\eta^*}{\eta^* - \sigma + 1} (Z^{D*})^{\sigma-1} \\
&= M^* \left(\frac{1}{\rho} \right)^{-(\sigma-1)} \left(\frac{\tau_C^*}{\tau_R^*} \right)^{-(\sigma-1)} \frac{\eta^*}{\eta^* - \sigma + 1} \left(\frac{f^{D*}}{(\tau_R^*)^\sigma (\frac{1}{\sigma})(\rho)^{\sigma-1} A^{F*}} \right) \\
&= M^* \left(\frac{1}{\rho} \right)^{-(\sigma-1)} \left(\frac{\tau_C^*}{\tau_R^*} \right)^{-(\sigma-1)} \frac{\eta^*}{\eta^* - \sigma + 1} \left(\frac{f^{D*}}{(\tau_R^*)^\sigma (\frac{1}{\sigma})(\rho)^{\sigma-1} (\tau_C^*)^{-\sigma} (C^{F*})^{\theta\sigma+1-\sigma}} \right) \\
&= M^* \left(\sigma \frac{\tau_C^*}{\tau_R^*} \right) \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \left(\frac{f^{D*}}{(C^{F*})^{\theta\sigma+1-\sigma}} \right) \\
\therefore \quad & (C^{F*})^{\theta\sigma+1-\sigma+(\sigma-1)(1-\theta)} = (C^{F*})^\theta = M^* \left(\sigma \frac{\tau_C^*}{\tau_R^*} \right) \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) f^{D*} \\
i.e. \quad & C^{F*} = \left(M^* \left(\sigma \frac{\tau_C^*}{\tau_R^*} \right) \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) f^{D*} \right)^{\frac{1}{\theta}}
\end{aligned}$$

where $\sigma \equiv \frac{1}{1-\rho}$, $1 - \sigma = \frac{-\rho}{1-\rho}$, and $\rho = \frac{\sigma-1}{\sigma}$ with $\sigma > 1$ and $0 < \rho < 1$.

Hence, we obtain:

$$\begin{aligned}
A^{F*} &= (\tau_C^*)^{-\sigma} (C^{F*})^{\theta\sigma+1-\sigma} \\
Z^{D*} &= \left(\frac{f^{D*}}{(\tau_R^*)^\sigma (\frac{1}{\sigma})(\rho)^{\sigma-1} A^{F*}} \right)^{\frac{1}{\sigma-1}}
\end{aligned}$$

Find C^F , A^F , Z^{X*} , Z^{I*} : C^F can be solved out by

$$\begin{aligned}
(C^F)^{(\sigma-1)(1-\theta)} &= \left(\begin{array}{l} + M^{X*} \left(\frac{\tau^*}{\rho} \frac{\tau_C}{\tau_R^*} \right)^{-(\sigma-1)} \left(\tilde{Z}^{X*} \right)^{\sigma-1} \\ + M^{I*} \left(\frac{1-s_V}{\rho} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)} \left(\tilde{Z}^{I*} \right)^{\sigma-1} \end{array} \right) \\
&= \left(\begin{array}{l} + M^* \left(\frac{(Z^{X*})^{-\eta^*} - (Z^{I*})^{-\eta^*}}{(Z^{D*})^{-\eta^*}} \right) \left(\frac{\tau^*}{\rho} \frac{\tau_C}{\tau_R^*} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \left(\frac{(Z^{X*})^{-(\eta^* - \sigma + 1)} - (Z^{I*})^{-(\eta^* - \sigma + 1)}}{(Z^{X*})^{-\eta^*} - (Z^{I*})^{-\eta^*}} \right) \\ + M^* \left(\frac{Z^{I*}}{Z^{D*}} \right)^{-\eta^*} \left(\frac{1-s_V}{\rho} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) (Z^{I*})^{\sigma-1} \end{array} \right) \\
&= \left(\begin{array}{l} + M^* \left(\frac{\tau^*}{\rho} \frac{\tau_C}{\tau_R^*} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \left(\frac{(Z^{X*})^{-(\eta^* - \sigma + 1)} - (Z^{I*})^{-(\eta^* - \sigma + 1)}}{(Z^{D*})^{-\eta^*}} \right) \\ + M^* \left(\frac{1-s_V}{\rho} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \left(\frac{(Z^{I*})^{-\eta^* + \sigma - 1}}{(Z^{D*})^{-\eta^*}} \right) \end{array} \right) \\
&= \left(\begin{array}{l} + M^* \left(\frac{\tau^*}{\rho} \frac{\tau_C}{\tau_R^*} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \\ \left(\frac{\left(\frac{f^{X*}}{(\tau_R^*)^\sigma (\frac{1}{\tau^*})^{\sigma-1} (\frac{1}{\sigma}) (\rho)^{\sigma-1} A^F} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}} - \left(\frac{f^I - f^{X*}}{\left[(\tau_R)^\sigma \left(\frac{1}{1-s_V} \right)^{\sigma-1} A^F - (\tau_R^*)^\sigma (\frac{1}{\tau^*})^{\sigma-1} A^F \right] (\frac{1}{\sigma}) (\rho)^{\sigma-1}} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}}}{(Z^{D*})^{-\eta^*}} \right) \\ + M^* \left(\frac{1-s_V}{\rho} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \left(\frac{\left(\frac{f^I - f^{X*}}{\left[(\tau_R)^\sigma \left(\frac{1}{1-s_V} \right)^{\sigma-1} A^F - (\tau_R^*)^\sigma (\frac{1}{\tau^*})^{\sigma-1} A^F \right] (\frac{1}{\sigma}) (\rho)^{\sigma-1}} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}}}{(Z^{D*})^{-\eta^*}} \right) \end{array} \right) \\
&\therefore (C^F)^{\frac{\theta(\sigma-1) - (\theta\sigma+1-\sigma)\eta^*}{\sigma-1}} = \\
&\left(\begin{array}{l} + M^* \left(\frac{\tau^*}{\rho} \frac{\tau_C}{\tau_R^*} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \\ \left(\frac{\left(\frac{f^{X*}}{(\frac{1}{\tau^*})^{\sigma-1} (\frac{1}{\sigma}) (\rho)^{\sigma-1} \left(\frac{\tau_C}{\tau_R^*} \right)^{-\sigma}} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}} - \left(\frac{f^I - f^{X*}}{\left[\left(\frac{1}{1-s_V} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R} \right)^{-\sigma} - \left(\frac{1}{\tau^*} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R^*} \right)^{-\sigma} \right] (\frac{1}{\sigma}) (\rho)^{\sigma-1}} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}}}{(Z^{D*})^{-\eta^*}} \right) \\ + M^* \left(\frac{1-s_V}{\rho} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \left(\frac{\left(\frac{f^I - f^{X*}}{\left[\left(\frac{1}{1-s_V} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R} \right)^{-\sigma} - \left(\frac{1}{\tau^*} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R^*} \right)^{-\sigma} \right] (\frac{1}{\sigma}) (\rho)^{\sigma-1}} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}}}{(Z^{D*})^{-\eta^*}} \right) \end{array} \right).
\end{aligned}$$

Hence we obtain

$$\begin{aligned}
A^F &= (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma}, \\
Z^{X*} &= \left(\frac{f^{X*}}{(\tau_R^*)^\sigma (\frac{1}{\tau^*})^{\sigma-1} (\frac{1}{\sigma}) (\rho)^{\sigma-1} A^F} \right)^{\frac{1}{\sigma-1}}, \quad Z^{I*} = \left(\frac{f^I - f^{X*}}{\left[(\tau_R)^\sigma \left(\frac{1}{1-s_V} \right)^{\sigma-1} A^F - (\tau_R^*)^\sigma (\frac{1}{\tau^*})^{\sigma-1} A^F \right] (\frac{1}{\sigma}) (\rho)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}}.
\end{aligned}$$

Find C^{H*} , A^{H*} , Z^X , Z^I :

$$\begin{aligned}
(C^{H*})^{(\sigma-1)(1-\theta)} &= \left(\begin{array}{l} + M^X \left(\frac{\tau}{\rho} \frac{\tau_C^*}{\tau_R} \right)^{-(\sigma-1)} \left(\tilde{Z}^X \right)^{\sigma-1} \\ + M^I \left(\frac{1-s_V^*}{\rho} \frac{\tau_C^*}{\tau_R^*} \right)^{-(\sigma-1)} \left(\tilde{Z}^I \right)^{\sigma-1} \end{array} \right) \\
&= \left(\begin{array}{l} + M \left(\frac{(Z^X)^{-\eta} - (Z^I)^{-\eta}}{(Z^D)^{-\eta}} \right) \left(\frac{\tau}{\rho} \frac{\tau_C^*}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) \left(\frac{(Z^X)^{-(\eta-\sigma+1)} - (Z^I)^{-(\eta-\sigma+1)}}{(Z^X)^{-\eta} - (Z^I)^{-\eta}} \right) \\ + M \left(\frac{Z^I}{Z^D} \right)^{-\eta} \left(\frac{1-s_V^*}{\rho} \frac{\tau_C^*}{\tau_R^*} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) (Z^I)^{\sigma-1} \end{array} \right) \\
&= \left(\begin{array}{l} + M \left(\frac{\tau}{\rho} \frac{\tau_C^*}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) \left(\frac{(Z^X)^{-(\eta-\sigma+1)} - (Z^I)^{-(\eta-\sigma+1)}}{(Z^D)^{-\eta}} \right) \\ + M \left(\frac{1-s_V^*}{\rho} \frac{\tau_C^*}{\tau_R^*} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) \frac{(Z^I)^{-\eta+\sigma-1}}{(Z^D)^{-\eta}} \end{array} \right) \\
&\quad \left(\begin{array}{l} + M \left(\frac{\tau}{\rho} \frac{\tau_C^*}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) \\ \left(\frac{\left(\frac{f^X}{(\tau_R)^{\sigma} \left(\frac{1}{\tau} \right)^{\sigma-1} \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} A^{H*}} \right)^{\frac{-\eta+\sigma-1}{\sigma-1}} - \left(\frac{f^{I*} - f^X}{\left[\left(\frac{\tau_R^*}{1-s_V^*} \right)^{\sigma-1} A^{H*} - (\tau_R)^{\sigma} \left(\frac{1}{\tau} \right)^{\sigma-1} A^{H*} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{-(\eta-\sigma+1)}{\sigma-1}} \end{array} \right) \\
&= \left(\begin{array}{l} + M \left(\frac{1-s_V^*}{\rho} \frac{\tau_C^*}{\tau_R^*} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) \frac{\left(\frac{f^{I*} - f^X}{\left[\left(\frac{\tau_R^*}{1-s_V^*} \right)^{\sigma-1} A^{H*} - (\tau_R)^{\sigma} \left(\frac{1}{\tau} \right)^{\sigma-1} A^{H*} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{-(\eta-\sigma+1)}{\sigma-1}} }{(Z^D)^{-\eta}} \end{array} \right) \\
&\therefore (C^{H*})^{\frac{\theta(\sigma-1) - (\theta\sigma+1-\sigma)\eta}{\sigma-1}} = \left(\begin{array}{l} + M \left(\frac{\tau}{\rho} \frac{\tau_C^*}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) \\ \left(\frac{\left(\frac{f^X}{\left(\frac{1}{\tau} \right)^{\sigma-1} \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma}} \right)^{\frac{-(\eta-\sigma+1)}{\sigma-1}} - \left(\frac{f^{I*} - f^X}{\left[\left(\frac{1}{1-s_V^*} \right)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R^*} \right)^{-\sigma} - \left(\frac{1}{\tau} \right)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{-(\eta-\sigma+1)}{\sigma-1}} }{(Z^D)^{-\eta}} \end{array} \right) \\
&\quad \left(\begin{array}{l} + M \left(\frac{1-s_V^*}{\rho} \frac{\tau_C^*}{\tau_R^*} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) \frac{\left(\frac{f^{I*} - f^X}{\left[\left(\frac{1}{1-s_V^*} \right)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma} - \left(\frac{1}{\tau} \right)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{-(\eta-\sigma+1)}{\sigma-1}} }{(Z^D)^{-\eta}} \end{array} \right)
\end{aligned}$$

In turn, we obtain

$$A^{H*} = (\tau_C^*)^{-\sigma} (C^{H*})^{\theta\sigma+1-\sigma},$$

$$Z^X = \left(\frac{f^X}{(\tau_R)^\sigma (\frac{1}{\tau})^{\sigma-1} (\frac{1}{\sigma})(\rho)^{\sigma-1} A^{H*}} \right)^{\frac{1}{\sigma-1}}, \quad Z^I = \left(\frac{f^{I*}-f^X}{\left[(\tau_R^*)^\sigma \left(\frac{1}{1-s_V^*} \right)^{\sigma-1} A^{H*} - (\tau_R)^\sigma (\frac{1}{\tau})^{\sigma-1} A^{H*} \right] (\frac{1}{\sigma})(\rho)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}}.$$

Therefore, given M and M^* , and given solutions for C^H , C^{H*} , C^{F*} , C^F , Z^D , Z^X , Z^I , Z^{D*} , Z^{X*} , and Z^{I*} , we can find all the other endogenous variables: consumption of homogeneous goods, masses of entrants, locals, exporters, and multinationals, taxes, and welfare.

Mass of Firms:

$$M^E = \frac{\delta M}{1-G(Z^D)}, \quad M^{E*} = \frac{\delta M^*}{1-G^*(Z^{D*})}$$

$$M^L = \left(\frac{G(Z^X)-G(Z^D)}{1-G(Z^D)} \right) M = \left(\frac{(Z^D)^{-\eta}-(Z^X)^{-\eta}}{(Z^D)^{-\eta}} \right) M, \quad M^{L*} = \left(\frac{G^*(Z^{X*})-G^*(Z^{D*})}{1-G^*(Z^{D*})} \right) M^* = \left(\frac{(Z^{D*})^{-\eta^*}-(Z^{X*})^{-\eta^*}}{(Z^{D*})^{-\eta^*}} \right) M^*$$

$$M^X = \left(\frac{G(Z^I)-G(Z^X)}{1-G(Z^D)} \right) M = \left(\frac{(Z^X)^{-\eta}-(Z^I)^{-\eta}}{(Z^D)^{-\eta}} \right) M, \quad M^{X*} = \left(\frac{G^*(Z^{I*})-G^*(Z^{X*})}{1-G^*(Z^{D*})} \right) M^* = \left(\frac{(Z^{X*})^{-\eta^*}-(Z^{I*})^{-\eta^*}}{(Z^{D*})^{-\eta^*}} \right) M^*$$

$$M^I = \left(\frac{1-G(Z^I)}{1-G(Z^D)} \right) M = \left(\frac{Z^I}{Z^D} \right)^{-\eta} M, \quad M^{I*} = \left(\frac{1-G^*(Z^{I*})}{1-G^*(Z^{D*})} \right) M^* = \left(\frac{Z^{I*}}{Z^{D*}} \right)^{-\eta^*} M^*$$

Indirect Utility:

$$V = L\tau_L + \left(\frac{1}{\theta} - 1 \right) (C^H)^\theta + \left(\frac{1}{\theta} - 1 \right) (C^F)^\theta$$

$$V^* = L^*\tau_L^* + \left(\frac{1}{\theta} - 1 \right) (C^{F*})^\theta + \left(\frac{1}{\theta} - 1 \right) (C^{H*})^\theta$$

Consumption on Nontradable Homogeneous Goods:

$$L_0 = C_0 = \tau_L L - (C^H)^\theta - (C^F)^\theta$$

$$L_0^* = C_0^* = \tau_L^* L^* - (C^{F*})^\theta - (C^{H*})^\theta$$

Home Government Budget Balance: Tax variables are numerically pinned down by the government budget balance given a FDI subsidy rate. The government budget balance is given by

$$\begin{aligned}
& \left((1 - \tau_L)L + M(\tau_C - 1) \frac{J(Z^D)}{1-G(Z^D)} \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^H)^{\theta\sigma+1-\sigma} \right. \\
& + M^{X*}(\tau_C - 1) \frac{J^*(Z^{X*}, Z^{I*})}{G^*(Z^{I*}) - G^*(Z^{X*})} \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R^*}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \\
& + M^{I*}(\tau_C - 1) \frac{J^*(Z^{I*})}{1-G^*(Z^{I*})} \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} (1 - s_V)^{1-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \\
& + M(1 - \tau_R) \frac{J(Z^D)}{1-G(Z^D)} \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^H)^{\theta\sigma+1-\sigma} \\
& + M(1 - \tau_R) \frac{J(Z^X, Z^I)}{1-G(Z^D)} \left(\frac{1}{\tau}\right)^{\sigma-1} \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} (\tau_C^*)^{-\sigma} (C^{H*})^{\theta\sigma+1-\sigma} \\
& \left. + M^*(1 - \tau_R) \left(\frac{J^*(Z^{I*})}{1-G^*(Z^{D*})}\right) \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} (1 - s_V)^{1-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right) \\
= & \left(+ M^{I*} \frac{J^*(Z^{I*})}{1-G^*(Z^{I*})} s_V \left(\frac{1}{\rho}\right)^{-\sigma} \left(\frac{1}{\tau_R}\right)^{-\sigma} (1 - s_V)^{-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right)
\end{aligned}$$

Lump-Sum Tax: If the government imposes tax on labor income, then we can pin down τ_L algebraically.

$$L(1 - \tau_L) = \left(+ M^{I*} \frac{J^*(Z^{I*})}{1-G^*(Z^{I*})} s_V \left(\frac{1}{\rho}\right)^{-\sigma} \left(\frac{1}{\tau_R}\right)^{-\sigma} (1 - s_V)^{-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right)$$

Consumption Tax: Suppose the government imposes consumption tax. Then τ_C can be found numerically by solving

$$\begin{aligned}
& \left(+ M(\tau_C - 1) \frac{J(Z^D)}{1-G(Z^D)} \left(\frac{1}{\rho} \frac{1}{\tau_R}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^H)^{\theta\sigma+1-\sigma} \right. \\
& + M^{X*}(\tau_C - 1) \frac{J^*(Z^{X*}, Z^{I*})}{G^*(Z^{I*}) - G^*(Z^{X*})} \left(\frac{\tau^*}{\rho} \frac{1}{\tau_R^*}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \\
& \left. + M^{I*}(\tau_C - 1) \frac{J^*(Z^{I*})}{1-G^*(Z^{I*})} \left(\frac{1-s_V}{\rho} \frac{1}{\tau_R}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right) \\
= & \left(+ M^{I*} \frac{J^*(Z^{I*})}{1-G^*(Z^{I*})} s_V \left(\frac{1}{\rho}\right)^{-\sigma} \left(\frac{1}{\tau_R}\right)^{-\sigma} (1 - s_V)^{-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right)
\end{aligned}$$

Tax on Firm Revenue: Suppose the government imposes firm-revenue tax. Then τ_R can be found numerically by solving

$$\begin{aligned}
& \left(+ M(1 - \tau_R) \frac{J(Z^D)}{1-G(Z^D)} \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} (\tau_C)^{-\sigma} (C^H)^{\theta\sigma+1-\sigma} \right. \\
& + M(1 - \tau_R) \frac{J(Z^X, Z^I)}{1-G(Z^D)} \left(\frac{1}{\tau}\right)^{\sigma-1} \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} (\tau_C^*)^{-\sigma} (C^{H*})^{\theta\sigma+1-\sigma} \\
& \left. + M^*(1 - \tau_R) \left(\frac{J^*(Z^{I*})}{1-G^*(Z^{D*})}\right) \left(\frac{1}{\rho}\right)^{1-\sigma} \left(\frac{1}{\tau_R}\right)^{1-\sigma} (1 - s_V)^{1-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right) \\
= & \left(+ M^{I*} \frac{J^*(Z^{I*})}{1-G^*(Z^{I*})} s_V \left(\frac{1}{\rho}\right)^{-\sigma} \left(\frac{1}{\tau_R}\right)^{-\sigma} (1 - s_V)^{-\sigma} (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right)
\end{aligned}$$

2.2 Find the equilibrium under endogenous firm mass

Now we endogenize masses of firms and close the equilibrium.

Free Entries: When we endogenize total firm masses, M and M^* , they can be found from free entry conditions, given by

$$\delta F^D = \begin{pmatrix} + J(Z^D) (\tau_R)^\sigma \left(\frac{1}{\sigma}\right) (\rho)^{\sigma-1} \left[(\tau_C)^{-\sigma} (C^H)^{\theta\sigma+1-\sigma} \right] \\ + J(Z^X, Z^I) (\tau_R)^\sigma \left(\frac{1}{\sigma}\right) \left(\frac{\rho}{\tau}\right)^{\sigma-1} \left[(\tau_C^*)^{-\sigma} (C^{H*})^{\theta\sigma+1-\sigma} \right] \\ + J(Z^I) (\tau_R^*)^\sigma \left(\frac{1}{\sigma}\right) \left(\frac{\rho}{(1-s_V)}\right)^{\sigma-1} \left[(\tau_C^*)^{-\sigma} (C^{H*})^{\theta\sigma+1-\sigma} \right] \\ - f^D (1 - G(Z^D)) \\ - f^X (G(Z^I) - G(Z^X)) \\ - f^{I*} (1 - G(Z^I)) \end{pmatrix}$$

where

$$\begin{aligned} J(Z^D) &= \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} (Z^D)^{-\eta+\sigma-1}, \\ J(Z^I) &= \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} (Z^I)^{-\eta+\sigma-1}, \quad J(Z^X, Z^I) = \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} \left[(Z^X)^{-(\eta-\sigma+1)} - (Z^I)^{-(\eta-\sigma+1)} \right], \\ (1 - G(Z^D)) &= (z_{min})^\eta (Z^D)^{-\eta}, \\ (1 - G(Z^I)) &= (z_{min})^\eta (Z^I)^{-\eta}, \quad (G(Z^I) - G(Z^X)) = (z_{min})^\eta \left[(Z^X)^{-\eta} - (Z^I)^{-\eta} \right]. \end{aligned}$$

$$\delta F^{D*} = \begin{pmatrix} + J^* (Z^{D*}) (\tau_R^*)^\sigma \left(\frac{1}{\sigma}\right) (\rho)^{\sigma-1} \left[(\tau_C^*)^{-\sigma} (C^{F*})^{\theta\sigma+1-\sigma} \right] \\ + J^* (Z^{X*}, Z^{I*}) (\tau_R^*)^\sigma \left(\frac{1}{\sigma}\right) \left(\frac{\rho}{\tau^*}\right)^{\sigma-1} \left[(\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right] \\ + J^* (Z^{I*}) (\tau_R)^\sigma \left(\frac{1}{\sigma}\right) \left(\frac{\rho}{(1-s_V)}\right)^{\sigma-1} \left[(\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} \right] \\ - f^{D*} (1 - G^*(Z^{D*})) \\ - f^{X*} (G^*(Z^{I*}) - G^*(Z^{X*})) \\ - f^I (1 - G^*(Z^{I*})) \end{pmatrix}$$

where

$$\begin{aligned}
J^*(Z^{D*}) &= \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^* - \sigma + 1} (Z^{D*})^{-\eta^* + \sigma - 1}, \\
J^*(Z^{I*}) &= \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^* - \sigma + 1} (Z^{I*})^{-\eta^* + \sigma - 1}, \quad J^*(Z^{X*}, Z^{I*}) = \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^* - \sigma + 1} \left[(Z^{X*})^{-(\eta^* - \sigma + 1)} - (Z^{I*})^{-(\eta^* - \sigma + 1)} \right], \\
(1 - G^*(Z^{D*})) &= (z_{min}^*)^{\eta^*} (Z^{D*})^{-\eta^*}, \\
(1 - G^*(Z^{I*})) &= (z_{min}^*)^{\eta^*} (Z^{I*})^{-\eta^*}, \quad (G^*(Z^{I*}) - G^*(Z^{X*})) = (z_{min}^*)^{\eta^*} \left[(Z^{X*})^{-\eta^*} - (Z^{I*})^{-\eta^*} \right].
\end{aligned}$$

For Home consumption, demand, cutoff in Home, we can obtain

$$\begin{aligned}
C^H &= \left(\left(\sigma \frac{\tau_C}{\tau_R} \right) \left(\frac{\eta}{\eta - \sigma + 1} \right) f^D \right)^{\frac{1}{\theta}} (M)^{\frac{1}{\theta}} \\
A^H &= (\tau_C)^{-\sigma} \left(\left(\sigma \frac{\tau_C}{\tau_R} \right) \left(\frac{\eta}{\eta - \sigma + 1} \right) f^D \right)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} (M)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
&= \Xi_{A^H} (M)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
Z^D &= \left(\frac{f^D}{(\tau_R)^\sigma \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} (A^H)^{\frac{-1}{\sigma-1}} \\
&= \left(\frac{f^D}{(\tau_R)^\sigma \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} (\Xi_{A^H})^{\frac{-1}{\sigma-1}} (M)^{\frac{-(\theta\sigma+1-\sigma)}{\theta(\sigma-1)}} \\
&= \Xi_{Z^D} (M)^{\frac{-(\theta\sigma+1-\sigma)}{\theta(\sigma-1)}}
\end{aligned}$$

where $\theta\sigma + 1 - \sigma < 0$ and $\sigma > 1$.

For Foreign consumption, demand, cutoff in Foreign, we can obtain

$$\begin{aligned}
C^{F*} &= \left(\left(\sigma \frac{\tau_C^*}{\tau_R^*} \right) \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) f^{D*} \right)^{\frac{1}{\theta}} (M^*)^{\frac{1}{\theta}} \\
A^{F*} &= (\tau_C^*)^{-\sigma} \left(\left(\sigma \frac{\tau_C^*}{\tau_R^*} \right) \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) f^{D*} \right)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} (M^*)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
&= \Xi_{A^{F*}} (M^*)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
Z^{D*} &= \left(\frac{f^{D*}}{(\tau_R^*)^\sigma \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} (A^{F*})^{\frac{-1}{\sigma-1}} \\
&= \left(\frac{f^{D*}}{(\tau_R^*)^\sigma \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} (\Xi_{A^{F*}})^{\frac{-1}{\sigma-1}} (M^*)^{\frac{-(\theta\sigma+1-\sigma)}{\theta(\sigma-1)}} \\
&= \Xi_{Z^{D*}} (M^*)^{\frac{-(\theta\sigma+1-\sigma)}{\theta(\sigma-1)}}
\end{aligned}$$

where $\theta\sigma + 1 - \sigma < 0$ and $\sigma > 1$.

For Foreign consumption, demand, cutoff in Home, we can obtain

$$\begin{aligned}
& (C^F)^{\frac{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta^*}{\sigma-1}} \\
&= \left(+ M^* \left(\frac{\tau^*}{\rho} \frac{\tau_C}{\tau_R^*} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \right. \\
&\quad \left(\left(\frac{f^{X*}}{\left(\frac{1}{\tau^*} \right)^{\sigma-1} \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \left(\frac{\tau_C}{\tau_R^*} \right)^{-\sigma}} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}} - \left(\left[\left(\frac{1}{1-s_V} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R} \right)^{-\sigma} - \left(\frac{1}{\tau^*} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R^*} \right)^{-\sigma} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}} \right) \\
&\quad \left. \left(+ M^* \left(\frac{1-s_V}{\rho} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \frac{\left(\left[\left(\frac{1}{1-s_V} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R} \right)^{-\sigma} - \left(\frac{1}{\tau^*} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R^*} \right)^{-\sigma} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}} }{(Z^{D*})^{-\eta^*}} \right. \right. \\
&\quad \left. \left. + \left(\frac{\tau^*}{\rho} \frac{\tau_C}{\tau_R^*} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \left(\left(\frac{f^{X*}}{\left(\frac{1}{\tau^*} \right)^{\sigma-1} \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \left(\frac{\tau_C}{\tau_R^*} \right)^{-\sigma}} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}} \right) \right. \right. \\
&\quad \left. \left. - \left(\frac{\tau^*}{\rho} \frac{\tau_C}{\tau_R^*} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \left(\left(\left[\left(\frac{1}{1-s_V} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R} \right)^{-\sigma} - \left(\frac{1}{\tau^*} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R^*} \right)^{-\sigma} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}} \right) \right. \right. \\
&\quad \left. \left. + \left(\frac{1-s_V}{\rho} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \left(\left(\left[\left(\frac{1}{1-s_V} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R} \right)^{-\sigma} - \left(\frac{1}{\tau^*} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R^*} \right)^{-\sigma} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}} \right) \right. \right. \\
&= M^* (Z^{D*})^{\eta^*} \left(\left(\frac{1-s_V}{\rho} \frac{\tau_C}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta^*}{\eta^* - \sigma + 1} \right) \frac{\left(\left[\left(\frac{1}{1-s_V} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R} \right)^{-\sigma} - \left(\frac{1}{\tau^*} \right)^{\sigma-1} \left(\frac{\tau_C}{\tau_R^*} \right)^{-\sigma} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \right)^{\frac{-(\eta^* - \sigma + 1)}{\sigma-1}} \right) \right. \\
&= M^* (Z^{D*})^{\eta^*} \Xi_{CF} = M^* (\Xi_{Z^{D*}})^{\eta^*} (M^*)^{\frac{-(\theta\sigma+1-\sigma)\eta^*}{\theta(\sigma-1)}} \Xi_{CF} = \Xi_{CF} (\Xi_{Z^{D*}})^{\eta^*} (M^*)^{\frac{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta^*}{\theta(\sigma-1)}} \\
&\therefore C^F = \left(\Xi_{CF} (\Xi_{Z^{D*}})^{\eta^*} (M^*)^{\frac{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta^*}{\theta(\sigma-1)}} \right)^{\frac{\sigma-1}{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta^*}} = \left[\Xi_{CF} (\Xi_{Z^{D*}})^{\eta^*} \right]^{\frac{\sigma-1}{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta^*}} (M^*)^{\frac{1}{\theta}} \\
A^F &= (\tau_C)^{-\sigma} (C^F)^{\theta\sigma+1-\sigma} = (\tau_C)^{-\sigma} \left[\Xi_{CF} (\Xi_{Z^{D*}})^{\eta^*} \right]^{\frac{(\sigma-1)(\theta\sigma+1-\sigma)}{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta^*}} (M^*)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} = \Xi_{AF} (M^*)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
Z^{X*} &= \left(\frac{f^{X*}}{\left(\tau_R^* \right)^\sigma \left(\frac{1}{\tau^*} \right)^{\sigma-1} \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} (A^F)^{\frac{-1}{\sigma-1}} \\
&= \left(\frac{f^{X*}}{\left(\tau_R^* \right)^\sigma \left(\frac{1}{\tau^*} \right)^{\sigma-1} \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} (\Xi_{AF})^{\frac{-1}{\sigma-1}} (M^*)^{\frac{-(\theta\sigma+1-\sigma)}{\theta(\sigma-1)}} = \Xi_{Z^{X*}} (M^*)^{\frac{-(\theta\sigma+1-\sigma)}{\theta(\sigma-1)}} \\
Z^{I*} &= \left(\left(\left[\left(\tau_R \right)^\sigma \left(\frac{1}{1-s_V} \right)^{\sigma-1} A^F - \left(\tau_R^* \right)^\sigma \left(\frac{1}{\tau^*} \right)^{\sigma-1} A^F \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \right. \\
&\quad \left. \left(\left[\left(\tau_R \right)^\sigma \left(\frac{1}{1-s_V} \right)^{\sigma-1} \Xi_{AF} - \left(\tau_R^* \right)^\sigma \left(\frac{1}{\tau^*} \right)^{\sigma-1} \Xi_{AF} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \right)^{\frac{1}{\sigma-1}} (M^*)^{\frac{-(\theta\sigma+1-\sigma)}{\theta(\sigma-1)}} = \Xi_{Z^{I*}} (M^*)^{\frac{-(\theta\sigma+1-\sigma)}{\theta(\sigma-1)}} \right)
\end{aligned}$$

where $\theta\sigma + 1 - \sigma < 0$ and $\sigma > 1$.

For Home consumption, demand, cutoff in Foreign, we can obtain

$$\begin{aligned}
& (C^{H*})^{\frac{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta}{\sigma-1}} \\
&= \left(+ M \left(\frac{\tau}{\rho} \frac{\tau_C^*}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) \right. \\
&\quad \left. \left(\frac{\left(\frac{f^X}{\left(\frac{1}{\tau} \right)^{\sigma-1} \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma}} \right)^{\frac{-(\eta-\sigma+1)}{\sigma-1}} - \left(\frac{f^{I*}-f^X}{\left[\left(\frac{1}{1-s_V^*} \right)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma} - \left(\frac{1}{\tau} \right)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} }{(Z^D)^{-\eta}} \right)^{\frac{-(\eta-\sigma+1)}{\sigma-1}} \right) \\
&\quad + M \left(\frac{1-s_V^*}{\rho} \frac{\tau_C^*}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) \frac{\left(\frac{f^{I*}-f^X}{\left[\left(\frac{1}{1-s_V^*} \right)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma} - \left(\frac{1}{\tau} \right)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{-(\eta-\sigma+1)}{\sigma-1}}}{(Z^D)^{-\eta}} \\
&= M (Z^D)^\eta \left(+ \left(\frac{\tau}{\rho} \frac{\tau_C^*}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) \left(\frac{f^X}{\left(\frac{1}{\tau} \right)^{\sigma-1} \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma}} \right)^{\frac{-(\eta-\sigma+1)}{\sigma-1}} \right. \\
&\quad \left. - \left(\frac{\tau}{\rho} \frac{\tau_C^*}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) \left(\frac{f^{I*}-f^X}{\left[\left(\frac{1}{1-s_V^*} \right)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma} - \left(\frac{1}{\tau} \right)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{-(\eta-\sigma+1)}{\sigma-1}} \right. \\
&\quad \left. + \left(\frac{1-s_V^*}{\rho} \frac{\tau_C^*}{\tau_R} \right)^{-(\sigma-1)} \left(\frac{\eta}{\eta-\sigma+1} \right) \left(\frac{f^{I*}-f^X}{\left[\left(\frac{1}{1-s_V^*} \right)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma} - \left(\frac{1}{\tau} \right)^{\sigma-1} \left(\frac{\tau_C^*}{\tau_R} \right)^{-\sigma} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{-(\eta-\sigma+1)}{\sigma-1}} \right) \\
&= M (Z^D)^\eta \Xi_{C^{H*}} = M (\Xi_{Z^D})^\eta (M)^{\frac{-(\theta\sigma+1-\sigma)\eta}{\theta(\sigma-1)}} \Xi_{C^{H*}} = \Xi_{C^{H*}} (\Xi_{Z^D})^\eta (M)^{\frac{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta}{\theta(\sigma-1)}}
\end{aligned}$$

$$\begin{aligned}
\therefore C^{H*} &= \left(\Xi_{C^{H*}} (\Xi_{Z^D})^\eta (M)^{\frac{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta}{\theta(\sigma-1)}} \right)^{\frac{\sigma-1}{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta}} = [\Xi_{C^{H*}} (\Xi_{Z^D})^\eta]^{\frac{\sigma-1}{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta}} (M)^{\frac{1}{\theta}} \\
A^{H*} &= (\tau_C^*)^{-\sigma} (C^{H*})^{\theta\sigma+1-\sigma} = (\tau_C^*)^{-\sigma} [\Xi_{C^{H*}} (\Xi_{Z^D})^\eta]^{\frac{(\sigma-1)(\theta\sigma+1-\sigma)}{\theta(\sigma-1)-(\theta\sigma+1-\sigma)\eta}} (M)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} = \Xi_{A^{H*}} (M)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
Z^X &= \left(\frac{f^X}{(\tau_R)^\sigma \left(\frac{1}{\tau} \right)^{\sigma-1} \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} (A^{H*})^{\frac{-1}{\sigma-1}} \\
&= \left(\frac{f^X}{(\tau_R)^\sigma \left(\frac{1}{\tau} \right)^{\sigma-1} \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} (\Xi_{A^{H*}})^{\frac{-1}{\sigma-1}} (M)^{\frac{-(\theta\sigma+1-\sigma)}{\theta(\sigma-1)}} = \Xi_{Z^X} (M)^{\frac{-(\theta\sigma+1-\sigma)}{\theta(\sigma-1)}} \\
Z^I &= \left(\frac{f^{I*}-f^X}{\left[(\tau_R^*)^\sigma \left(\frac{1}{1-s_V^*} \right)^{\sigma-1} A^{H*} - (\tau_R)^\sigma \left(\frac{1}{\tau} \right)^{\sigma-1} A^{H*} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} \\
&= \left(\frac{f^{I*}-f^X}{\left[(\tau_R^*)^\sigma \left(\frac{1}{1-s_V^*} \right)^{\sigma-1} \Xi_{A^{H*}} - (\tau_R)^\sigma \left(\frac{1}{\tau} \right)^{\sigma-1} \Xi_{A^{H*}} \right] \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} (M)^{\frac{-(\theta\sigma+1-\sigma)}{\theta(\sigma-1)}} = \Xi_{Z^I} (M)^{\frac{-(\theta\sigma+1-\sigma)}{\theta(\sigma-1)}}
\end{aligned}$$

where $\theta\sigma + 1 - \sigma < 0$ and $\sigma > 1$.

Therefore, observe that

$$\begin{aligned}
J(Z^D)A^H &= \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} (Z^D)^{-\eta+\sigma-1} \Xi_{A^H}(M)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
&= \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} (\Xi_{Z^D})^{-\eta+\sigma-1} (M)^{\frac{(\theta\sigma+1-\sigma)(-\eta+\sigma-1)}{\theta(1-\sigma)}} \Xi_{A^H}(M)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
&= \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} (\Xi_{Z^D})^{-\eta+\sigma-1} \Xi_{A^H}(M)^{\frac{(\theta\sigma+1-\sigma)\eta}{\theta(\sigma-1)}} \\
J(Z^X, Z^I)A^{H*} &= \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} \left[(Z^X)^{-(\eta-\sigma+1)} - (Z^I)^{-(\eta-\sigma+1)} \right] \Xi_{A^{H*}}(M)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
&= \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} \left[\left(\Xi_{Z^X}(M)^{\frac{(\theta\sigma+1-\sigma)}{\theta(1-\sigma)}} \right)^{-(\eta-\sigma+1)} - \left(\Xi_{Z^I}(M)^{\frac{(\theta\sigma+1-\sigma)}{\theta(1-\sigma)}} \right)^{-(\eta-\sigma+1)} \right] \Xi_{A^{H*}}(M)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
&= \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} \left[(\Xi_{Z^X})^{-(\eta-\sigma+1)} - (\Xi_{Z^I})^{-(\eta-\sigma+1)} \right] \Xi_{A^{H*}}(M)^{\frac{(\theta\sigma+1-\sigma)\eta}{\theta(\sigma-1)}} \\
J(Z^I)A^{H*} &= \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} (Z^I)^{-\eta+\sigma-1} \Xi_{A^{H*}}(M)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
&= \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} (\Xi_{Z^I})^{-\eta+\sigma-1} \Xi_{A^{H*}}(M)^{\frac{(\theta\sigma+1-\sigma)\eta}{\theta(\sigma-1)}} \\
(1 - G(Z^D)) &= (z_{min})^\eta (Z^D)^{-\eta} &= (z_{min})^\eta (\Xi_{Z^D})^{-\eta} (M)^{\frac{(\theta\sigma+1-\sigma)\eta}{\theta(\sigma-1)}} \\
(1 - G(Z^I)) &= (z_{min})^\eta (Z^I)^{-\eta} &= (z_{min})^\eta (\Xi_{Z^I})^{-\eta} (M)^{\frac{(\theta\sigma+1-\sigma)\eta}{\theta(\sigma-1)}} \\
(G(Z^I) - G(Z^X)) &= (z_{min})^\eta \left[(Z^X)^{-\eta} - (Z^I)^{-\eta} \right] &= (z_{min})^\eta \left[(\Xi_{Z^X})^{-\eta} - (\Xi_{Z^I})^{-\eta} \right] (M)^{\frac{(\theta\sigma+1-\sigma)\eta}{\theta(\sigma-1)}}
\end{aligned}$$

Hence, the free entry condition for Home firms can be rewritten as

$$\delta F^D = (M)^{\frac{(\theta\sigma+1-\sigma)\eta}{\theta(\sigma-1)}} \left(\begin{array}{l} + (\tau_R)^\sigma \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} (\Xi_{Z^D})^{-\eta+\sigma-1} \Xi_{A^H} \\ + (\tau_R)^\sigma \left(\frac{1}{\sigma} \right) \left(\frac{\rho}{\tau} \right)^{\sigma-1} \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} \left[(\Xi_{Z^X})^{-(\eta-\sigma+1)} - (\Xi_{Z^I})^{-(\eta-\sigma+1)} \right] \Xi_{A^{H*}} \\ + (\tau_R^*)^\sigma \left(\frac{1}{\sigma} \right) \left(\frac{\rho}{(1-s_V^*)} \right)^{\sigma-1} \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} (\Xi_{Z^I})^{-\eta+\sigma-1} \Xi_{A^{H*}} \\ - f^D (z_{min})^\eta (\Xi_{Z^D})^{-\eta} \\ - f^X (z_{min})^\eta \left[(\Xi_{Z^X})^{-\eta} - (\Xi_{Z^I})^{-\eta} \right] \\ - f^{I*} (z_{min})^\eta (\Xi_{Z^I})^{-\eta} \end{array} \right)$$

For the counterpart for Foreign, we have

$$\begin{aligned}
J^*(Z^{D*}) A^{F*} &= \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^* - \sigma + 1} (Z^{D*})^{-\eta^* + \sigma - 1} \Xi_{A^{F*}}(M^*)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
&= \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^* - \sigma + 1} (\Xi_{Z^{D*}})^{-\eta^* + \sigma - 1} \Xi_{A^{F*}}(M^*)^{\frac{(\theta\sigma+1-\sigma)\eta^*}{\theta(\sigma-1)}} \\
J^*(Z^{X*}, Z^{I*}) A^F &= \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^* - \sigma + 1} \left[(Z^{X*})^{-(\eta^* - \sigma + 1)} - (Z^{I*})^{-(\eta^* - \sigma + 1)} \right] \Xi_{A^F}(M^*)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
&= \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^* - \sigma + 1} \left[(\Xi_{Z^{X*}})^{-(\eta^* - \sigma + 1)} - (\Xi_{Z^{I*}})^{-(\eta^* - \sigma + 1)} \right] \Xi_{A^F}(M^*)^{\frac{(\theta\sigma+1-\sigma)\eta^*}{\theta(\sigma-1)}} \\
J^*(Z^{I*}) A^F &= \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^* - \sigma + 1} (Z^{I*})^{-\eta^* + \sigma - 1} \Xi_{A^F}(M^*)^{\frac{(\theta\sigma+1-\sigma)}{\theta}} \\
&= \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^* - \sigma + 1} (\Xi_{Z^{I*}})^{-\eta^* + \sigma - 1} \Xi_{A^F}(M^*)^{\frac{(\theta\sigma+1-\sigma)\eta^*}{\theta(\sigma-1)}} \\
(1 - G^*(Z^{D*})) &= (z_{min}^*)^{\eta^*} (Z^{D*})^{-\eta^*} = (z_{min}^*)^{\eta^*} (\Xi_{Z^{D*}})^{-\eta^*} (M^*)^{\frac{(\theta\sigma+1-\sigma)\eta^*}{\theta(\sigma-1)}} \\
(1 - G^*(Z^{I*})) &= (z_{min}^*)^{\eta^*} (Z^{I*})^{-\eta^*} = (z_{min}^*)^{\eta^*} (\Xi_{Z^{I*}})^{-\eta^*} (M^*)^{\frac{(\theta\sigma+1-\sigma)\eta^*}{\theta(\sigma-1)}} \\
(G^*(Z^{I*}) - G^*(Z^{X*})) &= (z_{min}^*)^{\eta^*} \left[(Z^{X*})^{-\eta^*} - (Z^{I*})^{-\eta^*} \right] = (z_{min}^*)^{\eta^*} \left[(\Xi_{Z^{X*}})^{-\eta^*} - (\Xi_{Z^{I*}})^{-\eta^*} \right] (M^*)^{\frac{(\theta\sigma+1-\sigma)\eta^*}{\theta(\sigma-1)}}
\end{aligned}$$

Hence, the free entry condition for Foreign firms can be rewritten as

$$\delta F^{D*} = (M^*)^{\frac{(\theta\sigma+1-\sigma)\eta^*}{\theta(\sigma-1)}} \left(\begin{array}{l} + (\tau_R^*)^\sigma \left(\frac{1}{\sigma} \right) (\rho)^{\sigma-1} \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^* - \sigma + 1} (\Xi_{Z^{D*}})^{-\eta^* + \sigma - 1} \Xi_{A^{F*}} \\ + (\tau_R^*)^\sigma \left(\frac{1}{\sigma} \right) \left(\frac{\rho}{\tau^*} \right)^{\sigma-1} \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^* - \sigma + 1} \left[(\Xi_{Z^{X*}})^{-(\eta^* - \sigma + 1)} - (\Xi_{Z^{I*}})^{-(\eta^* - \sigma + 1)} \right] \Xi_{A^F} \\ + (\tau_R)^{\sigma} \left(\frac{1}{\sigma} \right) \left(\frac{\rho}{(1-s_V)} \right)^{\sigma-1} \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^* - \sigma + 1} (\Xi_{Z^{I*}})^{-\eta^* + \sigma - 1} \Xi_{A^F} \\ - f^{D*} (z_{min}^*)^{\eta^*} (\Xi_{Z^{D*}})^{-\eta^*} \\ - f^{X*} (z_{min}^*)^{\eta^*} \left[(\Xi_{Z^{X*}})^{-\eta^*} - (\Xi_{Z^{I*}})^{-\eta^*} \right] \\ - f^I (z_{min}^*)^{\eta^*} (\Xi_{Z^{I*}})^{-\eta^*} \end{array} \right)$$

In sum, the two free entry conditions pin down M and M^* :

$$(M)^{\frac{(\sigma-1-\theta\sigma)\eta}{\theta(\sigma-1)}} = \frac{1}{\delta F^D} \left(\begin{array}{l} + (\tau_R)^\sigma \left(\frac{1}{\sigma}\right) (\rho)^{\sigma-1} \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} (\Xi_{Z^D})^{-\eta+\sigma-1} \Xi_{A^H} \\ + (\tau_R)^\sigma \left(\frac{1}{\sigma}\right) \left(\frac{\rho}{\tau}\right)^{\sigma-1} \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} \left[(\Xi_{Z^X})^{-(\eta-\sigma+1)} - (\Xi_{Z^I})^{-(\eta-\sigma+1)} \right] \Xi_{A^{H*}} \\ + (\tau_R^*)^\sigma \left(\frac{1}{\sigma}\right) \left(\frac{\rho}{(1-s_V^*)}\right)^{\sigma-1} \frac{\eta(z_{min})^\eta}{\eta-\sigma+1} (\Xi_{Z^I})^{-\eta+\sigma-1} \Xi_{A^{H*}} \\ - f^D(z_{min})^\eta (\Xi_{Z^D})^{-\eta} \\ - f^X(z_{min})^\eta \left[(\Xi_{Z^X})^{-\eta} - (\Xi_{Z^I})^{-\eta} \right] \\ - f^{I*}(z_{min})^\eta (\Xi_{Z^I})^{-\eta} \end{array} \right)$$

$$(M^*)^{\frac{(\sigma-1-\theta\sigma)\eta^*}{\theta(\sigma-1)}} = \frac{1}{\delta F^{D*}} \left(\begin{array}{l} + (\tau_R^*)^\sigma \left(\frac{1}{\sigma}\right) (\rho)^{\sigma-1} \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^*-\sigma+1} (\Xi_{Z^{D*}})^{-\eta^*+\sigma-1} \Xi_{A^{F*}} \\ + (\tau_R^*)^\sigma \left(\frac{1}{\sigma}\right) \left(\frac{\rho}{\tau^*}\right)^{\sigma-1} \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^*-\sigma+1} \left[(\Xi_{Z^{X*}})^{-(\eta^*-\sigma+1)} - (\Xi_{Z^{I*}})^{-(\eta^*-\sigma+1)} \right] \Xi_{A^F} \\ + (\tau_R)^\sigma \left(\frac{1}{\sigma}\right) \left(\frac{\rho}{(1-s_V)}\right)^{\sigma-1} \frac{\eta^*(z_{min}^*)^{\eta^*}}{\eta^*-\sigma+1} (\Xi_{Z^{I*}})^{-\eta^*+\sigma-1} \Xi_{A^F} \\ - f^{D*}(z_{min}^*)^{\eta^*} (\Xi_{Z^{D*}})^{-\eta^*} \\ - f^{X*}(z_{min}^*)^{\eta^*} \left[(\Xi_{Z^{X*}})^{-\eta^*} - (\Xi_{Z^{I*}})^{-\eta^*} \right] \\ - f^I(z_{min}^*)^{\eta^*} (\Xi_{Z^{I*}})^{-\eta^*} \end{array} \right)$$